**True/False:** Indicate by "+' or "o" whether each statement is "true" or "false", respectively:

1. If a primal LP constraint is slack at the optimal solution, then the optimal value of the dual variable for that same constraint must be positive. ___ o ___
2. In reference to LP, the terms "dual variable" and "simplex multiplier" are synonymous. ___ + ___
3. If you make a mistake in choosing the pivot row in the simplex method, the next basic solution will have one or more negative basic variables. ___ + ___
4. An assignment problem is a special type of linear programming problem. ___ + ___
5. Every basic feasible solution of an assignment problem is degenerate. ___ + ___
6. A degenerate solution of an LP is one which has more nonbasic than basic variables. ___ o ___
7. If a basic feasible solution of a transportation problem is not degenerate, the next iteration must result in an improvement of the objective. ___ + ___
8. The two-phase simplex method solves for the primal variables in phase one, and then solves for the dual variables in phase two. ___ o ___
9. In a "balanced" transportation problem, the number of sources equals the number of destinations. ___ + ___
10. The minimum expected regret is never less than the expected value of perfect information. ___ o ___
11. A dual variable for an equality constraint is always zero. ___ o ___
12. In a maximization LP problem, if the right-hand-side of a "greater-than-or-equal" constraint is increased, the objective function will either remain the same or increase. ___ o ___

Match the four hypothetical graphs of optimal value vs right-hand-side to the appropriate combination of min/max and inequality type, by writing the correct letter (A,B,C,D) in the blanks.

---

13. Min $cx$ st $Ax \geq b$ ___ B ___
14. Min $cx$ st $Ax \leq b$ ___ C ___
15. Max $cx$ st $Ax \geq b$ ___ D ___
16. Max $cx$ st $Ax \leq b$ ___ A ___

---

17. If, in the optimal **primal** solution of an LP problem (min $cx$ st $Ax \geq b$, $x \geq 0$), there is zero slack in constraint #1, then in the optimal dual solution,
   a. dual variable #1 must be zero
   b. dual variable #1 must be positive
   c. slack variable for dual constraint #1 must be zero
   d. dual constraint #1 must be slack
   e. **NOTA** ___ e ___
18. If, in the optimal solution of the **dual** of an LP problem (min $cx$ subject to: $Ax \geq b$, $x \geq 0$), dual variable #2 is positive, then in the optimal **primal** solution,
   a. variable #2 must be zero
   b. variable #2 must be positive
   c. slack variable for constraint #2 must be zero
   d. constraint #2 must be slack
   e. **NOTA** ___ e ___
19. If there is a tie in the "minimum-ratio test" of the simplex method, the solution in the next tableau
   a. will be nonbasic
   b. will be nonfeasible
   c. will have a worse objective value
   d. will be degenerate
   e. **NOTA** ___ d ___
20. Bayes’ Rule is used to compute
   a. the joint probability of a “state of nature” and the outcome of an experiment
   b. the conditional probability of a “state of nature” given the outcome of an experiment
   c. the conditional probability of the outcome of an experiment given a “state of nature”
   d. **NOTA** ___ b ___
The problems below refer to the following LP:

\[
\begin{align*}
\text{Minimize} & \quad 8X_1 + 4X_2 \\
\text{subject to} & \quad 3X_1 + 4X_2 \leq 6 \\
& \quad X_1 + 4X_2 \geq 4 \\
& \quad 5X_1 + 2X_2 \leq 10 \\
& \quad X_1 \geq 0, X_2 \geq 0
\end{align*}
\]

\[
\begin{align*}
\text{Minimize} & \quad 8X_1 + 4X_2 \\
\text{subject to} & \quad 3X_1 + 4X_2 + X_3 = 6 \\
& \quad X_1 + 4X_2 - X_4 = 4 \\
& \quad 5X_1 + 2X_2 + X_5 = 10 \\
& \quad X_j \geq 0, j=1,2,3,4,5
\end{align*}
\]

21. The feasible region includes points
   a. B, F, & G
   b. A, B, C, & F
   c. C, E, & F
   d. E, F, & G
   e. B, D, & G
   f. NOTA

22. At point F, the basic variables include the variables
   a. X_2 & X_3
   b. X_3 & X_4
   c. X_4 & X_5
   d. X_1 & X_4
   e. X_2 & X_5
   f. NOTA

23. The dual of the original LP (before introducing slack & surplus variables) has the following constraints (not including nonnegativity or nonpositivity constraints):
   a. 2 constraints of type (≥)
   b. one each of type ≤ & ≥
   c. 2 constraints of type (≤)
   d. one each of type ≥ & =
   e. NOTA
   f. NOTA

24. The dual of the LP has the following types of variables:
   a. two non-negative variables and one non-positive variable
   b. one non-negative and two non-positive variables
   c. two non-negative variables and one unrestricted in sign
   d. three non-negative variables
   e. three non-positive variables
   f. NOTA

25. If point F is optimal, then which dual variables must be zero, according to the Complementary Slackness Theorem?
   a. Y_1 and Y_2
   b. Y_1 and Y_3
   c. Y_2 and Y_3
   d. Y_1 only
   e. Y_2 only
   f. Y_3 only

26. When using the Hungarian method to solve assignment problems, if the number of lines drawn to cover the zeroes in the reduced matrix is equal to the number of rows,
   a. this indicates that no solution exists.
   b. a dummy row or column must be introduced
   c. a mistake occurred; one should review previous steps.
   d. this means an optimal solution has been reached.

The following is a transportation tableau, with an initial set of shipments indicated:

\[
\begin{array}{cccc|c}
1 & 2 & 3 & 4 & \text{supply} \\
1 & \begin{array}{cccc}
4 & 5 & 3 & 1 \\
6 & 1 & 2 & 3 \\
3 & 3 & 2 & 1 \\
\end{array} & 7 \\
2 & \begin{array}{cccc}
\end{array} & 5 \\
3 & \begin{array}{cccc}
\end{array} & 3 \\
\end{array}
\]

27. Is the solution above a basic feasible solution? yes
Complete the computation of a set of dual variables for the above transportation tableau, starting by assigning the dual variable for source #1 equal to zero:

28. Dual variables for the supply constraints: \( U_1 = 0, U_2 = +1, U_3 = +2 \)

29. Dual variables for demand constraints: \( V_1 = +5, V_2 = +1, V_3 = +2 \)

30. Compute the reduced costs for \( X_{21} \):
   \( \Delta c_{21} = 6 - (1+5) = 6 - 6 = 0 \)

31. \( \Delta c_{31} = 3 - (2+5) = 3 - 7 = -4 \)

32. Which of these two variables should enter the basis?
   a. \( X_{21} \)
   b. \( X_{31} \)
   c. both
   d. neither

33. Which basic variable should leave the basis?
   a. \( X_{11} \)
   b. \( X_{12} \)
   c. \( X_{22} \)
   d. \( X_{23} \)
   e. \( X_{33} \)
   f. NOTA

34. If \( x_{ij} > 0 \) in the transportation problem, then dual variables \( U \) and \( V \) must satisfy
   a. \( C_{ij} > U_i + V_j \)
   b. \( C_{ij} = U_i + V_j \)
   c. \( C_{ij} < U_i + V_j \)
   d. \( C_{ij} + U_i + V_j = 0 \)
   e. \( C_{ij} = U_i - V_j \)
   f. NOTA

Sensitivity Analysis in LP. Consult the LINDO output below to answer the questions:

35. The number of optimal solutions of this LP is
   a. exactly one
   b. exactly two
   c. infinite
   d. none

36. The resulting increase in cost is (in dollars)
   a. between 0 and 100 \((10\times0.333)\)
   b. between 100 and 200
   c. between 200 and 300
   d. between 400 and 500
   e. between 500 and 1000
   f. greater than 500

37. Taking into account this failure is equivalent to
   a. increasing the variable \( WN1 \) by 10
   b. decreasing the variable \( WN1 \) by 10
   c. increasing the variable \( SLK2 \) by 10
   d. decreasing the variable \( SLK2 \) by 10
   e. NOTA

38. If a pivot were performed to enter \( SLK2 \) into the basis, the variable leaving the basis would be
   a. \( SLK3 \) by min ratio test
   b. \( SLK7 \)
   c. \( WN1 \)
   d. \( RG1 \)
   e. \( RN1 \)
   f. \( NOTA \)

39. If a pivot were performed to enter \( SLK2 \) into the basis, the resulting value of \( SLK2 \) would be
   a. between 0 and 100
   b. between 200 and 300 \((278.667/1.067)\)
   c. between 400 and 500
   d. between 500 and 1000
   e. greater than 500

40-41. The effect on variable \( WN1 \) of an increase of 10 in the variable \( IG2 \) would be to
   a. increase
   b. decrease
   c. neither the basis nor the values of the variables will change
   d. both the basis and the values of the variables will change since the basis B changes
   e. the basis will not change, but the values of the variables will change
   f. insufficient information is available to answer this question
   g. NOTA

42. Suppose that the monthly storage cost of a nylon tire at the end of June were to increase from 10 cents to 15 cents. Then…
   a. neither the basis nor the values of the variables will change
   b. both the basis and the values of the variables will change
   c. the basis will not change, but the values of the variables will change since \( x_{ab} = (A_B)^{-1} \) & \( b \) changes
   d. insufficient information is available to answer this question
   e. NOTA

43. Suppose that the demand for fiberglass tires in June were to double. Then…
   a. neither the basis nor the values of the variables will change
   b. both the basis and the values of the variables will change
   c. the basis will not change, but the values of the variables will change since \( x_{ab} = (A_B)^{-1} \) & \( b \) changes
   d. insufficient information is available to answer this question
   e. NOTA

An automobile tire manufacturer has the ability to produce both nylon and fiberglass tires. During the next 3 months, they have agreed to deliver the following quantities:

<table>
<thead>
<tr>
<th>Date</th>
<th>Nylon</th>
<th>Fiberglass</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 30</td>
<td>4000</td>
<td>1000</td>
</tr>
<tr>
<td>July 31</td>
<td>8000</td>
<td>5000</td>
</tr>
<tr>
<td>August 31</td>
<td>3000</td>
<td>5000</td>
</tr>
</tbody>
</table>
The company has two presses, referred to as the Wheeling and Regal machines, and appropriate molds which can be used to produce these tires, with the following production hours available in the upcoming months:

<table>
<thead>
<tr>
<th>Month</th>
<th>Wheeling</th>
<th>Regal</th>
</tr>
</thead>
<tbody>
<tr>
<td>June</td>
<td>700</td>
<td>1500</td>
</tr>
<tr>
<td>July</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>August</td>
<td>1000</td>
<td>300</td>
</tr>
</tbody>
</table>

The production rates for each machine-and-tire combination, in terms of hours per tire, are as follows:

<table>
<thead>
<tr>
<th>Tire</th>
<th>Wheeling</th>
<th>Regal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nylon</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>Fiberglass</td>
<td>0.12</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Production costs per tire, including labor and materials, are

<table>
<thead>
<tr>
<th>Tire</th>
<th>Wheeling</th>
<th>Regal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nylon</td>
<td>0.75</td>
<td>0.80</td>
</tr>
<tr>
<td>Fiberglass</td>
<td>0.60</td>
<td>0.70</td>
</tr>
</tbody>
</table>

The inventory carrying charge is $0.10 per tire per month.

Decision variables:
- WNt = number of nylon tires to be produced on the Wheeling machine during month t, t=1,2,3
- RNt = number of nylon tires to be produced on the Regal machine during month t, t=1,2,3
- WGt = number of fiberglass tires to be produced on the Wheeling machine during month t, t=1,2,3
- RGt = number of fiberglass tires to be produced on the Regal machine during month t, t=1,2,3
- INt = number of nylon tires put into inventory at the end of month t, t=1,2
- IGt = number of fiberglass tires put into inventory at the end of month t, t=1,2

MIN 0.75 WN1 + 0.8 RN1 + 0.6 WG1 + 0.7 RG1 + 0.1 IN1 + 0.1 IG1 + 0.75 WN2 + 0.8 RN2 + 0.6 WG2 + 0.7 RG2 + 0.1 IN2 + 0.1 IG2 + 0.75 WN3 + 0.8 RN3 + 0.6 WG3 + 0.7 RG3

SUBJECT TO

2) 0.15 WN1 + 0.12 WG1 <= 700
3) 0.16 RN1 + 0.14 RG1 <= 1500
4) 0.15 WN2 + 0.12 WN2 <= 300
5) 0.16 RN2 + 0.14 RG2 <= 400
6) 0.15 WN3 + 0.12 WG3 <= 1000
7) 0.16 RN3 + 0.14 RG3 <= 300
8) WN1 + RN1 - IN1 = 4000
9) WG1 + RG1 - IG1 = 1000
10) IN1 + WN2 + RN2 - IN2 = 8000
11) IG1 + WG2 + RG2 - IG2 = 5000
12) IN2 + WN3 + RN3 = 3000
13) IG2 + WG3 + RG3 = 5000

END

OBJECTIVE FUNCTION VALUE
1) 19173.33

VARIABLE VALUE REDUCED COST
WN1 1866.666626 0.000000
RN1 7633.333496 0.000000
WG1 3500.000000 0.000000
RG1 0.000000 0.060000
IN1 5500.000000 0.000000
IG1 2500.000000 0.000000
WN2 0.000000 0.025000
RN2 2500.000000 0.000000
WG2 2500.000000 0.000000
RG2 0.000000 0.047500
IN2 0.000000 0.200000
IG2 0.000000 0.200000
WN3 2666.666748 0.000000
RN3 333.333344 0.000000
WG3 5000.000000 0.000000
RG3 0.000000 0.060000

ROW SLACK OR SURPLUS DUAL PRICES
2) 0.000000 0.333333
3) 278.666656 0.000000
4) 0.000000 1.166667
5)  0.000000  0.625000
6)  0.000000  0.333333
7)  246.666672  0.000000
8)  0.000000  -0.800000
9)  0.000000  -0.640000
10) 0.000000  -0.900000
11) 0.000000  -0.740000
12) 0.000000  -0.800000
13) 0.000000  -0.640000

RANGES IN WHICH THE BASIS IS UNCHANGED:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CURRENT COEF</th>
<th>ALLOWABLE INCREASE</th>
<th>ALLOWABLE DECREASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>WN1</td>
<td>0.750000</td>
<td>0.075000</td>
<td>0.059375</td>
</tr>
<tr>
<td>RN1</td>
<td>0.800000</td>
<td>0.075000</td>
<td>0.050000</td>
</tr>
<tr>
<td>WG1</td>
<td>0.600000</td>
<td>0.047500</td>
<td>0.020000</td>
</tr>
<tr>
<td>RG1</td>
<td>0.700000</td>
<td>INFINITY</td>
<td>0.060000</td>
</tr>
<tr>
<td>IN1</td>
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<td>0.025000</td>
<td>0.059375</td>
</tr>
<tr>
<td>IG1</td>
<td>0.100000</td>
<td>0.047500</td>
<td>0.020000</td>
</tr>
<tr>
<td>WN2</td>
<td>0.750000</td>
<td>INFINITY</td>
<td>0.050000</td>
</tr>
<tr>
<td>RN2</td>
<td>0.800000</td>
<td>0.054286</td>
<td>INFINITY</td>
</tr>
<tr>
<td>WG2</td>
<td>0.600000</td>
<td>0.020000</td>
<td>INFINITY</td>
</tr>
<tr>
<td>RG2</td>
<td>0.700000</td>
<td>INFINITY</td>
<td>0.047500</td>
</tr>
<tr>
<td>IN2</td>
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<td>INFINITY</td>
<td>0.200000</td>
</tr>
<tr>
<td>IG2</td>
<td>0.100000</td>
<td>INFINITY</td>
<td>0.200000</td>
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<tr>
<td>WN3</td>
<td>0.750000</td>
<td>0.050000</td>
<td>0.075000</td>
</tr>
<tr>
<td>RN3</td>
<td>0.800000</td>
<td>0.075000</td>
<td>0.050000</td>
</tr>
<tr>
<td>WG3</td>
<td>0.600000</td>
<td>0.060000</td>
<td>INFINITY</td>
</tr>
<tr>
<td>RG3</td>
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<td>0.060000</td>
</tr>
</tbody>
</table>

RIGHTHAND SIDE RANGES

<table>
<thead>
<tr>
<th>ROW</th>
<th>CURRENT</th>
<th>ALLOWABLE INCREASE</th>
<th>ALLOWABLE DECREASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>700.000000</td>
<td>1145.000000</td>
<td>261.249969</td>
</tr>
<tr>
<td>3</td>
<td>1500.000000</td>
<td>INFINITY</td>
<td>278.666656</td>
</tr>
<tr>
<td>4</td>
<td>300.000000</td>
<td>300.000000</td>
<td>261.249969</td>
</tr>
<tr>
<td>5</td>
<td>400.000000</td>
<td>880.000000</td>
<td>278.666656</td>
</tr>
<tr>
<td>6</td>
<td>1000.000000</td>
<td>50.000000</td>
<td>231.250000</td>
</tr>
<tr>
<td>7</td>
<td>300.000000</td>
<td>INFINITY</td>
<td>246.666672</td>
</tr>
<tr>
<td>8</td>
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</tr>
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<td>3500.000000</td>
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<td>5500.000000</td>
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<td>5000.000000</td>
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<td>12</td>
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<td>13</td>
<td>5000.000000</td>
<td>1927.083252</td>
<td>416.666687</td>
</tr>
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</table>

THE TABLEAU

<table>
<thead>
<tr>
<th>ROW (BASIS)</th>
<th>WN1</th>
<th>RN1</th>
<th>WG1</th>
<th>RG1</th>
<th>IN1</th>
<th>IG1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.060</td>
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<td>WN1</td>
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<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>SLK 3</td>
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<td>0.000</td>
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<tr>
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<td>WG2</td>
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</tr>
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<td>RN2</td>
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<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
</tr>
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<td>WN3</td>
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<td>SLK 7</td>
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<td>RN1</td>
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<td>0.800</td>
<td>0.000</td>
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<td>1.000</td>
<td>0.000</td>
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<td>IN1</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
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<td>11</td>
<td>IG1</td>
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<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
</tr>
<tr>
<td>12</td>
<td>RN3</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>13</td>
<td>WG3</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
\textbf{Decision Analysis}

The government is attempting to determine whether immigrants should be tested for a certain contagious disease. Let's assume that the decision will be made strictly on a financial basis.

Assume that each immigrant who is allowed into the country and has the disease (event $D$) costs the U.S. $200,000, and each immigrant who enters and does not have the disease (event $N$) will contribute $10,000 to the national economy. Assume that 5\% of all potential immigrants have this disease.

The government's goal is to maximize (per potential immigrant) expected benefits minus expected costs.

A tree representing the government's decision appears below.

\begin{center}
\begin{tikzpicture}
\node (root) {D \nodepart{second} 5\%};
\node (admit) [below right of=root] {Admit \nodepart{second} 95\%};
\node (deny) [below left of=root] {Deny \nodepart{second} 0};
\draw (root) edge node [above] {-200,000} (admit);
\draw (root) edge node [above] {+10,000} (deny);
\end{tikzpicture}
\end{center}
44. The optimal decision is
   a. admit all immigrants
   b. deny admission to all immigrants
   c. the government is indifferent
   d. NOTA

Suppose that there is a medical test which may be administered before determining whether a potential immigrant should be admitted. The cost of this test is $500 per person. The test result is either positive (event +) indicating presence of the disease or negative (event −) indicating absence of the disease, but the test is somewhat unreliable: 10% of all people with the disease test negative, and 5% of the persons without the disease test positive.

45-46. Complete the following blanks

\[
\begin{align*}
P\{D\} & \quad (prior\ probability) \quad 0.05 \\
P\{+\mid D\} & \quad 0.90 \\
P\{N\} & \quad (prior\ probability) \quad 0.95 \\
P\{-\mid N\} & \quad 0.10 \\
P\{+\} & \quad 0.0925 \\
P\{D\mid +\} & \quad 0.4865 \\
P\{-\} & \quad 0.5135 \\
P\{+\mid -\} & \quad 0.0055 \\
P\{N\mid +\} & \quad 0.9075 \\
P\{-\mid -\} & \quad 0.9945 \\
\end{align*}
\]

The decision tree below includes the decision as to whether or not administer the medical test. Note that the $500 cost of the test has not been incorporated in the “payoffs” at the far right.

47-49. "Fold back" nodes 2 through 8, and write the missing values of the nodes below:

<table>
<thead>
<tr>
<th>Node</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>+8845</td>
</tr>
<tr>
<td>7</td>
<td>−92165</td>
</tr>
<tr>
<td>6</td>
<td>−500</td>
</tr>
<tr>
<td>5</td>
<td>+8026</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>7526</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

50. The expected value of the test result (in $) is (Choose nearest value):
   a. ≤ 0
   b. 500
   c. 1000
   d. 5,000
   e. 7,500 (\$8026)
   f. 10,000
   g. 20,000
   h. ≥ 20,000