Do Part One, one from Part Two, and three from Part Three (20 pts. each)

Score: Part One: I. _______ Multiple Choice
Part Two: II. _______ LINDO Analysis
III. _______ Simplex Algorithm
Part Three: IV. _______ Decision trees
V. _______ Markov chain model: Student Progress
VI. _______ Markov chain model: Inventory System
VII. _______ Continuous-time Markov chain
VIII. _______ Dynamic programming
TOTAL: _______ (of 100 possible)

I. Multiple Choice

1. If you make a mistake in choosing the pivot row in the simplex method, the solution in the next tableau
   a. will be nonbasic
   b. will be nonfeasible
   c. will have a worse objective value
   d. will be degenerate

2. Consider a discrete-time Markov chain with transition probability matrix:
   \[
   \begin{bmatrix}
   0.2 & 0.8 \\
   0.9 & 0.1 
   \end{bmatrix}
   \]
   a. 0.1
   b. 0.64
   c. 0.8
   d. 0.24
   e. none of the above

3. If the Markov chain in the previous problem was initially in state #1, the probability that the system will still be in state 1 after two steps is
   a. 0.64
   b. 0.24
   c. 0
   d. 0.26
   e. 0.76
   f. none of the above

4. A transient state of a Markov chain is one in which the probability of returning to that state is zero.
   a. returning to that state is zero.
   b. returning to that state is less than 1.
   c. returning to that state is 1.
   d. none of the above.

5. A recurrent state of a Markov chain is one in which the transition rates are positive for every state.
   a. which is not transient.
   b. which is not absorbing.
   c. which communicates with an absorbing state.
   d. none of the above.

6. The steady-state probability vector \( \pi \) of a discrete Markov chain with transition probability matrix \( P \) satisfies the matrix equation
   a. \( P^T \pi = 0 \)
   b. \( P \pi = 0 \)
   c. \( P \pi = \pi \)
   d. \( \pi P = \pi \)
   e. none of the above

7. For a continuous-time Markov chain, let \( \Lambda \) be the matrix of transition rates. The sum of each...
   a. column is 1
   b. column is 0
   c. row is 1
   d. row is 0
   e. none of the above

8. For a discrete-time Markov chain, let \( P \) be the matrix of transition probabilities. The sum of each...
   a. column is 1
   b. column is 0
   c. row is 1
   d. row is 0
   e. none of the above
9. To compute the steady state distribution $\pi$ of a continuous-time Markov chain, one must solve (in addition to sum of components $0f \pi$ equal to 1) the matrix equation (where $\Lambda^t$ is the transpose of $\Lambda$):

a. $\pi \Lambda = 1$

b. $\Lambda^t \pi = 1$

c. $\Lambda^t \pi = \pi$

d. $\pi \Lambda = \pi$

e. $\pi \Lambda = 0$

f. none of the above

10. In PERT, the completion time for the project is assumed to

a. have the Beta distribution

b. have the Normal distribution

c. be constant

d. have the exponential distribution

e. none of the above

11. If you make a mistake in choosing the pivot column in the simplex method, the solution in the next tableau

a. will be nonbasic

b. will be nonfeasible

c. will have a worse objective value

d. will be degenerate

e. None of the above

12. If, in the optimal dual solution of an LP problem (min $cx$ st $Ax \leq b$, $x \geq 0$), dual variable #3 is zero, then in the optimal primal solution,

a. primal variable #3 must be zero

b. primal variable #3 must be positive

c. slack variable for primal constraint #3 must be zero

d. primal constraint #3 must be slack

e. None of the above

13. Consider the following queueing model:

The notation for this type of queue is:

a. M/M/1

b. M/M/2

c. M/M/4

d. M/M/2/4

e. M/M/1/4

f. none of the above

14. Consider the following queueing model:

The notation for this type of queue is:

a. M/M/1

b. M/M/2

c. M/M/1/4

d. M/M/4

e. M/M/1/4/4

f. none of the above

15. If there is a "tie" in the Minimum-Ratio Test in the simplex method, the solution in the next tableau

a. will be nonbasic

b. will be optimal

c. will have a non-improved objective value

d. will be degenerate

e. None of the above

16. In an M/M/1 queue, if $\lambda > \mu$,

a. $\pi_0 = 1$ in steady state

b. no steady state exists

c. $\pi_i > 0$ for all $i$

d. $\pi_0 = 0$ in steady state

e. None of the above

17. In an M/M/1/N queue, if $\lambda > \mu$,

a. $\pi_0 = 1$ in steady state

b. no steady state exists

c. $\pi_i > 0$ for all $i$

d. $\pi_0 = 0$ in steady state

e. None of the above

18. The probabilities in the transition probability matrix $P$ of a discrete-time Markov chain are actually
19. An absorbing state in a Markov chain is one in which the probability of
   a. moving into that state is zero
   b. moving out of that state is 1
   c. moving out of that state is zero
   d. None of the above

20. Consider the following queueing model:

   The notation for this type of queue is:
   a. M/M/1
   b. M/M/2
   c. M/M/1/4
   d. M/M/4
   e. M/M/1/4/4
   f. None of the above

II. LINDO analysis

Problem Statement: The Classic Stone Cutter Company produces four types of stone sculptures: figures, figurines, free forms, and statues. Each product requires the following hours of work for cutting and chiseling stone and polishing the final product:

<table>
<thead>
<tr>
<th>Operation</th>
<th>FIGURES</th>
<th>FIGURINES</th>
<th>FREE FORMS</th>
<th>STATUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting</td>
<td>30</td>
<td>5</td>
<td>45</td>
<td>60</td>
</tr>
<tr>
<td>Chiseling</td>
<td>20</td>
<td>8</td>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td>Polishing</td>
<td>0</td>
<td>20</td>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>Profit ($/ unit)</td>
<td>280</td>
<td>40</td>
<td>500</td>
<td>510</td>
</tr>
</tbody>
</table>

The company's current work force has production capacity sufficient to allocate 300 hours to cutting, 180 hours to chiseling, and 300 hours to polishing each week.

Define the variables:

- FIGURES = # of figures to be produced each week,
- FIGURINES = # figurines to be produced each week,
- FREE FORMS = # free forms to be produced each week,
- STATUES = # statues to be produced each week.

The LINDO output for solving this problem follows:

MAX 280 FIGURE + 40 FIGURINE + 500 FREEFORM + 510 STATUE
SUBJECT TO
2) 30 FIGURE + 5 FIGURINE + 45 FREEFORM + 60 STATUE <= 300
3) 20 FIGURE + 8 FIGURINE + 60 FREEFORM + 30 STATUE <= 300
4) 20 FIGURINE + 120 STATUE <= 300

END

OBJECTIVE FUNCTION VALUE
1) 2700.00000

VARIABLE VALUE REDUCED COST
FIGURE 6.000000 0.000000
FIGURINE 0.000000 30.000000
FREEFORM 0.000000 70.000000
STATUE 2.000000 0.000000

ROW SLACK OR SURPLUS DUAL PRICES
2) 0.000000 6.000000
3) 0.000000 5.000000
4) 60.000000 0.000000
RANGES IN WHICH THE BASIS IS UNCHANGED

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CURRENT</th>
<th>ALLOWABLE INCREASE</th>
<th>ALLOWABLE DECREASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIGURE</td>
<td>280.000000</td>
<td>60.000000</td>
<td>9.333333</td>
</tr>
<tr>
<td>FIGURINE</td>
<td>40.000000</td>
<td>30.000000</td>
<td>INFINITY</td>
</tr>
<tr>
<td>FREEFORM</td>
<td>500.000000</td>
<td>70.000000</td>
<td>INFINITY</td>
</tr>
<tr>
<td>STATUE</td>
<td>510.000000</td>
<td>23.333336</td>
<td>89.999992</td>
</tr>
</tbody>
</table>

RIGHTHAND SIDE RANGES

<table>
<thead>
<tr>
<th>ROW</th>
<th>CURRENT</th>
<th>ALLOWABLE INCREASE</th>
<th>ALLOWABLE DECREASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>300.000000</td>
<td>7.500000</td>
<td>30.000000</td>
</tr>
<tr>
<td>3</td>
<td>180.000000</td>
<td>20.000000</td>
<td>5.000000</td>
</tr>
<tr>
<td>4</td>
<td>300.000000</td>
<td>INFINITY</td>
<td>60.000000</td>
</tr>
</tbody>
</table>

THE TABLEAU

<table>
<thead>
<tr>
<th>ROW (BASIS)</th>
<th>FIGURE</th>
<th>FIGURINE</th>
<th>FREEFORM</th>
<th>STATUE</th>
<th>SLK 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ART</td>
<td>0.000</td>
<td>30.000</td>
<td>70.000</td>
<td>0.000</td>
<td>6.000</td>
</tr>
<tr>
<td>2 FIGURE</td>
<td>1.000</td>
<td>1.100</td>
<td>7.500</td>
<td>0.000</td>
<td>-0.100</td>
</tr>
<tr>
<td>3 STATUE</td>
<td>0.000</td>
<td>-0.467</td>
<td>-3.000</td>
<td>1.000</td>
<td>0.067</td>
</tr>
<tr>
<td>4 SLK 4</td>
<td>0.000</td>
<td>76.000</td>
<td>360.000</td>
<td>0.000</td>
<td>-8.000</td>
</tr>
</tbody>
</table>

Ignoring the restriction that the numbers of items produced per week must be integer, answer the following questions:

1. The optimal solution above is (check as many as apply):
   ___ basic  ___ degenerate  ___ unique

2. The number of basic variables in this optimal solution (not including z, the objective value) is
   a. one  b. two  c. three  d. four  e. none of the above

3. In every basic solution of this problem
   a. not every product will be included
   b. exactly two products will be included
   c. at least one slack variable will be >0
   d. none of the above

4. If it were required to make one figurine as a salesman's sample, the profit will decrease by (choose the nearest value)
   a. zero  b. $5  c. $6  d. $30  e. $40  f. $50  g. cannot be determined  h. none of the above

5. If it were required to make one figurine as a salesman's sample, the production of statues would
   a. be unchanged  b. increase by less than 1  c. decrease by less than 1  d. increase by more than 1  e. decrease by more than 1  f. cannot be determined  g. none of the above

6. If it were required to make one additional statue, the profit will decrease by (choose the nearest value)
   a. zero  b. $5  c. $6  d. $25  e. $100  f. $510  g. cannot be determined  h. none of the above
7. If the profit of free forms were to be $550 per unit,
a. the profit would be unchanged  b. the profit would increase by $100
c. the production of free forms should increase  d. none of the above

8. If five additional hours of cutting were available, the profit would increase by
(choose the nearest value)
a. less than $10  b. $10  c. $20
d. $30  e. more than $40  f. cannot be determined

9. If five additional hours of cutting were available, the number of figures would
a. be unchanged  b. increase by 0.5  c. decrease by 0.5
d. increase by 1  e. decrease by 1  f. none of the above

10. The value of the second variable in the optimal dual solution
a. is zero  b. is positive  c. is negative
d. cannot be determined  e. none of the above

11. The value of the optimal objective value of the dual problem is
a. zero  b. 2700  c. -2700
d. cannot be determined  e. none of the above

III. Simplex Algorithm for LP: At an intermediate step of the simplex algorithm, in which
the objective is to be minimized, the tableau is:

\[
\begin{array}{cccccc|c}
\text{-z} & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & \text{RHS} \\
1 & 0 & 0 & 0 & -2 & 0 & 6 & -10 \\
0 & 2 & 0 & 1 & -4 & 0 & 1 & 4 \\
0 & 0 & 1 & 0 & 1 & 0 & -1 & 3 \\
0 & -2 & 0 & 0 & 2 & 1 & 3 & 1 \\
\end{array}
\]

1. What are the basic variables for this tableau? (circle):
- Z X 1 X 2 X 3 X 4 X 5 X 6

2. The current value of the cost for this basic solution is (circle: +10 or -10)

3. The current value of X 1 for this basic solution is
a. 0  b. 1  c. 3  d. 4  e. 10

4. The current value of X 2 for this basic solution is
a. 0  b. 1  c. 3  d. 4  e. 10

5. The current value of X 3 for this basic solution is
a. 0  b. 1  c. 3  d. 4  e. 10

6. Increasing X 4 would (circle: increase / decrease) the objective function.

7. What is the substitution rate of X 4 for X 5?
 a. 0  b. 1  c. -1  d. 2  e. -2

8. If X 4 were increased by 2 units, the value of X 5 will
a. not change  b. increase by 2  c. decrease by 2
d. increase by 4  e. decrease by 4  f. none of the above

9. If the original constraints were all of type "≤" and X 4, X 5, and X 6 are slack variables,
the value of the first dual variable π 1 corresponding to the tableau given above is
a. 0  b. 1  c. -1  d. 2  e. -2
f. none of the above  g. cannot be determined

10. If the original constraints were of type "≥" and X 4, X 5, and X 6 are surplus variables,
the value of the second dual variable π 2 corresponding to the tableau above is
a. 0  b. 1  c. -1  d. 2  e. -2
f. none of the above  g. cannot be determined

11. Perform a pivot to improve the objective function, and complete the blank entries in the
tableau below:
12. The improvement in the objective resulting from the pivot in (11) is (choose the nearest value)
   a. zero  b. 1  c. 2  d. 3  e. 4  f. $\geq 5$

### PART THREE

#### IV. Decision Theory & Decision Trees:
General Custard Corporation is being sued by Sue Smith. Sue can settle out of court and win $60,000, or she can go to court. If she goes to court, there is a 25% chance that she will win the case (event $W$) and a 75% chance she will lose (event $L$). If she wins, she will receive $200,000, and if she loses, she will net $0$. A decision tree representing her situation appears below, where payoffs are in thousands of dollars:

1. What is the decision which maximizes the expected value? a. settle  b. go to court

For $20,000, Sue can hire a consultant who will predict the outcome of the trial, i.e., either he predicts a loss of the suit (event $PL$), or he predicts a win (event $PW$). The consultant is correct 80% of the time.

2. The probability that the consultant will predict a win, i.e. $P\{PW\}$ is (choose nearest value)
   a. $\leq 25\%$  b. 30%  c. 35%  d. 40%  e. 45%  f. $\geq 50\%$

3. According to Bayes' theorem, the probability that, if the consultant predicts a win, then in fact Sue will win, i.e. $P\{W \mid PW\}$ is (choose nearest value)
   a. $\leq 25\%$  b. 30%  c. 35%  d. 40%  e. 45%  f. $\geq 50\%$

The decision tree below includes Sue's decision as to whether or not to hire the consultant.
4. "Fold back" nodes 2 through 8, and write the value of each node below:

<table>
<thead>
<tr>
<th>Node</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>94.286</td>
</tr>
</tbody>
</table>

5. Should Sue hire the consultant? Circle: Yes  No

6. The expected value of the consultant's opinion is (in thousands of $) (Choose nearest value):
   a. ≤16  b. 17  c. 18  d. 19
   e. 20  f. 21  g. 22  h. ≥23

7. What would be the expected value of "perfect information" which is given to Sue, i.e., a prediction which is 100% accurate, so that the portion of the tree containing nodes 4, 5, 6, 7, etc., would appear as below? (Choose nearest value, in thousands of $)
   a. ≤15  b. 20  c. 25  d. 30
   e. 35  f. 40  g. 45  h. ≥50
V. Absorption Analysis of Markov Chain. The college admissions officer has modeled the path of a student through the Engineering College as a Markov chain:

Each student's state is observed at the beginning of each fall semester. For example, if a student is a junior at the beginning of the current fall semester, there is an 80% chance that he will be a senior at the beginning of the next fall semester, a 15% chance that he will still be a junior, and a 5% chance that he will have quit. (Assume that once a student quits, he never re-enrolls.)

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Freshman</td>
</tr>
<tr>
<td>2</td>
<td>Sophomore</td>
</tr>
<tr>
<td>3</td>
<td>Junior</td>
</tr>
<tr>
<td>4</td>
<td>Senior</td>
</tr>
<tr>
<td>5</td>
<td>Drop-out</td>
</tr>
<tr>
<td>6</td>
<td>Graduate</td>
</tr>
</tbody>
</table>

Consult the computer output below to answer the questions that follow.

1. Complete the diagram for this Markov chain, including the transition probabilities.

2. Which states are transient? Circle: 1 2 3 4 5 6
3. Which states are recurrent? Circle: 1 2 3 4 5 6
4. Which states are absorbing? Circle: 1 2 3 4 5 6
5. Does this system have a steady-state probability distribution?
   a. Yes b. No c. Maybe
6. If a student enters the college as a freshman, how many years can he or she expect to spend as a student in the college before either dropping out or graduating? (Choose nearest value)
   a. 3.0 years b. 3.5 years c. 4 years d. 4.5 years e. 5 years
7. The probability that, at the beginning of the fourth year in the college, he or she is classified as a senior is (Choose nearest value)
   a. 50% b. 60% c. 70% d. 80% e. 90%
8. The probability that he or she eventually will graduate is (Choose nearest value)
   a. 30% b. 50% c. 70% d. 90% e. >90%
9. If a student has survived to the point that he or she has been classified as a junior, the probability that he or she eventually graduates is (Choose nearest value)
   a. 30% b. 50% c. 70% d. 90% e. >90%

<table>
<thead>
<tr>
<th>Transition probabilities (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>from</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

\[ E = \text{Expected No. Visits to Transient States} \]

\[
\begin{array}{llll}
\text{from} & 1 & 2 & 3 & 4 \\
1 & 1.11111111 & 0.99765432 & 0.99765432 & 0.87791495 \\
2 & 0 & 1.11111111 & 1.11111111 & 0.99765432 \\
3 & 0 & 0 & 1.1702706 & 1.0457516 \\
4 & 0 & 0 & 0 & 1.11111111 \\
\end{array}
\]

\[
\begin{array}{llll}
\text{from} & 1 & 2 & 3 & 4 \\
1 & 0.01 & 0.18 & 0.88 & 0 & 0.15 & 0 \\
2 & 0 & 0.01 & 0.2125 & 0.88 & 0.0975 & 0 \\
3 & 0 & 0 & 0.0225 & 0.2 & 0.0975 & 0.88 \\
4 & 0 & 0 & 0 & 0.01 & 0.035 & 0.936 \\
5 & 0 & 0 & 0 & 0 & 1 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{array}
\]
VI. Markov Chain Model--(s,S) Inventory System: Consider the following inventory system for a certain spare part for a company's 2 production lines. A maximum of four parts may be kept on the shelf. At the end of each day, the parts in use are inspected and, if worn, replaced with one off the shelf. The probability distribution of the number replaced each day is:

\[
\begin{array}{c|ccc}
 n & 0 & 1 & 2 \\
 P\{n\} & 0.3 & 0.5 & 0.2 \\
\end{array}
\]

To avoid shortages, the current policy is to restock the shelf at the end of each day (after spare parts have been removed) so that the shelf is again filled to its limit (i.e., 4) if there are fewer than 2 parts on the shelf.

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions: \textbf{Note that in the computer output, state #1 is inventory level 0, state #2 is inventory level 1, etc.}

\[
P = \begin{bmatrix}
0 & 0 & 0.2 & 0.5 & 0.3 \\
0 & 0 & 0.2 & 0.5 & 0.3 \\
0.2 & 0.5 & 0 & 0 \\
0 & 0.2 & 0.5 & 0.3 \\
0 & 0 & 0.2 & 0.5 & 0.3 \\
\end{bmatrix}, P^2 = \begin{bmatrix}
0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\
0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\
0.06 & 0.15 & 0.23 & 0.35 & 0.24 \\
0.1 & 0.31 & 0.34 & 0.19 & 0.06 \\
0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\
\end{bmatrix}
\]

\[
P^3 = \begin{bmatrix}
0.074 & 0.245 & 0.327 & 0.295 & 0.099 \\
0.074 & 0.245 & 0.327 & 0.295 & 0.099 \\
0.046 & 0.185 & 0.328 & 0.315 & 0.125 \\
0.068 & 0.208 & 0.291 & 0.292 & 0.141 \\
0.074 & 0.245 & 0.327 & 0.295 & 0.099 \\
\end{bmatrix}, P^4 = \begin{bmatrix}
0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\
0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\
0.065 & 0.227 & 0.327 & 0.273 & 0.107 \\
0.068 & 0.208 & 0.316 & 0.296 & 0.125 \\
0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\
\end{bmatrix}
\]

\[
\sum_{n=1}^{\infty} P^n = \begin{bmatrix}
0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\
0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\
0.3716 & 1.062 & 1.1853 & 0.938 & 0.4431 \\
0.2262 & 0.9319 & 1.4477 & 1.0781 & 0.3281 \\
0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\
\end{bmatrix}
\]
1. The value $P_{5,3}$ is
   a. $P\{\text{demand}=0\}$
   b. $P\{\text{demand}=1\}$
   c. $P\{\text{demand}=2\}$
   d. $P\{\text{demand} \leq 1\}$
   e. $P\{\text{demand} \geq 1\}$
   f. none of the above

2. The value $P_{1,4}$ is
   a. $P\{\text{demand}=0\}$
   b. $P\{\text{demand}=1\}$
   c. $P\{\text{demand}=2\}$
   d. $P\{\text{demand} \leq 1\}$
   e. $P\{\text{demand} \geq 1\}$
   f. none of the above

3. The value $P_{3,1}$ is
   a. $P\{\text{demand}=0\}$
   b. $P\{\text{demand}=1\}$
   c. $P\{\text{demand}=2\}$
   d. $P\{\text{demand} \leq 1\}$
   e. $P\{\text{demand} \geq 1\}$
   f. none of the above

4. The numerical value $A$ in the matrix above is (select nearest value)
   a. 0
   b. 0.1
   c. 0.2
   d. 0.3
   e. 0.4
   f. 0.5

5. The numerical value $B$ in the mean-first-passage time matrix ($M$) above is (select nearest value)
   a. 1
   b. 2
   c. 4
   d. 6
   e. 8
   f. 10

6. If the shelf is full Monday morning, the expected number of days until a stockout occurs is (select nearest value):
   a. 2
   b. 5
   c. 10
   d. 15
   e. 20
   f. more than 20

7. If the shelf is full Monday morning, the probability that the shelf is full Thursday night is (select nearest value):
   a. 7%
   b. 8%
   c. 9%
   d. 10%
   e. 11%
   f. more than 12%

8. If the shelf is full Monday morning, the probability that the shelf is restocked Thursday night is (select nearest value):
   a. 10%
   b. 15%
   c. 20%
   d. 25%
   e. 30%
   f. more than 30%

9. If the shelf is full Monday morning, the expected number of nights that the shelf is restocked before Friday morning is (select nearest value):
   a. 0.6
   b. 0.7
   c. 0.8
   d. 0.9
   e. more than once but less than twice
   f. more than 2

10. The number of transient states in this Markov chain model is
    a. zero
    b. 1
    c. 2
    d. 5
    e. none of the above

11. The number of recurrent states in this Markov chain model is
    a. zero
    b. 1
    c. 2
    d. 5
    e. none of the above

12. The number of absorbing states in this Markov chain model is
    a. zero
    b. 1
    c. 2
    d. 5
    e. none of the above

13. Which (one or more) of the following equations are among those solved to compute the steady state probability distribution?
    a. $\pi_1 = 0.2\pi_3$
    b. $\pi_1 = 0.2\pi_3 + 0.5\pi_4 + 0.3\pi_5$
c. $\pi_3 = 0.2\pi_1 + 0.2\pi_2 + 0.3\pi_3 + 0.5\pi_4 + 0.2\pi_5$
d. $\pi_4 = 0.2\pi_2 + 0.5\pi_3 + 0.3\pi_4$
e. $\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1$

VII. **Continuous-Time Markov Chain** Two mechanics work in an auto repair shop, with a **maximum capacity** of 3 cars, so that any cars arriving when there are already 3 in the shop are turned away. Each mechanic works individually, completing the repair of a car in an average of 6 hours (the actual time being random with exponential distribution). (If there is only one car in the shop, only one mechanic works on it, while the other takes a break.) Cars arrive randomly, according to a Poisson process, at the rate of one every **two** hours when there are no cars in the shop, but one every **three** hours when one or more cars are in the shop. (If 3 cars are in the shop, of course, no cars will enter the shop.)

1. Draw a transition diagram below, with transition rates included, for this system.

   ![Transition Diagram](image)

2. What is the name of the distribution of the time between arrivals when the shop is empty?
   a. Markov  
b. Poisson  
c. Uniform  
d. Exponential  
e. Normal  
f. Weibull  
g. None of the above

3. The steady-state probability that the shop is empty is **(choose nearest value):**
   a. 10%  
b. 20%  
c. 30%  
d. 40%  
e. 50%  
f. 60%  
g. 70%  
h. >80%

4. In steady state, the fraction of the day that **exactly one** car will be in the shop is **(choose nearest value):**
   a. 10%  
b. 20%  
c. 30%  
d. 40%  
e. 50%  
f. 60%  
g. 70%  
h. >80%

5. In steady state, the average number of cars in the shop is **(choose nearest value):**
   a. .5  
b. 1  
c. 1.5  
d. 2  
e. 2.5  
f. 3

6. The average arrival rate in steady state is approximately one every 4 hours, i.e., 0.25/hour. According to Little's Formula, the average total time spent by a car in the shop (including both waiting and repair time) is **(choose nearest value):**
   a. 6 hours  
b. 7 hours  
c. 8 hours  
d. 9 hours  
e. 10 hours  
f. 11 hours

VIII. **Dynamic Programming** We wish to plan production of an expensive, low-demand item for the next three months (January, February, & March).
   - the cost of production is $15 for setup, plus $5 per unit produced, up to a maximum of 4 units.
   - the storage cost for inventory is $2 per unit, based upon the level at the **beginning** of the month.
   - a maximum of 3 units may be kept in inventory at the end of each month; any excess inventory is simply discarded.
   - the demand each month is random, with the same probability distribution each month:
     - $d = 0, 1, 2$
     - $P\{D = d\} = 0.3, 0.4, 0.3$
   - there is a penalty of $25 per unit for any demand which cannot be satisfied.
     - Backorders are not allowed.
   - there is **zero inventory** at the end of December.
   - a salvage value of $4 per unit is received for any inventory remaining at the end of the last month (March)

Consult the computer output which follows to answer the following questions: **Note that in the computer output, stage 3 = January, stage 2 = February, etc.** (i.e., $n = \#$ months remaining in planning period.)
1. What is the optimal production quantity for January (starting with zero inventory)? 

2. What is the total expected cost for the three months? 

3. If, during January, the demand is 2 units, what should be produced in February? 

4. Three values have been blanked out in the computer output. What are they?
   i. the optimal value $f_2(1)$ 
   ii. the optimal decision $x_2^*(1)$ 
   iii. the cost associated with the decision to produce 1 unit in February when the inventory is 0 at the end of January. 

The table of costs for each combination of state & decision at stage 2 is:

<table>
<thead>
<tr>
<th>$s$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>46.00</td>
<td>31.62</td>
<td>34.62</td>
<td>31.89</td>
<td>33.60</td>
</tr>
<tr>
<td>1</td>
<td>26.62</td>
<td>31.62</td>
<td>28.89</td>
<td>30.80</td>
<td>35.00</td>
</tr>
<tr>
<td>2</td>
<td>13.62</td>
<td>25.89</td>
<td>27.60</td>
<td>32.00</td>
<td>37.00</td>
</tr>
<tr>
<td>3</td>
<td>7.89</td>
<td>24.60</td>
<td>29.00</td>
<td>34.00</td>
<td>39.00</td>
</tr>
</tbody>
</table>

The tables of the optimal value function $f_n(S_n)$ at each stage are:

**Stage 3**

<table>
<thead>
<tr>
<th>State</th>
<th>Optimal</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>44.61</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>39.83</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>28.33</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>21.82</td>
<td>0</td>
</tr>
</tbody>
</table>

**Stage 2**

<table>
<thead>
<tr>
<th>State</th>
<th>Optimal</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>31.89</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>13.62</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>7.89</td>
<td>0</td>
</tr>
</tbody>
</table>

**Stage 1**

<table>
<thead>
<tr>
<th>State</th>
<th>Optimal</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>21.00</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>8.30</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-2.00</td>
<td>0</td>
</tr>
</tbody>
</table>