Inferences about a Covariance Matrix

Testing the Hypothesis That a Covariance Matrix Is Equal to a Given Matrix

Consider the hypothesis: $H_0: \Sigma = \Sigma_0$ vs. $H_1: \Sigma \neq \Sigma_0$. The hypothesized covariance matrix $\Sigma_0$ is a target value for $\Sigma$ or a nominal value from previous experience.

Obtain a random sample of $n$ observations, $X_1, X_2, \ldots, X_n$ from $N_p(\mu, \Sigma)$ and calculate sample covariance matrix $S$. To see if $S$ is significantly different from $\Sigma_0$, the likelihood ratio test statistic is:

$$W = -n p + p n \ln(n) - n \ln(\mid A \mid \mid \Sigma_0 \mid) + \text{tr}(\Sigma_0^{-1}A)$$

where $A = (n-1)S$.

The likelihood ratio test is biased, a modified likelihood ratio test statistic $W'$ can be obtained by replacing all $n$ in $W$ with $n-1$ (keep $A$ the same). The modified likelihood ratio test is unbiased.

$W$ and $W'$ are asymptotically (when $n$ is large) distributed as $\chi^2_{p(p+1)/2}$. So we reject $H_0$ if $W$ or $W'$ is greater than $\chi^2_{p(p+1)/2}(\alpha)$.
Generalized Variance

- Sometimes it is desirable to assign a *single* numerical value for the variation/dispersion expressed by $S$. One choice for a value is the determinant of $S$.
- Generalized sample variance $= |S|$
- $(\text{Volume of ellipsoid with constant generalized distance})^2 = (\text{constant})(\text{generalized sample variance})$
- Example 3.8 of J&W

Scatter Plots for Ex 3.8
The distribution of $|S|$ is the same as the distribution of $|A|/(n-1)^p$, where $A$ is as defined on pg. 43 of lecture notes.

Asymptotic Distribution:

$$\sqrt{n}(|S|/|\Sigma|-1) \sim N(0, 2p)$$