Testing $\mu=\mu_0$ Based on Hotelling's T^2

- Consider the test of the competing hypotheses:

  H_0: $\mu=\mu_0$ and H_1: $\mu\neq\mu_0$
- The following Hotelling's T^2 statistics is distributed as $(n\text{-}1)p/(n\text{-}p)*F_{p,\ n\text{-}p}$

$$T^2 = n(\overline{\mathbf{X}}-\boldsymbol{\mu}_0)^T \mathbf{S}^{-1}(\overline{\mathbf{X}}-\boldsymbol{\mu}_0)$$

- At the α-level of significance, we reject H_0 in favor of H_1 if

$$T^2 = n(\overline{\mathbf{X}}-\boldsymbol{\mu}_0)^T \mathbf{S}^{-1}(\overline{\mathbf{X}}-\boldsymbol{\mu}_0) > \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)$$

- The test based on T^2 is equivalent to the likelihood ratio test (not required)

Examples

- Example 5.1 of J&W (Evaluating T^2)
- Example 5.2 of J&W (Testing a multivariate mean vector with T^2):

Sweat Rate	Sodium	Potassium
3.7	48.5	9.3
5.7	65.1	8
3.8	47.2	10.9
3.2	53.2	12
3.1	55.5	9.7
4.6	36.1	7.9
2.4	24.8	14
7.2	33.1	7.6
6.7	47.4	8.5
5.4	54.1	11.3
3.9	36.9	12.7
4.5	58.8	12.3
3.5	27.8	9.8
4.5	40.2	8.4
1.5	13.5	10.1
8.5	56.4	7.1
4.5	71.6	8.2
6.5	52.8	10.9
4.1	44.1	11.2
5.5	40.9	9.4

Control Chart Basics

Motivation

How to do?

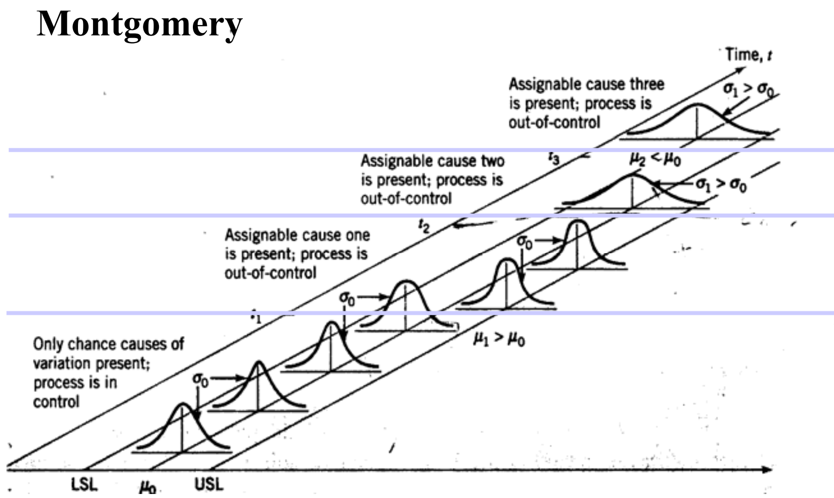
- Distinguish two process variations:
  - Chance causes and assignable causes
- Decide the status of a process
  - in control
  - out of control
- Continuously improve quality

Chance Cause & Assignable Cause

- Chance causes/common
  - system problems/inherent problems (natural variation/background noise)
  - “in statistical control”
- Assignable causes/special causes
  - problems arise in somewhat unpredictable fashion (operator error, material defects, machine failure), which often indicate a need for a timely repair or suggest improvements to the process
  - “out of statistical control”

Chance and assignable causes of variation

Figure 4.1 of Montgomery



Purpose of Using Control Charts

Improve Process and Reduce Process Variation

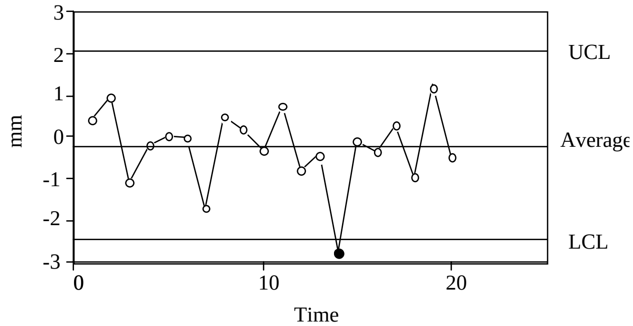
1. Most processes *do not* operate in a state of statistical control.
2. Consequently, the routine and attentive use of control charts will identify assignable causes. If these causes can be eliminated from the process, variability will be reduced and the process will be improved.
3. The control chart will only *detect* the occurrence of assignable causes. Management, operator, and engineering action will usually be necessary to *identify and eliminate* the assignable cause.

Objectives of Statistical Process Control (SPC)

- To quickly detect the occurrence of assignable causes or process shifts so that investigations of the process and corrective actions may be undertaken before many nonconforming units are manufactured.
- Process Variation Reduction

Concept of Control Charts

- Control Chart: is a graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number or time.
- Center Line – represents the value of the quality characteristic corresponding to the in-control state
- Upper Control Limit (UCL), Lower Control Limit (LCL) – are chosen so that if the process is in control, nearly all of the sample points will fall between them.



Samples (Subgroups) in Control Charts

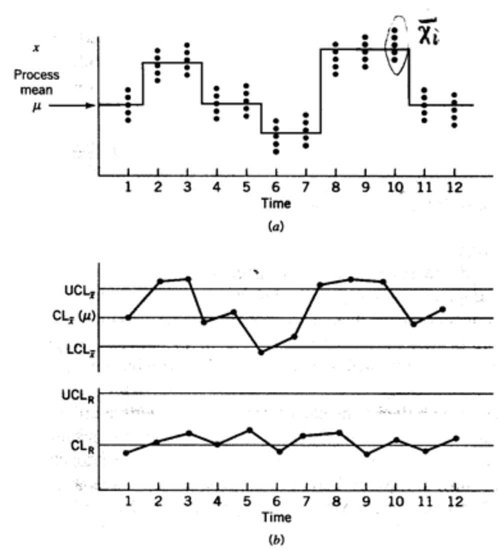


Figure 4-9 The "snapshot" approach to rational subgroups. *(a)* Behavior of the process mean. *(b)* Corresponding $\bar{x}$ and R control charts.