Inferences about a Mean Vector

Maximum Likelihood Estimation of the Mean Vector

- Maximum likelihood estimation is a technique to select the parameter values that maximize the joint density, or **likelihood**, evaluated at the observations.
- Let $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ be a random sample from a normal population with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$. Then

$$\hat{\boldsymbol{\mu}} = \overline{\mathbf{X}} \quad \text{and} \quad \hat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{j=1}^{n}(\mathbf{X}_j - \overline{\mathbf{X}})(\mathbf{X}_j - \overline{\mathbf{X}})^T = \frac{n-1}{n}\mathbf{S}$$

are the **maximum likelihood estimators** of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, respectively. Their observed values from a given sample are called the **maximum likelihood estimates** of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$.

Properties of the Maximum Likelihood Estimators (MLE)

- Maximum likelihood estimators are random quantities. The maximum likelihood estimates are their particular values for the given sample.
- MLE possess an **invariance property**.

30

The Sampling Distribution of X-bar

- Let $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ be a random sample of size n from a p-variate normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Then

$$\overline{\mathbf{X}} \sim N_p(\boldsymbol{\mu}, (1/n)\boldsymbol{\Sigma})$$

- Example: 4.21(c) of J&W

31