56:295 – 001
Multivariate Statistics and Advanced Quality Control Fall 05

Reading Assignments and Questions #1 Due: September 19 (Monday), 6:15pm

Reading Assignments:
If you are not familiar with basics of quality control, please read 3-3 (before 3-3.1), 3-3.6, 4-2, 4-3.1, 4-3.2, 4-3.4, 4-3.7 of Montgomery.
For all of you, please read 10-1, 10-2, and 10-3.1.

Reading Questions:

1. Answer the following problems from Montgomery:
   Exercises 10-1, 10-9, 10-10(a)(b)(c)

10-1.
Phase 2 T^2 control charts with $m = 50$ preliminary samples, $n = 25$ sample size, $p = 2$ characteristics. Let $\alpha = 0.001$.

$$\text{UCL} = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1}$$

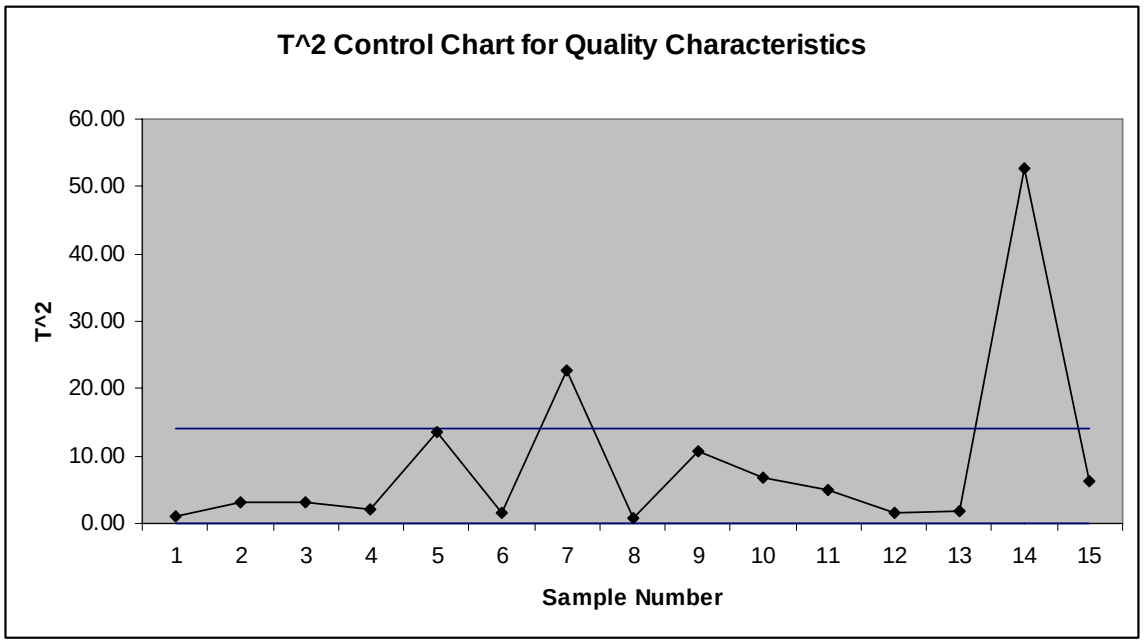
$$= \frac{2(50+1)(25-1)}{50(25)-50-2+1} F_{0.001,2,1199}$$

$$= \left(2448/1199\right)(6.948) = 14.186$$

LCL = 0

Excel : workbook Chap10.xls : worksheet Ex10-1

Sample No.	1	2	3	4	5	6	7	8	9	
xbar1	58	60	50	54	63	53	42	55	46	
xbar2	32	33	27	31	38	30	20	31	25	
diff1	3	5	-5	-1	8	-2	-13	0	-9	
diff2	2	3	-3	1	8	0	-10	1	-5	
matrix calc	0.0451	0.1268	0.1268	0.0817	0.5408	0.0676	0.9127	0.0282	0.4254	
t2 = n * calc	1.1268	3.1690	3.1690	2.0423	13.5211	1.6901	22.8169	0.7042	10.6338	
UCL =	14.1850	14.1850	14.1850	14.1850	14.1850	14.1850	14.1850	14.1850	14.1850	
LCL =	0	0	0	0	0	0	0	0	0	
OOC?	In control	In control	In control	In control	In control	In control	Above UCL	In control	In control	...



Process is out of control at samples 7 and 14.

10-9.

p = 10 quality characteristics, n = 3 sample size, m = 25 preliminary samples. Assume $\alpha = 0.01$.

Phase I UCL:

$$\text{UCL} = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1}$$
$$= \frac{10(25-1)(3-1)}{25(3)-25-10+1} F_{0.01,10,41}$$
$$= \left(\frac{480}{41}\right)(2.788)$$
$$= 32.638$$

Phase II UCL:

$$\text{UCL} = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1}$$
$$= \frac{10(25+1)(3-1)}{25(3)-25-10+1} F_{0.01,10,41}$$
$$= \left(\frac{520}{41}\right)(2.788)$$
$$= 35.360$$

10-10.

Excel : workbook Chap10.xls : worksheet Ex10-10

(a)

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 0.7 & 0.7 & 0.7 \\ 0.7 & 1 & 0.7 & 0.7 \\ 0.7 & 0.7 & 1 & 0.7 \\ 0.7 & 0.7 & 0.7 & 1 \end{bmatrix}$$

(b)

$$\text{UCL} = \chi^2_{\alpha,p} = \chi^2_{0.01,4} = 13.277$$

(c)

$$T^2 = n(\mathbf{y}-\boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathbf{y}-\boldsymbol{\mu})$$

$$= 1\left(\begin{bmatrix} 3.5 \\ 3.5 \\ 3.5 \\ 3.5 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right)' \begin{bmatrix} 1 & 0.7 & 0.7 & 0.7 \\ 0.7 & 1 & 0.7 & 0.7 \\ 0.7 & 0.7 & 1 & 0.7 \\ 0.7 & 0.7 & 0.7 & 1 \end{bmatrix}^{-1} \left(\begin{bmatrix} 3.5 \\ 3.5 \\ 3.5 \\ 3.5 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$= 15.806$$

Yes. Since $(T^2 = 15.806) > (\text{UCL} = 13.277)$, an out-of-control signal is generated.

2. Using the result covered in the class on multivariate normal distribution, explain briefly why the in-control region in Fig. 10-4 of Montgomery is an ellipse? What is the in-control region of chi-square control chart for $p>2$ quality characteristics?

It is the same as the contour of a bivariate normal distribution, which is an ellipse. The in-control region would be an ellipsoid for $p>2$.

3. Could you explain the meaning of *m* and *n* on page 495 of Montgomery?

m is the number of preliminary samples used in Phase I to construct the control chart. And *n* is the sample size for both Phase I and Phase II.

Reading Assignments and Questions #2 Due: September 26 (Monday), 6:15pm

Reading Assignments:

If you are not familiar with basics of control charts, I recommend (not require) you to read 5-1, 5-2, and 5-3 of Montgomery.

For all of you, please read 10-6

Reading Questions:

4. Apply the control chart based on Eq. (10-34) of Montgomery for the data in Example 10-1 (same data are used in Example 10-2). Please find the control limit and calculate the

statistic W_i for each sample, i=1, …, 20. You can use **S** to estimate the nominal covariance matrix **Σ** and follow Example 10.2 to estimate $|\mathbf{\Sigma}|$.

The test statistics W_i for each sample, i=1, …, 20 are:

S11	S22	S12	W
1.25	0.87	0.8	1.2167
1.26	0.85	0.81	1.245
1.3	0.9	0.82	1.217
1.02	0.85	0.81	3.4934
1.16	0.73	0.8	3.3359
1.01	0.8	0.76	2.8177
1.25	0.78	0.75	1.3121
1.4	0.83	0.8	1.3847
1.19	0.87	0.83	1.5729
1.17	0.86	0.95	8.6779
1.45	0.79	0.78	1.6416
1.24	0.82	0.81	1.4314
1.26	0.55	0.72	4.7562
1.17	0.76	0.75	1.58
1.23	0.89	0.82	1.261
1.24	0.91	0.83	1.2496
1.2	0.95	0.7	2.358
1.18	0.83	0.79	1.4485
1.31	0.89	0.76	1.602
1.29	0.85	0.68	2.2268

The control limit using α=0.05 is $\chi^2_{0.05,2(2+1)/2} = 7.81$.

5. Based on the results in Section 10-6 of Montgomery, what is Var[|**A**|], where **A** is from pg. 46 of lecture notes?

$Var[|\mathbf{A}|] = Var[(n-1)^p |\mathbf{S}|] = (n-1)^{2p} b_2 |\mathbf{\Sigma}|^2$

6. Please comment the relationship between the test statistic in Eq. (10-34) and the likelihood ratio or modified likelihood ratio test of covariance matrix.

It uses the likelihood ratio test instead of the modified likelihood ratio test.

7. Why should the lower control limit in Eq. (10-36) be replaced with zero if the calculated value is less than zero?

Since **S** is positive semidefinite, we always have $|\mathbf{S}| \geq 0$. So a negative lower control limit is meaningless.