56:295 – 001
Multivariate Statistics and Advanced Quality Control
Fall 05

HW7 Due: November 30 (Wednesday), 6:15pm

Note: Show solution procedures.

Solve the following problems from the textbook (J&W)
11.1(b)

$$R_1 : (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)^T \boldsymbol{\Sigma}^{-1}\mathbf{x}_0 - \frac{1}{2}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2) \geq \ln\left[\left(\frac{c(1|2)}{c(2|1)}\right)\left(\frac{p_2}{p_1}\right)\right] = \ln 1 = 0$$

LHS=4 at x_0=[2 7]^T. Because LHS>0, x_0 is classified as in π_1.

11.4

(**a**) The minimum ECM rule is given by assigning an observation $\boldsymbol{x}$ to π_1 if

$$\frac{f_1(\boldsymbol{x})}{f_2(\boldsymbol{x})} \geq \left(\frac{c(1|2)}{c(2|1)}\right)\left(\frac{p_1}{p_2}\right) = \left(\frac{100}{50}\right)\left(\frac{.2}{.8}\right) = .5$$

and assigning $\boldsymbol{x}$ to π_2 if

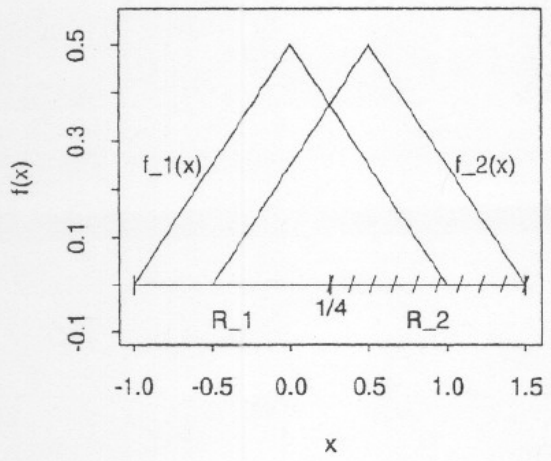
$$\frac{f_1(\boldsymbol{x})}{f_2(\boldsymbol{x})} < \left(\frac{c(1|2)}{c(2|1)}\right)\left(\frac{p_1}{p_2}\right) = \left(\frac{100}{50}\right)\left(\frac{.2}{.8}\right) = .5$$

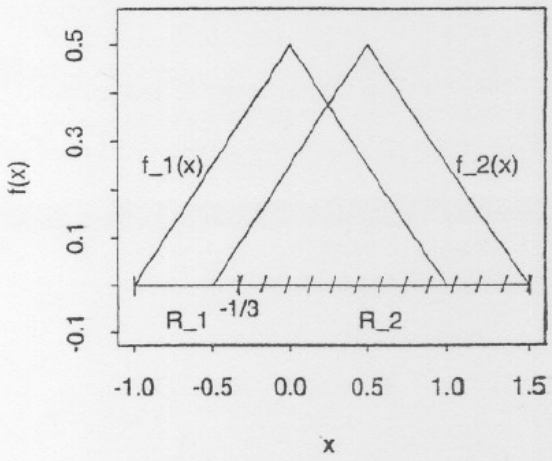
(**b**) Since $f_1(\boldsymbol{x}) = .3$ and $f_2(\boldsymbol{x}) = .5$,

$$\frac{f_1(\boldsymbol{x})}{f_2(\boldsymbol{x})} = .6 \geq .5$$

and assign $\boldsymbol{x}$ to π_1.

11.7





(b) When $p_1 = p_2$ and $c(1|2) = c(2|1)$, the classification regions are

$$R_1 : \frac{f_1(x)}{f_2(x)} \geq 1 \qquad R_2 : \frac{f_1(x)}{f_2(x)} < 1$$

These regions are given by $R_1 : -1 \leq x \leq .25$ and $R_2 : .25 < x \leq 1.5$.

(c) When $p_1 = .2$, $p_2 = .8$, and $c(1|2) = c(2|1)$, the classification regions are

$$R_1 : \frac{f_1(x)}{f_2(x)} \geq \frac{p_2}{p_1} = 4 \qquad R_2 : \frac{f_1(x)}{f_2(x)} < 4$$

These regions are given by $R_1 : -1 \leq x \leq -1/3$ and $R_2 : -1/3 < x \leq 1.5$.

11.11

Assuming equal prior probabilities $p_1 = p_2 = \frac{1}{2}$, and equal misclassification costs $c(2|1) = c(1|2) = \$10$:

c	$P(B1\|A2)$	$P(B2\|A1)$	$P(A2 \text{ and } B1)$	$P(A1 \text{ and } B2)$	$P(\text{error})$	Expected cost
9	.006	.691	.346	.003	.349	3.49
10	.023	.500	.250	.011	.261	2.61
11	.067	.309	.154	.033	.188	1.88
12	.159	.159	.079	.079	.159	1.59
13	.309	.067	.033	.154	.188	1.88
14	.500	.023	.011	.250	.261	2.61

Using (11-5), the expected cost is minimized for $c = 12$ and the minimum expected cost is \$1.59.