56:295 – 001
Multivariate Statistics and Advanced Quality Control
Fall 05

HW4 Due: October 12 (Wednesday), 6:15pm

Note: Show solution procedures.

1. Read the paper: George C. Runger "Projections and the U2 Multivariate Control Chart", Journal of Quality Technology, 28(3), 1996, which can be downloaded from homework page of course website.

2. In a process 10 variables are being monitored. However, the assignable cause anticipated only shifts the mean of the first variable and the last variable.
(a) Find the orthonormal matrix **U** corresponding to the mean shift subspace.
(b) If the control limit is selected such that the in-control ARL is equal to 200, find a mean shift $\boldsymbol{\mu}$ (from **0**) such that the ARL under this mean shift for the χ^2 chart is 31. What is the ARL under this mean shift for the U^2 chart? Assume the covariance matrix $\boldsymbol{\Sigma}$ of the process variables is a 10-by-10 identity matrix.

2. This question uses the data in the file Data_hw4.mat on homework page of the course website. It is a MAT format Matlab data file. You can use command "load" in Matlab to read the data. There is one variable, "Y", in the file. Y is a 150 by 20 matrix. The data are measurement data from the auto-body assembly process shown on page 65 of the lecture notes. Mean shifts in the subspace spanned by the columns of $\boldsymbol{\Gamma}$ on page 66 of lecture notes are present in the data. Matrix $\boldsymbol{\Gamma}$ is stored in the file Gamma_20by3.mat, which is downloadable on the homework page. Each row of the matrix Y represents a 20-dimension observation (10 points, 20 measurements for two directions of each point) on one product (car body). So Y contains a sample of 150 observations. Please refer to the Matlab help for more details on reading and using the data. Please use Matlab for computation.

(i) Let $\mathbf{Y}_1$ denote the vector of measurements from the first observation (first row of matrix Y). Fit a linear model as $\mathbf{Y}_1=\boldsymbol{\Gamma\beta}+\boldsymbol{\varepsilon}$, where $\boldsymbol{\Gamma}$ is the one given on page 66 of lecture notes and $\boldsymbol{\varepsilon}$ is a zero mean random vector with covariance matrix equal to $\boldsymbol{\Sigma}$, which is also given on slide 66. Please find the generalized least squares estimation of $\boldsymbol{\beta}$. Note that you should only use the data in the first row of Y here.

(ii) Calculate the U^2 statistics for each of the 150 observations to detect mean shifts in the subspace spanned by the columns of $\boldsymbol{\Gamma}$ on page 66 of lecture notes. Please evaluate $\boldsymbol{\Sigma}$ as shown on page 66 of lecture notes. Use α=0.005 to obtain the control limit for the U^2 chart. List all out-of-control observations when the U^2 chart is used.

(iii) Calculate the χ^2 statistics for each of the 150 observations. Use α=0.005 to obtain the control limit for the χ^2 chart. List all out-of-control observations when the χ^2 chart is used. Compare with part (ii) to comment on the advantage of U^2 chart to the χ^2 chart in detecting the mean shifts in this data set.