**Flow exergy**

If mass crosses the boundary of a CV, there is an exergy transfer due to mass flow. Also, there is exergy transfer accompanying flow work.

The *specific flow exergy* takes both effects into account:

\[
e_f = h - h_0 - T_0 (s - s_0) + \frac{V^2}{2} + g z
\]

[Exergy transfer accompanying work]: \( W - p_0 \Delta V \)

Between \( t \) and \( t + \Delta t \) the change in volume is: \( \Delta V = m_e v_e \), thus:

[Exergy transfer accompanying work]: \( W - p_0 m_e \Delta v_e \)

Dividing by \( \Delta t \) on the limit of \( \Delta t \to 0 \) we get:

[Rate of exergy transfer accompanying work]: \( \lim_{\Delta t \to 0} \frac{W}{\Delta t} - \lim_{\Delta t \to 0} \frac{m_e}{\Delta t} p_0 v_e \)

noting that the net work is \( W = m_e p_e v_e \) as \( \Delta t \to 0 \), we can write:

[Rate of exergy transfer accompanying work]: \( m_e (p_e v_e - p_0 v_e) \)
The energy transfer across the exit is:

[Rate of energy transfer due to mass flow]: \( \dot{m} e = \dot{m} \left( u + \frac{V^2}{2} + g z \right) \)

[Rate of exergy transfer due to mass flow]:

\[ \dot{m} e = \dot{m} \left[ (e - u_0) + p_0 (v - v_0) - T_0 (s - s_0) \right] \]

Accounting for the exergy transfer due to mass flow and flow work we have:

[Rate of exergy transfer due to mass flow and flow work]:

\[ \dot{m} e_f = \dot{m} \left[ (e - u_0) + p_0 (v - v_0) - T_0 (s - s_0) + (p v - p_0 v) \right] \]

since \( h = u + p v \) we obtain the specific flow exergy:

\[ e_f = h - h_0 - T_0 (s - s_0) + \frac{V^2}{2} + g z \]

compare with the closed system exergy.

Example:

A steam turbine develops a power output of 1000 kW. At steady state, the pressure at the inlet is 60 bar, the temperature is 400 °C and the velocity is 10 m/s. At the exit, the pressure is 0.1 bar, the velocity 50 m/s and the quality is 90 %. The change in level between inlet and exit is 10 m. The rate of heat transfer to the surroundings is – 61.3 kW. Find the exergy and flow exergy at the inlet and outlet. Use \( T_0 = 300 \text{ K}, \ p_0 = 1 \text{ bar} \), and \( z_0 \) the level at the exit.

Solution:

the rate of change of energy inside the turbine is zero, since the turbine operates at SS. Then
\[ 0 = Q_{cv} - W_{cv} + m \left( h_{in} + \frac{V_{in}^2}{2} + g \, z_{in} \right) - m \left( h_{out} + \frac{V_{out}^2}{2} + g \, z_{out} \right) \]

From table of superheated vapor at 60 bar:

\[ u_{in} = 2892.9 \, \text{kJ/kg} \]
\[ h_{in} = 3177.2 \, \text{kJ/kg} \]
\[ v_{in} = 0.04739 \, \text{m}^3/\text{kg} \]
\[ s_{in} = 6.5408 \, \text{kJ/kg K} \]

At the exit, the quality is 90 %, which means that we have a two-phase mixture. The properties are show in table A-3.

Using the lever rule to calculate a generic property \( y \), \( y = y_f + (y_g - y_f) \), we obtain:

\[ u_{out} = 121.45 + 0.9 \left( 2437.9 - 251.38 \right) = 2089.32 \, \text{kJ/kg} \]
\[ h_{out} = 191.83 + 0.9 \, 2392.8 = 2345.4 \, \text{kJ/kg} \]
\[ v_{out} = 0.0010102 + 0.9 \left( 14.674 - 0.0010102 \right) = 13.207 \, \text{m}^3/\text{kg} \]
\[ s_{out} = 0.6493 + 0.9 \left( 8.1502 - 0.6493 \right) = 7.4001 \, \text{kJ/kg K} \]
replacing into the energy equation we can calculate the mass flow rate:

\[
m = \frac{Q_{cv} - W_{cv}}{h_{\text{out}} + \frac{V_{\text{out}}^2}{2} + g \, z_{\text{out}}} - \left( h_{\text{in}} + \frac{V_{\text{in}}^2}{2} + g \, z_{\text{in}} \right) = 4599.4 \text{ kg/ hr}
\]

Table A-2 Properties of Saturated Water (Liquid–Vapor): Temperature Table

<table>
<thead>
<tr>
<th>Temp. C</th>
<th>Press. bar</th>
<th>Specific Volume m/kg</th>
<th>Internal Energy kJ/kg</th>
<th>Enthalpy kJ/kg</th>
<th>Entropy kJ/kg K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sat. Liquid ( e_i ) Sat. Vapor ( e_v )</td>
<td>Sat. Liquid ( u_i ) Sat. Vapor ( u_v )</td>
<td>Sat. Liquid ( h_i ) Sat. Vapor ( h_v ) Evap. ( h_b )</td>
<td>Sat. Vapor ( s_i ) Sat. Vapor ( s_v )</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>1.0062</td>
<td>206.136</td>
<td>0.00</td>
<td>2375.3</td>
<td>0.01</td>
</tr>
<tr>
<td>4</td>
<td>1.0081</td>
<td>157.232</td>
<td>16.77</td>
<td>2380.9</td>
<td>16.78</td>
</tr>
<tr>
<td>5</td>
<td>1.0072</td>
<td>147.120</td>
<td>20.97</td>
<td>2382.3</td>
<td>20.98</td>
</tr>
<tr>
<td>6</td>
<td>0.00935</td>
<td>137.734</td>
<td>25.19</td>
<td>2383.6</td>
<td>25.20</td>
</tr>
<tr>
<td>8</td>
<td>0.01072</td>
<td>120.917</td>
<td>33.59</td>
<td>2386.4</td>
<td>33.60</td>
</tr>
<tr>
<td>10</td>
<td>0.01228</td>
<td>106.379</td>
<td>42.00</td>
<td>2392.3</td>
<td>42.01</td>
</tr>
<tr>
<td>11</td>
<td>0.01312</td>
<td>99.857</td>
<td>46.20</td>
<td>2390.5</td>
<td>46.20</td>
</tr>
<tr>
<td>12</td>
<td>0.01402</td>
<td>93.784</td>
<td>50.41</td>
<td>2391.9</td>
<td>50.41</td>
</tr>
<tr>
<td>13</td>
<td>0.01497</td>
<td>88.124</td>
<td>54.60</td>
<td>2393.4</td>
<td>54.60</td>
</tr>
<tr>
<td>14</td>
<td>0.01398</td>
<td>82.848</td>
<td>58.79</td>
<td>2394.7</td>
<td>58.80</td>
</tr>
<tr>
<td>15</td>
<td>0.01705</td>
<td>77.926</td>
<td>62.99</td>
<td>2396.1</td>
<td>62.99</td>
</tr>
<tr>
<td>16</td>
<td>0.01818</td>
<td>73.333</td>
<td>67.18</td>
<td>2397.4</td>
<td>67.19</td>
</tr>
<tr>
<td>17</td>
<td>0.01938</td>
<td>69.044</td>
<td>71.38</td>
<td>2398.8</td>
<td>71.38</td>
</tr>
<tr>
<td>18</td>
<td>0.02064</td>
<td>65.038</td>
<td>75.57</td>
<td>2400.2</td>
<td>75.58</td>
</tr>
<tr>
<td>19</td>
<td>0.02198</td>
<td>61.293</td>
<td>79.76</td>
<td>2401.6</td>
<td>79.77</td>
</tr>
<tr>
<td>20</td>
<td>0.02339</td>
<td>57.791</td>
<td>83.95</td>
<td>2402.9</td>
<td>83.96</td>
</tr>
<tr>
<td>21</td>
<td>0.02487</td>
<td>54.514</td>
<td>88.14</td>
<td>2404.3</td>
<td>88.14</td>
</tr>
<tr>
<td>22</td>
<td>0.02645</td>
<td>51.447</td>
<td>92.32</td>
<td>2405.7</td>
<td>92.33</td>
</tr>
<tr>
<td>23</td>
<td>0.02810</td>
<td>48.574</td>
<td>96.51</td>
<td>2407.0</td>
<td>96.52</td>
</tr>
<tr>
<td>24</td>
<td>0.02985</td>
<td>45.883</td>
<td>100.70</td>
<td>2408.4</td>
<td>100.70</td>
</tr>
</tbody>
</table>

From table, the properties at the dead state are:

\[
u_0 = 83.95 \quad \text{kJ/kg}
\]
\[
h_0 = h_f + v_f (p_0 - p_{\text{sat}}) = 83.86 \quad \text{kJ/kg}
\]
\[
v_0 = 0.0010018 \quad \text{m}^3/\text{kg}
\]
\[
s_0 = 0.2966 \quad \text{kJ/kg K}
\]

The exergy at the inlet is:
\[
\textbf{e} = [(e - u_0) + p_0(v - v_0) - T_0(s - s_0)] = \\
2892.9 + \frac{10^2/2}{1000} + \frac{9.810}{1000} - 83.95 + \frac{10^5(0.04739 - 0.0010018)}{1000} - 300(6.5408 - 0.2966) = \\
2892.9 + 0.05 + 0.098 - 83.95 + 4.639 - 1873.26 = 940.48 \quad kJ / kg \\
\textbf{m} \textbf{e} = 1201.6 \quad kW
\]

The flow exergy at the inlet is:

\[
\textbf{e}_f = h - h_0 - T_0(s - s_0) + \frac{V^2}{2} + g z = \\
3177.2 - 83.86 - 1873.26 + 0.05 + 0.098 = 1220.23 \quad kJ / kg \\
\textbf{m} \textbf{e}_f = 1558.98 \quad kW
\]

At the exit we have:

\[
\textbf{e}_f = 132.8 \quad kJ / kg \\
\textbf{m} \textbf{e}_f = 169.7 \quad kW
\]