CONTINUUM STRUCTURAL TOPOLOGY OPTIMIZATION

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Introduction

Structural Optimization

- Size Optimization

- Shape Optimization

- Topology Optimization

“Continuum Structural Topology Optimization” presented to CEE/SMM Seminar, 31 March 2006
Alternative Approaches to Structural Topology Optimization

Discrete structural model of beam/truss elements

Continuum structural model

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OUTLINE:

• PRELIMINARY IDEAS
  • The analysis/design framework
    • How to capture all possible structural forms?
  • Model sparsity issue
  • Problem size reduction

• APPLICATIONS
  • Design for minimal compliance
  • Design for stability
The Analysis and Design Framework

\[ \phi(X) = \sum_{i=1}^{numnp} \phi_i N_i(X) \]

Applied Loads

Starting Design Domain

Supports

Intermediate Design

\[ \phi(X) = 0 \]

\[ \phi(X) = 1.0 \]

\[ 0 \leq \phi(X) \leq 1.0 \]

\[ E(X) = E(\phi(X)) \] [mixing rule]
Gradient-Based Optimization Algorithm

1. Initialization Starting Guess
2. Structural Analysis
   Find $u(b)$
3. Sensitivity Analysis (Linearization)
4. Optimization Step
   (Linear Programming Sub-Problem)
5. Optimal?
   - Yes: Stop
   - No: Update design

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Sparsity of Long-Span Bridges

Sunshine Skyway bridge cable-stayed bridge in Tampa, Florida

Akashi Bridge suspension bridge in Japan

Most long-span bridges occupy < 1% of their envelope volume.
Capturing SPARSITY in Continuum Topology Optimization

- Fixed-mesh model of full envelope volume;
  - must capture the form of the structure with realistic sparsity
  - must capture mechanical performance of the structure
- Fine meshes are required;
- Implies large computational expense;

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20 elements  50 elements  112 elements
40 elements  100 elements  224 elements
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50% Material constraint usage  10% Material constrain usage  3% Material constraint usage
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Nodes having vanishing design variable values are denoted with open circles; filled circles denote nodes associated with nonzero design values; nodes represented by open circles with X’s are “prime” nodes whose degrees of freedom are restrained in the size reduction method. Elements designated with “S” are at least partially solid and those with “V” are devoid of material. Those designated with “P” are prime and need not be considered during structural analysis since all of their degrees of freedom are restrained.
Strategy for Computational Efficiency:

1. Start with coarse structural model
2. Find associated optimal arrangement of material;
3. Construct a finer structural model;
4. Map the current design onto the finer structural model;
5. Reduce allowable material usage
6. Return to step #2.
• Structure modeled as linearly elastic system
  • Stability analysis performed via linearized buckling analysis

• Linear elastic problem: \( \mathbf{K}_L \cdot \mathbf{u} = \mathbf{f}^{ext} \)
  Compliance: \( \Pi = \frac{1}{2} \mathbf{f}^{ext} \cdot \mathbf{u} = \frac{1}{2} \mathbf{u} \cdot \mathbf{K}_L \cdot \mathbf{u} \)

• Eigenvalue problem: \( \mathbf{K}_L (\mathbf{b}) \psi + \lambda \mathbf{G} (\mathbf{u}, \mathbf{b}) \psi = 0 \)
  \( \lambda = \frac{\psi^T \mathbf{K}_L \psi}{\psi^T \mathbf{G} \psi} \)

• Objective function: \( L(\mathbf{u}, \mathbf{b}) = \frac{1}{\min(\lambda)} \)

• Design sensitivity analysis:
  \[
  \frac{dL}{db} = -\psi^T (\frac{\partial \mathbf{G}}{\partial \mathbf{b}} + \frac{1}{\lambda} \frac{\partial \mathbf{K}_L}{\partial \mathbf{b}}) \psi + (\mathbf{u}^a)^T (\frac{\partial \mathbf{K}_L}{\partial \mathbf{b}} \cdot \mathbf{u} - \frac{\partial \mathbf{f}^{ext}}{\partial \mathbf{b}}) \\
  \mathbf{K}_L \mathbf{u}^a = \psi^T \frac{\partial \mathbf{G}}{\partial \mathbf{u}} \psi
  \]
Design Constraints

• Bounds on individual design variables

\[ 0 \leq \phi_j \leq 1 \quad J = 1, 2, \ldots, \text{NUMNP} \]

• Material usage constraint

\[
\int_\Omega \phi(X) \, d\Omega \leq C \frac{\int_\Omega d\Omega}{\int_\Omega d\Omega}
\]
Fixed-End Beam Problem, Design for Stability

Design domain with 10% material constraint.

Resulting topology

First buckling mode.

Minimizing the general compliance.
Maximizing the min. critical buckling load, nonlinear formulation.
Maximizing the min. critical buckling load, linearized buckling.

\( F \)
\( d \)
\( F \)

\[ \Pi = 1.496 \times 10^6 \]

\[ \lambda = 10.0 \]

\[ \Pi = 5.09 \times 10^6 \]

\[ \lambda = 19.51 \]
Circle Problem

Design domain
With material
2.5% Constraint

Undeformed configuration

Undeformed configuration

\[ \lambda = 1.37 \times 10^5 \]
Minimizing structural Compliance

\[ \lambda = 3.28 \times 10^9 \]
Maximizing the min. critical buckling load, linearized buckling.

\[ \Pi = 4.09 \times 10^5 \]

\[ \Pi = 2.66 \times 10^5 \]
Long-Span Bridge Problem “2-Supports”

Design domain with 12.5% material constraint and 10 kPa applied load.

Compliance Minimization

Compliance Minimization (finer mesh)

Optimum linearized buckling stability

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Long-Span Bridge Problem “3-Supports”

Design domain with 12.5% material constraint and 10 kPa applied load.

Compliance Minimization

Optimum linearized buckling stability

\[ \Pi = 2.06 \times 10^4 \]
\[ \lambda = 5.29 \times 10^3 \]

\[ \Pi = 1.52 \times 10^5 \]
\[ \lambda = 5.15 \times 10^3 \]
Current Research Issues:

- Designing Structures in 3-Dimensions
  - Iterative Eigensolvers
    - How to reliably solve: $K\phi + \lambda G\phi = 0$ for the minimum eigenpair without ever forming or factorizing $K$ or $G$?
- Design of Compliant Mechanisms
  - Micro-devices