Challenges in Design Optimization of Textile–Reinforced Composites

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Background

• Structural FRP composites are being considered for usage in civil infrastructure applications.

• Perceived Advantages:
  • lightness
  • durability
  • damping characteristics

• Perceived Disadvantages
  • mechanical performance characteristics
Stiffnesses & Strengths of Aligned Fiber Composites are Highly Anisotropic

<table>
<thead>
<tr>
<th>Elastic Moduli (GPa)</th>
<th>Glass Epoxy (50/50)</th>
<th>Graphite Epoxy (50/50)</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{1111}$</td>
<td>38.29</td>
<td>129.0</td>
<td>268.8</td>
</tr>
<tr>
<td>$C_{2222}$, $C_{3333}$</td>
<td>8.81</td>
<td>10.4</td>
<td>268.8</td>
</tr>
<tr>
<td>$C_{1212}$, $C_{1313}$</td>
<td>3.32</td>
<td>3.57</td>
<td>76.9</td>
</tr>
<tr>
<td>$C_{2323}$</td>
<td>2.60</td>
<td>2.67</td>
<td>76.9</td>
</tr>
</tbody>
</table>
Research Objectives

- Find arrangements of fibers in composites which improve overall stiffness/strength.
- Explore possibilities systematically using analytical/computational methods.
- Improve analysis methods for unusual composites.
Primary Analysis Tool: Computational Homogenization

• What is computational homogenization?
   Using the computer to characterize the macroscopic response of heterogeneous systems.

• To what types of problems can it be applied?
   Examples:
   Characterization of existing composites
   Design of new composites
   Studies of Bio–tissues (bone, muscle, etc.)

• Strengths of the method
  Fairly general, although requires periodic or quasi–periodic material structures.
Domain for Homogenization Computations

In general, an RVE. domain. For *periodic* composites, domain is the unit cell.
Small Deformation Decompositions/Notation:

Micro–Stress
\[ \sigma(X) = S + \sigma^*(X) \]

Macro–Stress
\[ S = \langle \sigma \rangle \]

Micro–Strain
\[ \varepsilon(X) = E + \varepsilon^*(X) \]

Macro–Strain
\[ E = \langle \varepsilon \rangle \]

\[ \sigma^*, \varepsilon^* \text{ are inhomogeneous contributions.} \]

\[ \langle \sigma^* \rangle = 0; \quad \langle \varepsilon^* \rangle = 0; \]

\[ u(X) = E \cdot X + u^*_{\text{per}}(X) \]

\[ u(X) \text{ is the total displacement field.} \]
\[ EX \text{ is the homogeneous contribution.} \]
\[ u^*_\text{per} \text{ is the inhomogeneous contribution.} \]
General Decompositions/Notation:

**Micro−Stress (PK−II)**  **Macro−Stress (PK−II)**
\[ \sigma(X) = S + \sigma^*(X) \]  \[ S = \langle \sigma \rangle \]

**Micro−Deformation**  **Macro−Deformation**
\[ F(X) = I + \partial u/\partial X \]  \[ \Phi = \langle F \rangle \]  \[ = RU = U \]

**Local Strain (Green)**  **Macro−Strain (Green)**
\[ E = 1/2[F^T F - I] \]  \[ E = 1/2[\Phi^T \Phi - I] \]

\[ u(X) = (\Phi-I)\cdot X + u^*_{\text{per}}(X) \]

\[ u(X) \] is the *total* displacement field.
\[ (\Phi-I)\cdot X \] is the *homogeneous* contribution.
\[ u^*_{\text{per}} \] is the *inhomogeneous* contribution.
Procedure for Strain–Controlled Homogenization:

Impose a *homogeneous* displacement field:
\[ u = E \cdot X \text{ or } u = (\Phi - I) \cdot X \text{ on } \Omega_s. \]

Solve a variational problem for the *inhomogeneous* field \( u^*_\text{per} \).

Variational Equilibrium Statement:

\[ \int_{\Omega_s} (\nabla \cdot \sigma) \cdot \delta u \, d\Omega = 0 \]

Weak Form Solved:

\[ \int_{\Omega_s} \sigma(u) : \varepsilon^*(\delta u) \, d\Omega = 0. \]
B. Material Topology Optimization

- Optimize material arrangements to enhance mechanical performance.
- Properties associated with each arrangement are calculated using homogenization.

Flowchart:

1. Generate Initial Design
2. Calculate Properties (Homogenization)
3. Are Properties Optimal?
   - Yes: Stop
   - No: Modify the Design
Results of Material Topology Optimization

40% graphite
60% epoxy

50% graphite
50% epoxy

60% graphite
40% epoxy

\[ \begin{align*}
\text{C}_{2323} & = 2.09 \text{ GPa} \\
\text{C}_{3333} & = 7.96 \text{ GPa} \\
\text{C}_{1111} & = 104 \text{ GPa}
\end{align*} \]

\[ \begin{align*}
\text{C}_{2323} & = 2.67 \text{ GPa} \\
\text{C}_{2222}, \text{C}_{3333} & = 10.4 \text{ GPa} \\
\text{C}_{1111} & = 129 \text{ GPa}
\end{align*} \]

\[ \begin{align*}
\text{C}_{2323} & = 35.2 \text{ GPa} \\
\text{C}_{2222}, \text{C}_{3333} & = 48.2 \text{ GPa} \\
\text{C}_{1111} & = 135 \text{ GPa}
\end{align*} \]

\[ \begin{align*}
\text{C}_{2323} & = 47.3 \text{ GPa} \\
\text{C}_{2222}, \text{C}_{3333} & = 76.9 \text{ GPa} \\
\text{C}_{1111} & = 163 \text{ GPa}
\end{align*} \]
Example: Elasto–plastic Compliance Minimization of a Boron–Epoxy Composite

Original Design

Undeformed Cell

Deformed Cell

Optimal Design

$S_{12}$ (MPa)

Original Design

Optimal Design
Significance of Results

• Demonstrate necessity of getting fiber material to perform multi-axially.

• Demonstrate advantages of integration & continuity of fiber material in three orthogonal directions.

• Some material arrangements are fairly complex, and others are much simpler (more manufacturable).

Complex Arrangement

Simpler Arrangement
Manufacturability Concerns

- Re–designed composites contain continuous, monolithic, glass or graphite phases.
  - LCVD for small scale parts/structures
  - Infeasible for large scale structural composites

- Current trend is toward textile reinforcing
  - Gives 3–D reinforcing (weaker anisotropy)
  - Capabilities for producing 3–d weaves & meshes are developing rapidly

- Designed material arrangements are therefore approximated as textiles and re–analyzed.
Desired Material Arrangement (unit cell)

Textile Composite Approximation

a) Graphite plane weave with longitudinal infills.

b) Graphite–epoxy unit cell.
Comparative Axial Stiffnesses ($C_{1111}$)

Constrained Axial Moduli (GPa)

Graphite Volume Fractions

- Voigt Bound
- Reuss Bound
- Lattice composite
- Aligned fiber composite
- Textiled composite
Comparative Transverse Stiffnesses ($C_{2222}$, $C_{3333}$)

Graphite Volume Fractions

Constrained Transverse Moduli (GPa)

Voigt Bound

Reuss Bound

Lattice composite

Textiled composite

Aligned fiber composite
Comparative Shear Stiffnesses ($C_{2323}$)

Graphite Volume Fractions

Constrained Transverse Moduli (GPa)

Voigt Bound

Reuss Bound

Lattice composite

Aligned fiber composite

Textiled composite

Graphite Volume Fractions

0 0.5 1.0
Modeling of Textile–Reinforced Composites

• Individual yarns are actually aligned fiber composites themselves rather than pure glass or graphite.

• Must therefore model the yarns as transversely isotropic.

• Finite deformation effects need exploration.

  • Due to warp of initial warp of yarns we expect:

    (1) increasing stiffness under tensile loadings, as yarns straighten;

    (2) decreasing stiffness under compressive loadings as yarns "buckle";

    (3) pretensioning of yarns might be used to effectively increase all stiffnesses.
Stored Energy Functions

* Isotropic hyperelastic model

\[ W = \lambda (J^2 - 1)/4 - (\lambda/2 + \mu) \ln J + \mu/2(I_1 - 3) \]

* Transversely isotropic hyperelastic model

\[ W = \lambda (J^2 - 1)/4 - (\lambda/2 + \mu) \ln J + C_1(I_1 - 3) + C_2(I_2 - 3) + C_3(\exp(I_4 - 1) - I_4) \]

where \( I_1 = \text{tr} \ C \)

\[ I_2 = 1/2[(\text{tr} \ C)^2 - \text{tr} \ C^2] \]

\[ I_4 = a^0 \cdot C \cdot a^0 \]
Unit cell response

Effective medium response

\[ \langle S_{12} \rangle \text{ Pa} \]

\[ \langle E_{12} \rangle \]

- : Unit cell response
- \( \circ \circ \circ \circ \) : Effective medium response
Conclusion: Assumed form of strain energy function for transversely isotropic composite may be inappropriate.
Other challenges in unit cell modeling of textiles:

- Creation of traditional meshes which capture material arrangements:
  - time consuming (weeks–months of human time)
  - individual elements may have bad aspect ratios

- If textiles are to be analyzed/optimized, need rapid, automated techniques.

- Methods must also be self–adaptive, so that results produced are accurate (not limited by mesh resolution).
NOVEL APPROACH: Voxel–based meshing

• Voxel–based techniques are the basis of continuum topology optimization.

• Used in bio–mechanics to model trabecular bone from CT–scan data.

• Also being used by Nissan Motor Corp. to mesh complex automotive parts.

  → saves human time, but uses more computer time.
BASIC IDEAS OF VOXEL MESHING:

a) Develop a mathematical model to describe spatial location and shapes of objects in a model.

   -> For textiles, yarns are modeled as a sequence of elliptical cylinders.

   -> Based on spatial yarn model, any material point can be determined as either "inside" or "outside" of the yarn.

b) Construct a uniform mesh of volume elements (voxels):

   -> For each element, sample at a finite number of points ($\approx 10^3$) to determine the volume fraction of that element which is inside of a given yarn.

c) Impure voxels are treated with Voigt–Reuss type mixing rules.
YARN APPROXIMATION AS DISCRETE CYLINDER SEQUENCE.

$A(i-1)$

$A(i+1)$

$i$th cylinder

centerline

$a(x,y)=0$

$spheroidal junction$

$magnify$

$A_1$

$A_2$

$A_3$

$A_4$

$A(N+1)$

centerline

$b(x,y)=0$

$cross section$

$P(x_p, y_p, z_p)$
TEXTILE MESHES OF INCREASING REFINEMENT
Textile Models Created with tri–quadratic tetrahedrons.
SUMMARY:

• Moderate success. Many challenges.

• Major challenges in development of efficient and realistic models of textiles:
  
  a) Automated meshing techniques:
    - to capture material arrangements
    - adaptive refinement so that results are not mesh−dependent.
  
  b) Efficient computing (analysis problems are both large and nonlinear)
  
  c) Constitutive modeling:
    - matrix; fibers; yarns; textiles;