Challenges in Design Optimization of Textile−Reinforced Composites

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Background

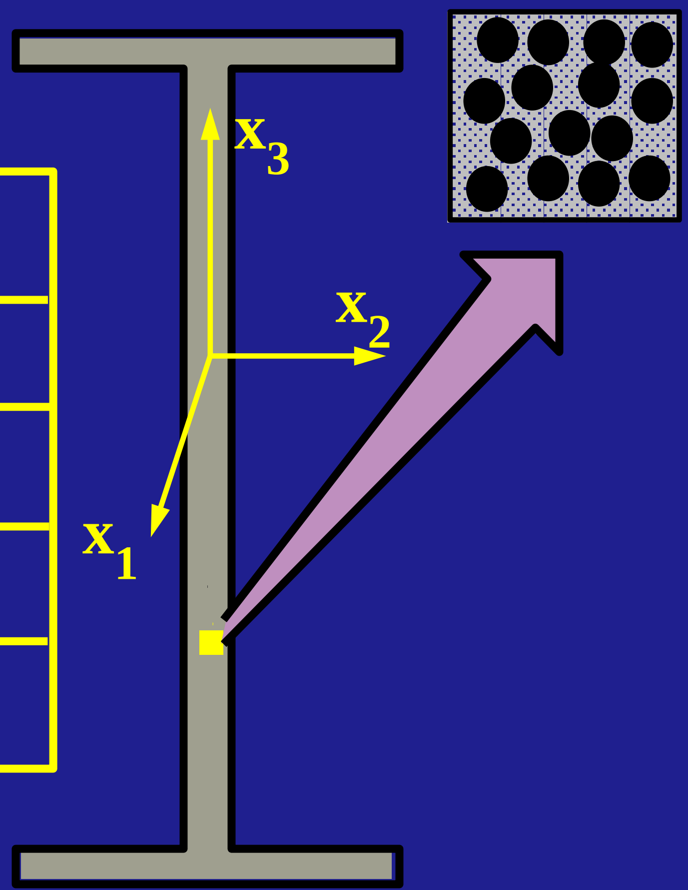
- Structural FRP composites are being considered for usage in civil infrastructure applications.

- Perceived Advantages:
  - lightness
  - durability
  - damping characteristics

- Perceived Disadvantages
  - mechanical performance characteristics

Stiffnesses & Strengths of Aligned Fiber Composites are Highly Anisotropic

Elastic Moduli (GPa)	Glass Epoxy (50/50)	Graphite Epoxy (50/50)	Steel
C_{1111}	38.29	129.0	268.8
C_{2222}, C_{3333}	8.81	10.4	268.8
C_{1212}, C_{1313}	3.32	3.57	76.9
C_{2323}	2.60	2.67	76.9



Research Objectives

- Find arrangements of fibers in composites which improve overall stiffness/strength.
- Explore possibilities systematically using analytical/computational methods.
- Improve analysis methods for unusual composites.

Primary Analysis Tool: Computational Homogenization

- **What is computational homogenization?**

  Using the computer to characterize the macroscopic response of heterogeneous systems.

- **To what types of problems can it be applied?**

  Examples:
  Characterization of existing composites
  Design of new composites
  Studies of Bio−tissues (bone, muscle, etc.)

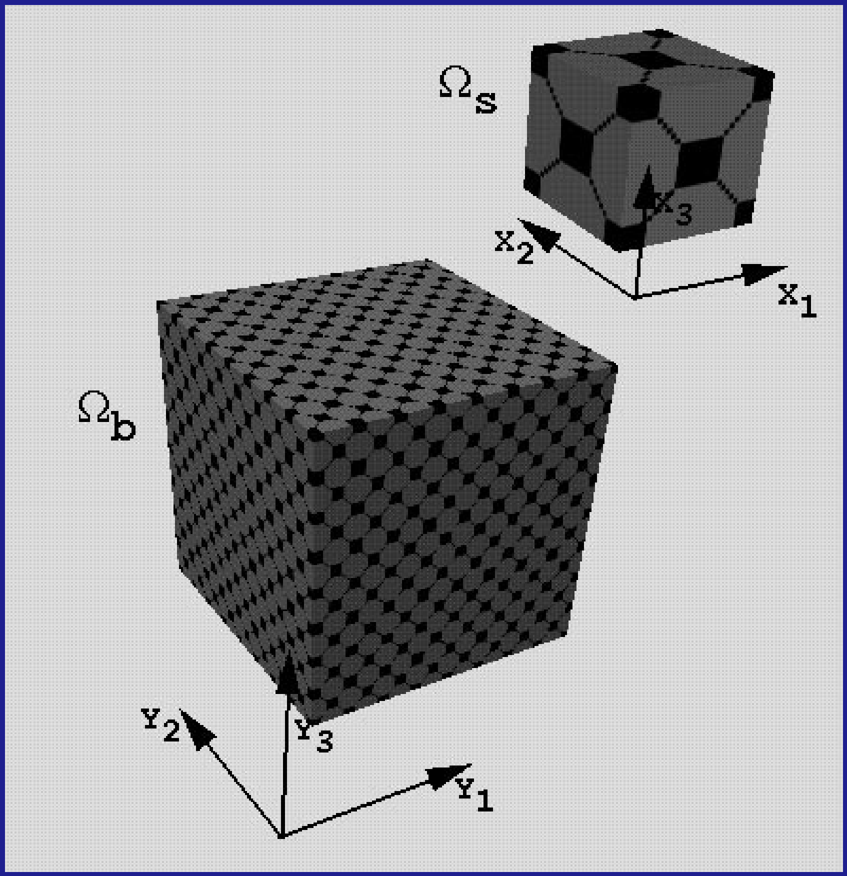
- **Strengths of the method**

  Fairly general, although requires periodic or quasi−periodic material structures.

Domain for Homogenization Computations

In general, an RVE. domain.

For *periodic* composites, domain is the unit cell.



Small Deformation Decompositions/Notation:

Micro−Stress

$\sigma(X) = S + \sigma^*(X)$

Macro−Stress

$S = <\sigma>$

Micro−Strain

$\varepsilon(X) = E + \varepsilon^*(X)$

Macro−Strain

$E = <\varepsilon>$

σ^*, ε^* are *inhomogeneous* contributions.

$<\sigma^*> = 0; \quad <\varepsilon^*> = 0;$

$u(X) = E \bullet X + u^*_{per}(X)$

$u(X)$ is the *total* displacement field.
EX is the *homogeneous* contribution.
u^*_{per} is the *inhomogeneous* contribution.

General Decompositions/Notation:

Micro−Stress (PK−II)

$\sigma(X) = S + \sigma^*(X)$

Macro−Stress (PK−II)

$S = < \sigma >$

Micro−Deformation

$F(X) = I + \partial u/\partial X$

Macro−Deformation

$\Phi = < F >$
$= RU = U$

Local Strain (Green)

$E = 1/2[F^T F - I]$

Macro−Strain (Green)

$E = 1/2[\Phi^T \Phi - I]$

$$u(X) = (\Phi - I) \bullet X + u^*_{per}(X)$$

$u(X)$ is the *total* displacement field.
$(\Phi - I) \bullet X$ is the *homogeneous* contribution.
u^*_{per} is the *inhomogeneous* contribution.

Procedure for Strain−Controlled Homogenization:

Impose a *homogeneous* displacement field:
$\mathbf{u} = \mathbf{E} \bullet \mathbf{X}$ or $\mathbf{u} = (\Phi - \mathbf{I}) \bullet \mathbf{X}$ on Ω_s.

Solve a variational problem for the *inhomogeneous* field $\mathbf{u}^*_{per}$.

Variational Equilibrium Statement:

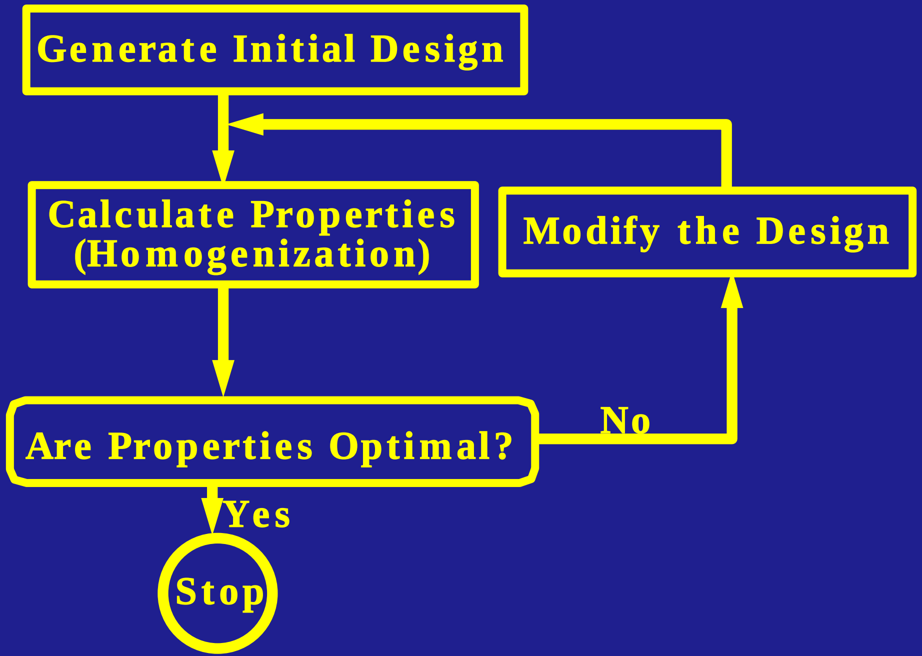
$$\int_{\Omega_s} (\nabla \cdot \boldsymbol{\sigma}) \cdot \delta \mathbf{u} \, d\Omega = 0$$

Weak Form Solved:

$$\int_{\Omega_s} \boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\epsilon}^*(\delta \mathbf{u}) d\Omega = \mathbf{0}.$$

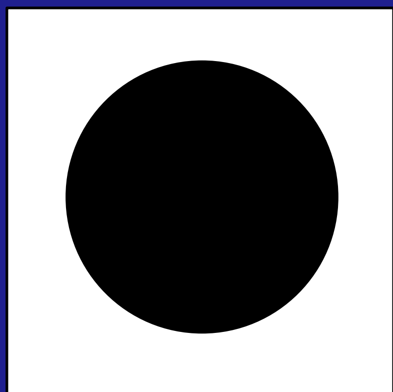
B. Material Topology Optimization

- Optimize material arrangements to enhance mechanical performance.
- Properties associated with each arrangement are calculated using homogenization.

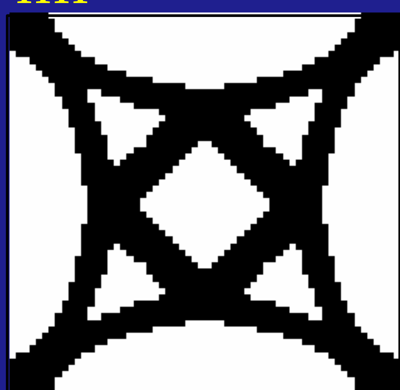


Results of Material Topology Optimization

40% graphite
60% epoxy

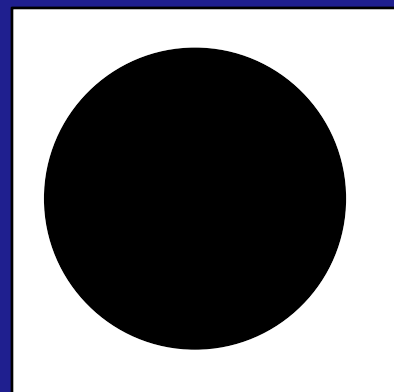


C_{2323} = 2.09GPa
C_{2222}, C_{3333} = 7.96GPa
C_{1111} = 104GPa



C_{2323} = 28.5GPa
C_{2222}, C_{333} = 39.5GPa
C_{1111} = 109GPa

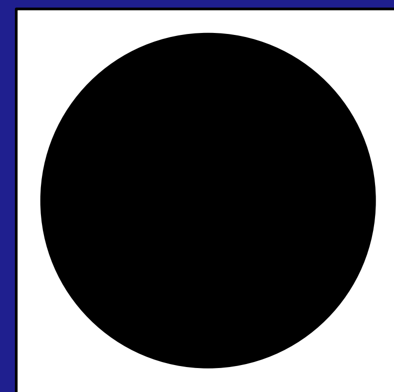
50% graphite
50% epoxy



C_{2323} = 2.67GPa
C_{2222}, C_{3333} = 10.4GPa
C_{1111} = 129GPa

C_{2323} = 35.2GPa
C_{2222}, C_{3333} = 48.2GPa
C_{1111} = 135GPa

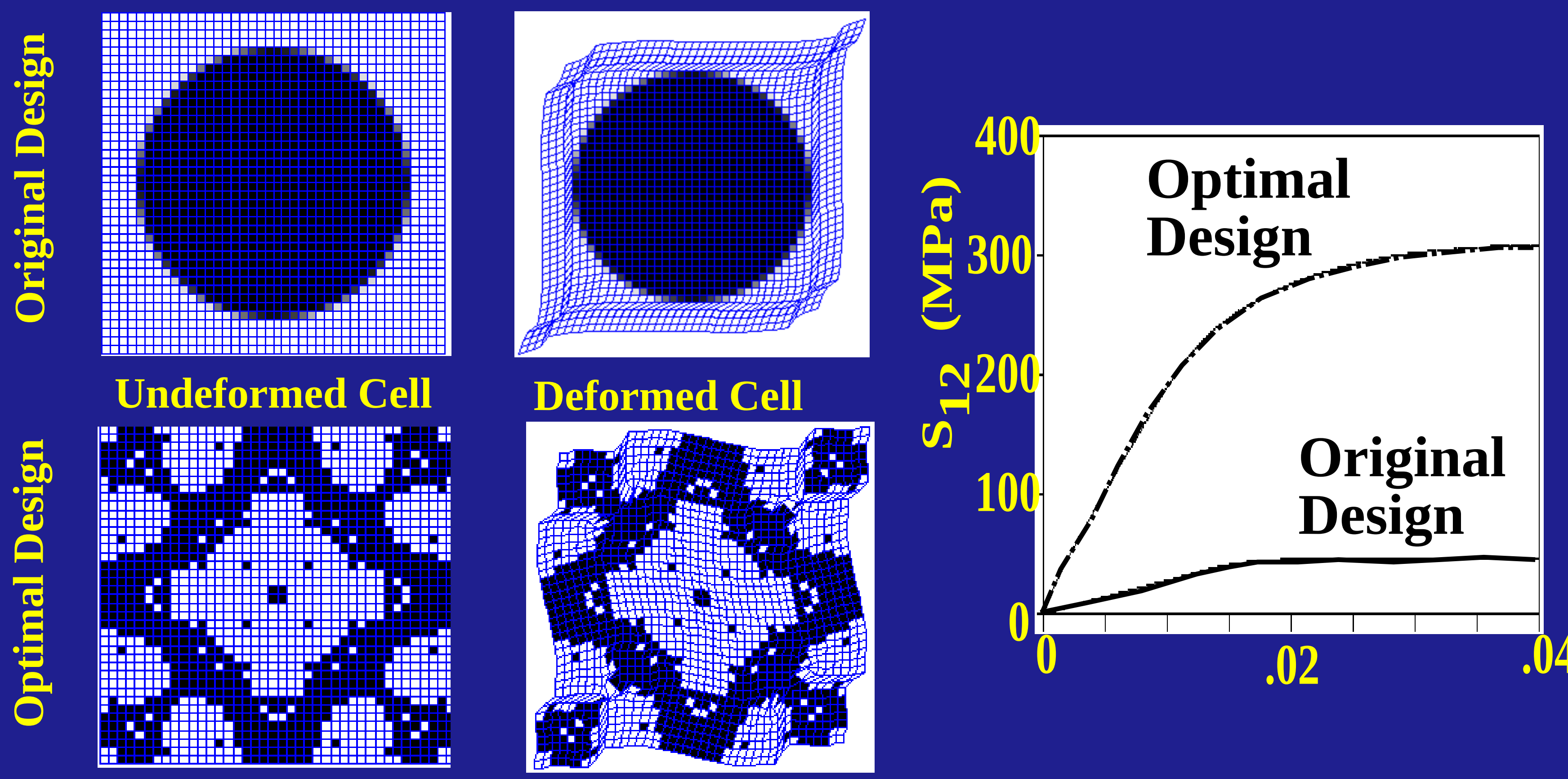
60% graphite
40% epoxy



C_{2323} = 3.60GPa
C_{2222}, C_{3333} = 15.1GPa
C_{1111} = 155GPa

C_{2323} = 47.30GPa
C_{2222}, C_{3333} = 76.9GPa
C_{1111} = 163GPa

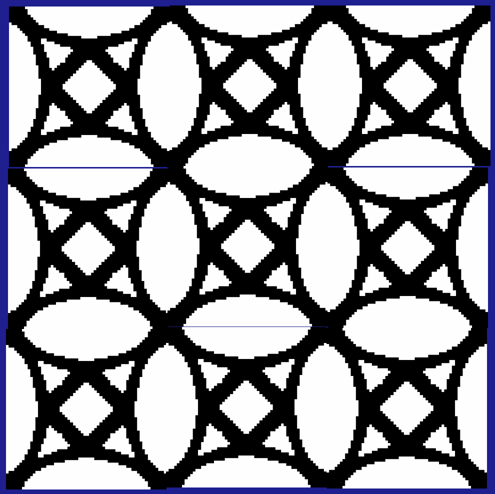
Example: Elasto−plastic Compliance Minimization of a Boron−Epoxy Composite



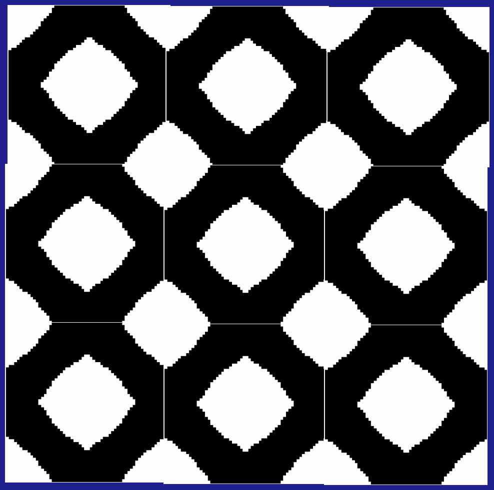
Significance of Results

- Demonstrate necessity of getting fiber material to perform <u>multi−axially.</u>
- Demonstrate advantages of integration & continuity of fiber material in three orthogonal directions.
- Some material arrangements are fairly complex, and others are much simpler (more manufacturable).

Complex Arrangement



Simpler Arrangement

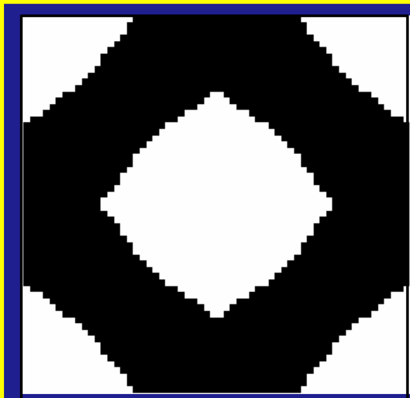


Manufacturability Concerns

- **Re−designed composites contain continuous, monolithic, glass or graphite phases.**
  - → *LCVD for small scale parts/structures*
  - → *Infeasible for large scale structural composites*

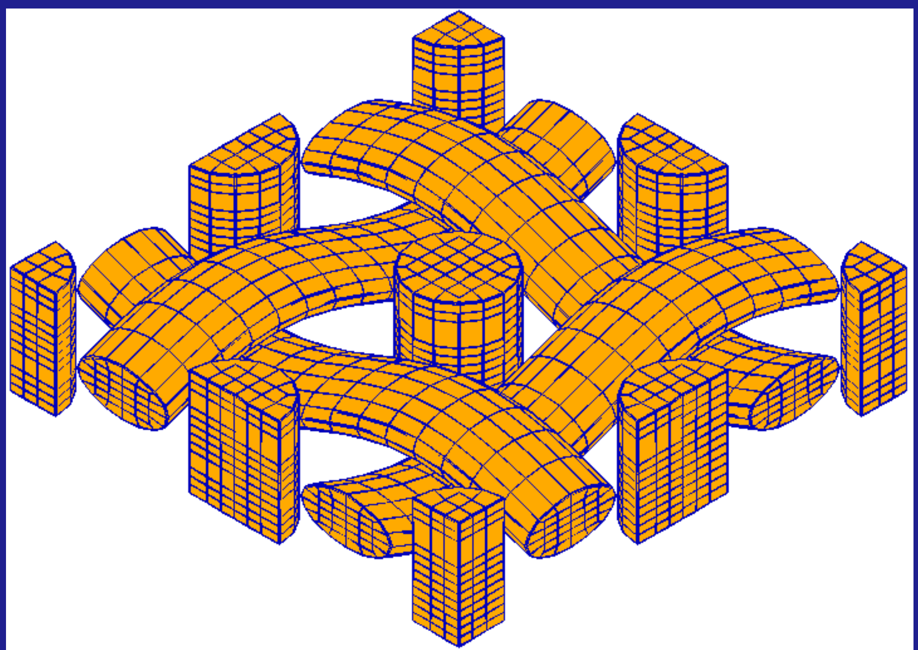
- **Current trend is toward textile reinforcing**
  - → *Gives 3−D reinforcing (weaker anisotropy)*
  - → *Capabilities for producing 3−d weaves & meshes are developing rapidly*

- **Designed material arrangements are therefore approximated as textiles and re−analyzed.**

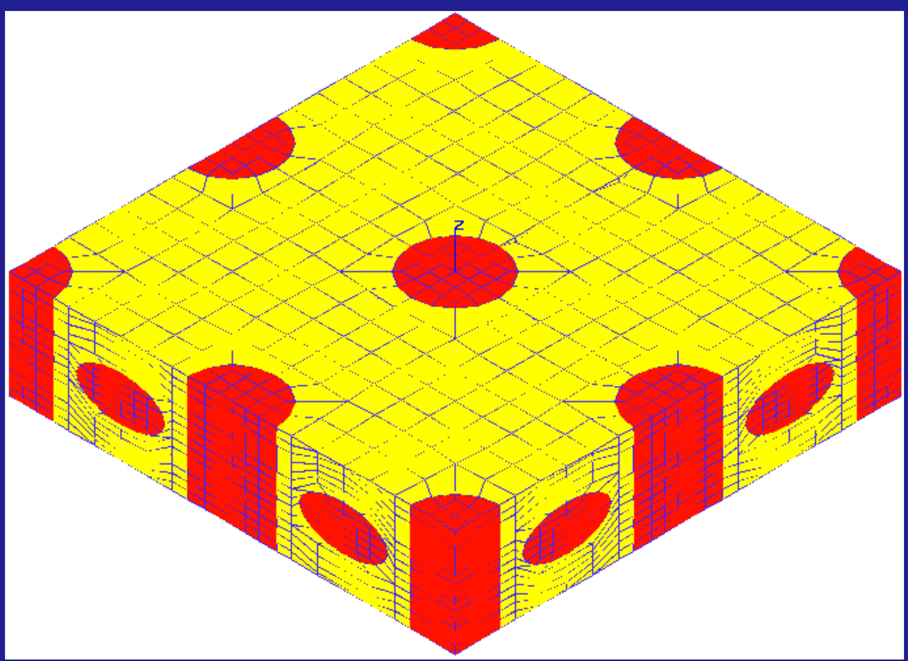


Desired Material Arrangement (unit cell)

Textile Composite Approximation

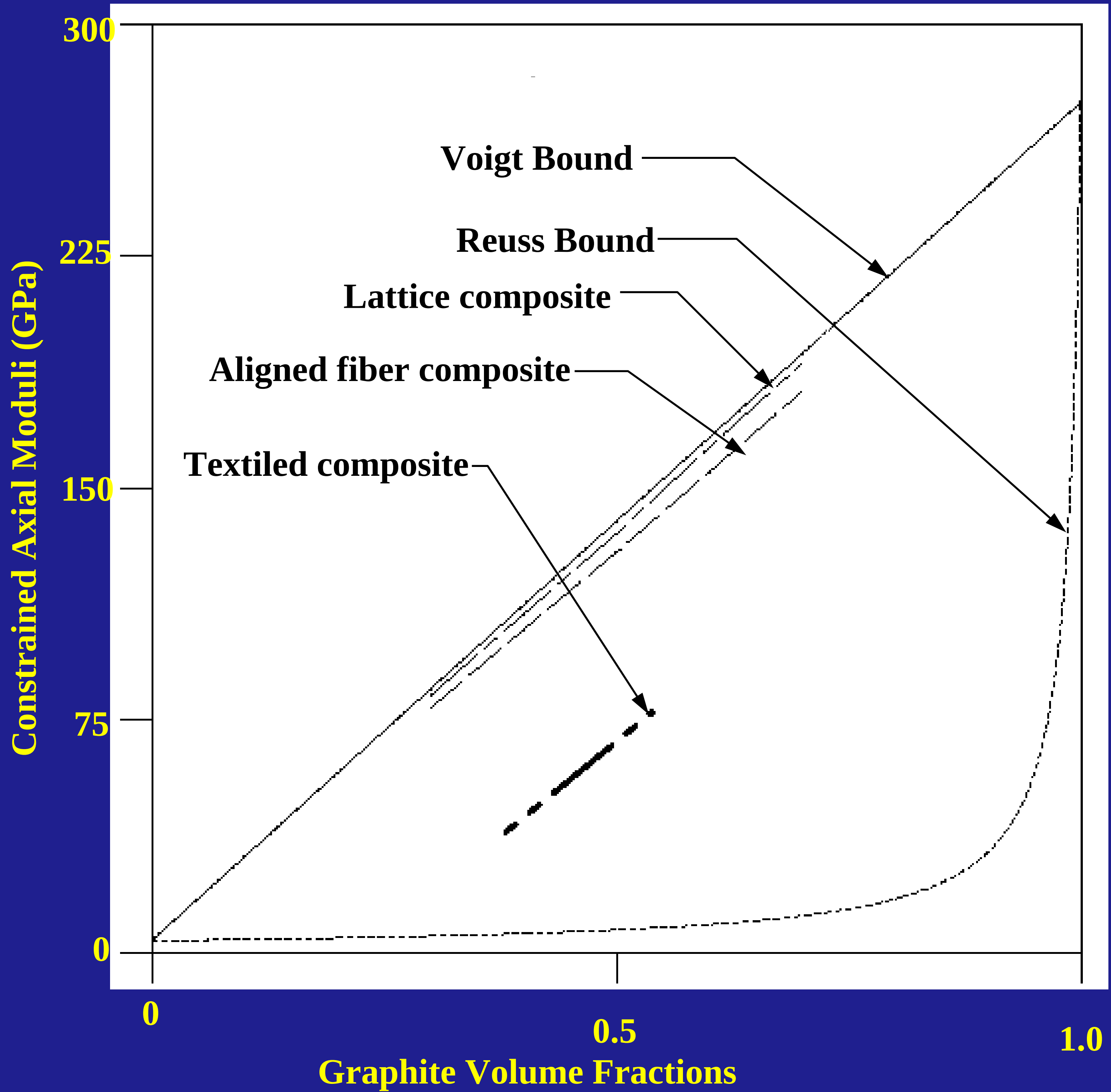


a) Graphite plane weave with longitudinal infills.

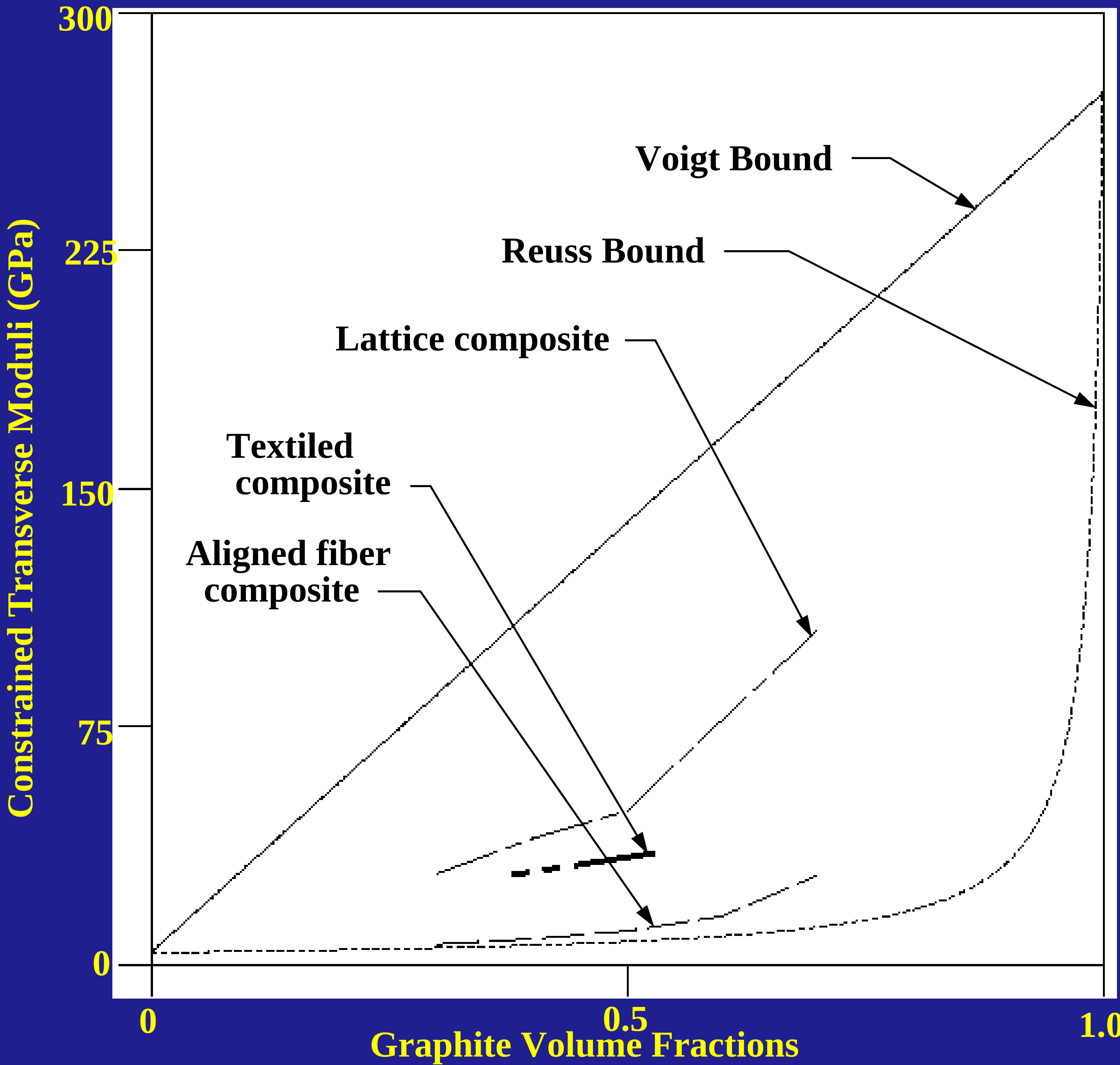


b) Graphite−epoxy unit cell.

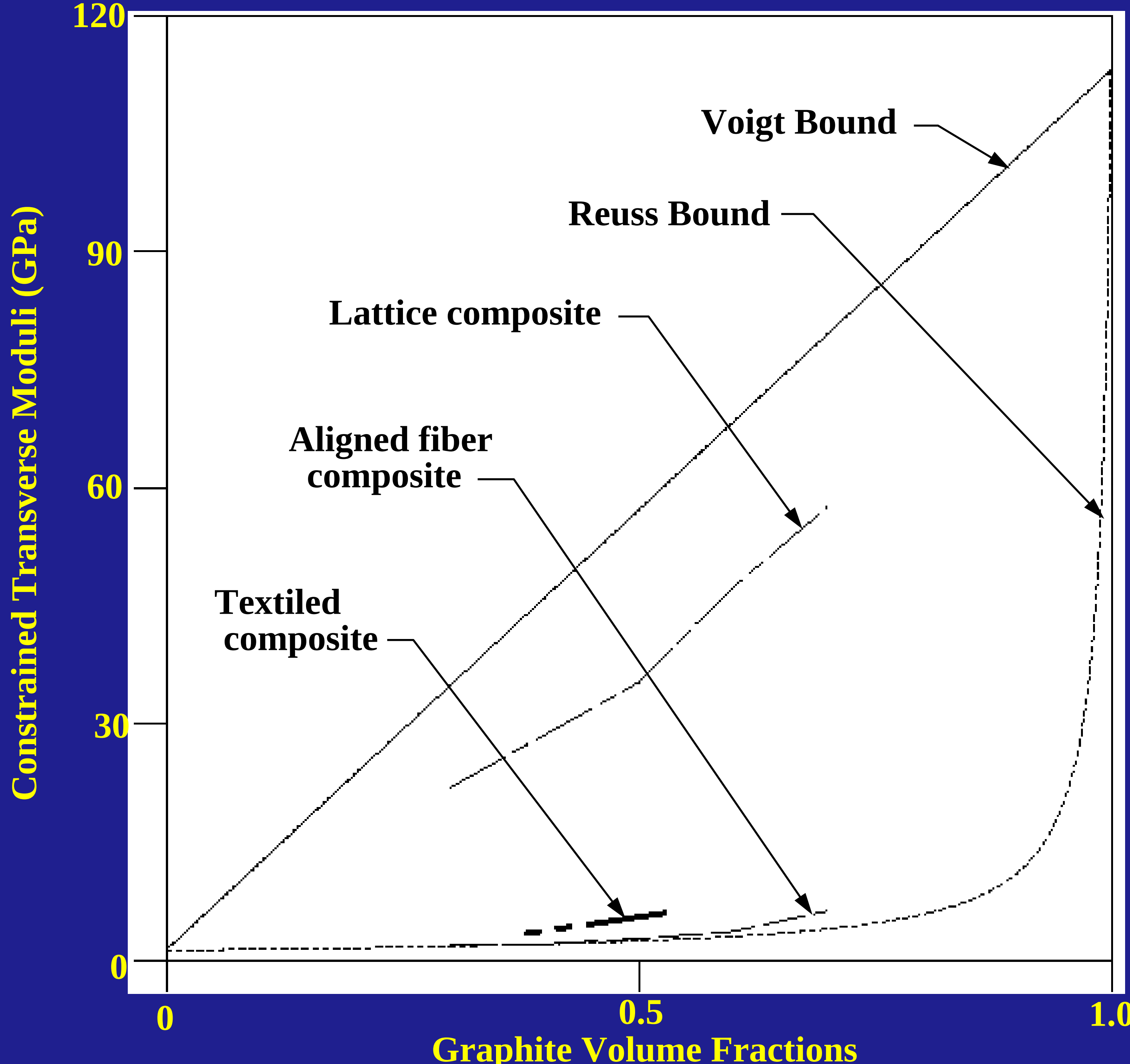
Comparative Axial Stiffnesses (C_{1111})



Comparative Transverse Stiffnesses (C_{2222}, C_{3333})



Comparative Shear Stiffnesses (C_{2323})



Modeling of Textile−Reinforced Composites

- **Individual yarns are actually aligned fiber composites themselves rather than pure glass or graphite.**
- **Must therefore model the yarns as transversely isotropic.**
- **Finite deformation effects need exploration.**
  - **Due to warp of initial warp of yarns we expect:**

(1) *increasing* stiffness under tensile loadings, as yarns straighten;

(2) *decreasing* stiffness under compressive loadings as yarns "buckle";

(3) pretensioning of yarns might be used to effectively increase all stiffnesses.

Stored Energy Functions

* Isotropic hyperelastic model

$$W=\lambda(J^2-1)/4-(\lambda/2+\mu)\ln J+\mu/2(I_1-3)$$

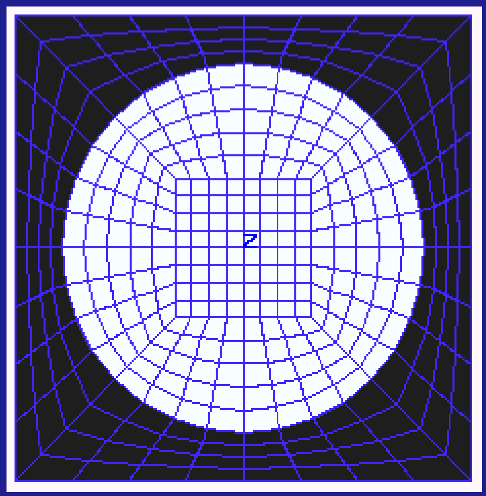
* Transversely isotropic hyperelastic model

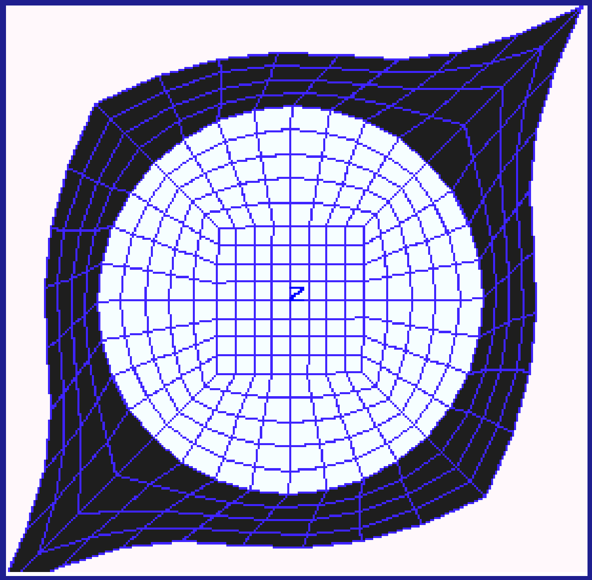
$$W=\lambda(J^2-1)/4-(\lambda/2+\mu)\ln J+C_1(I_1-3)+C_2(I_2-3)+C_3(\exp(I_4-1)-I_4)$$

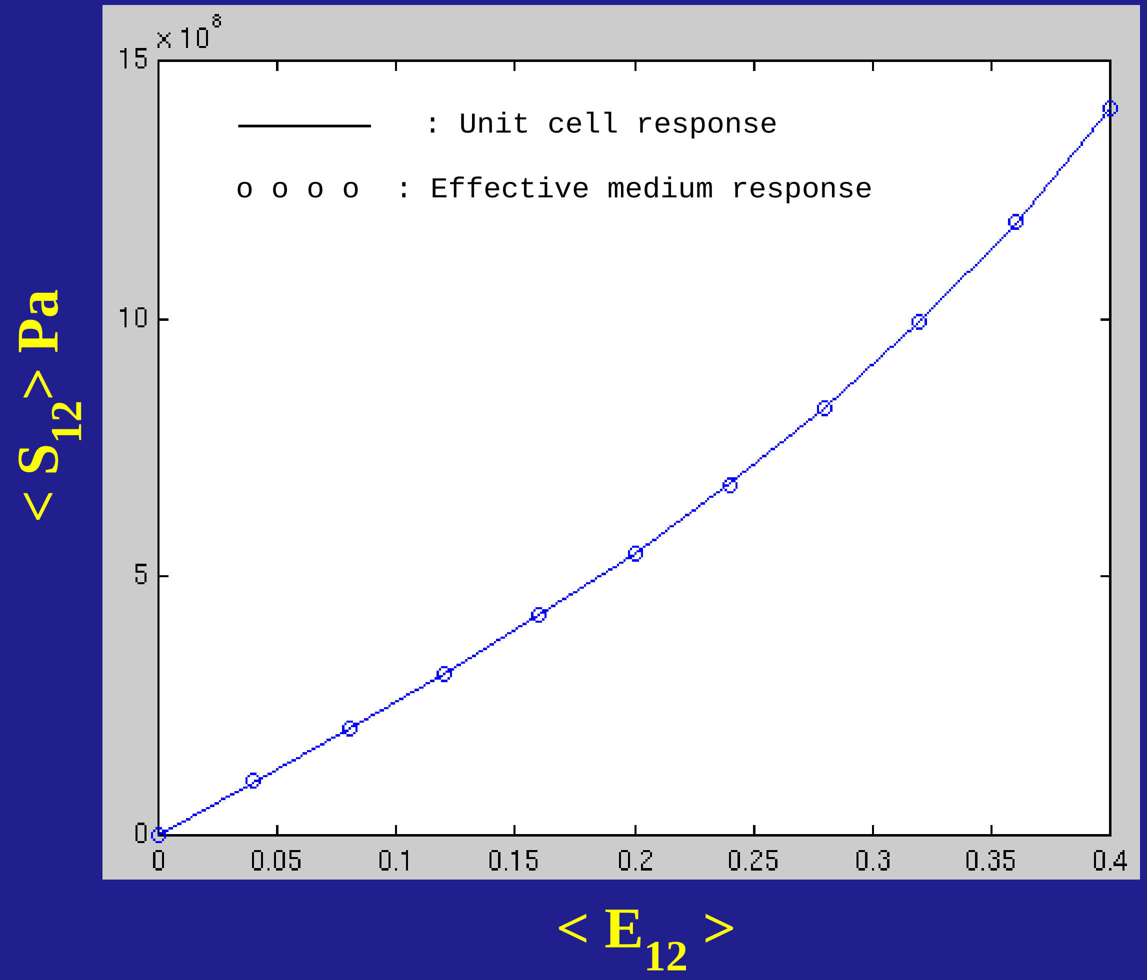
where $I_1=\mathrm{tr}\,C$

$I_2=1/2[(\mathrm{tr}\,C)^2-\mathrm{tr}\,C^2]$

$I_4=a^0\cdot C\cdot a^0$

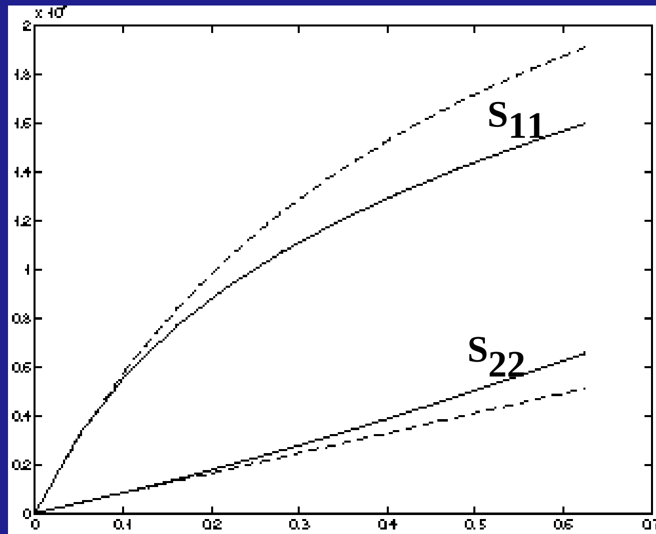






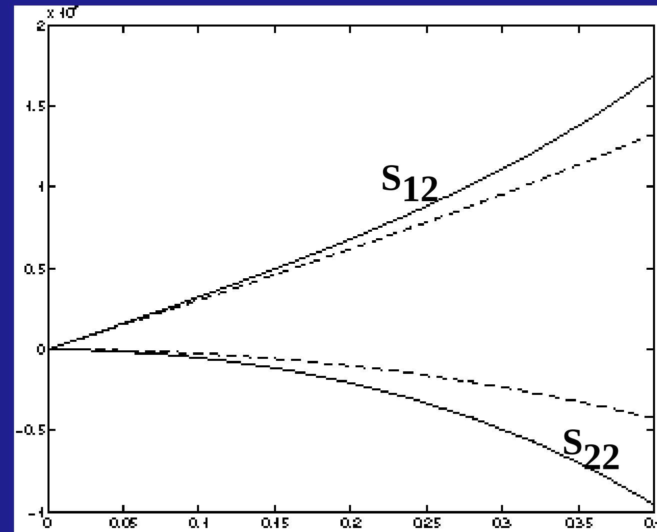
x 10^8
15
10
5
0
: Unit cell response
o o o o : Effective medium response
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
$< S_{12} >$ Pa
$< E_{12} >$

S_{11} & S_{22}



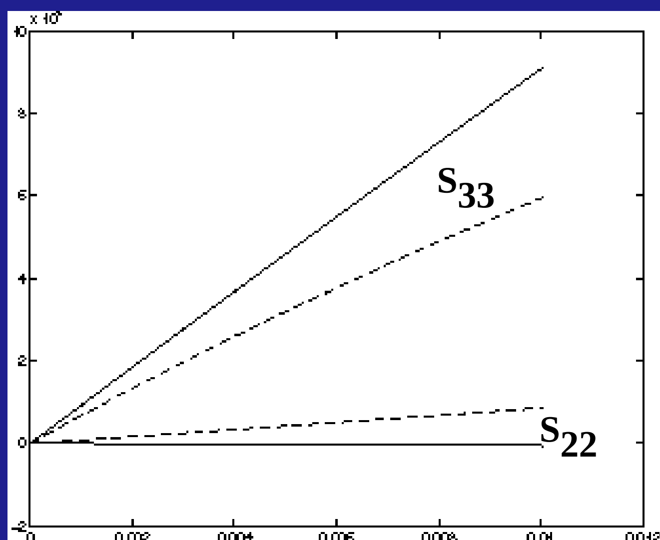
E11

S_{12} & S_{22}



E_{12}

S_{33} & S_{22}



E33

.... :transversely isotropic response

——— :unit cell response

<u>Conclusion:</u> Assumed form of strain energy function for transversely isotropic composite may be inappropriate.

Other challenges in unit cell modeling of textiles:

- Creation of traditional meshes which capture material arrangements:

  −> time consuming (weeks−months of human time)
  −> individual elements may have bad aspect ratios

- If textiles are to be analyzed/optimized, need rapid, automated techniques.

- Methods must also be self−adaptive, so that results produced are accurate (not limited by mesh resolution).

NOVEL APPROACH: Voxel−based meshing

- Voxel−based techniques are the basis of continuum topology optimization.
- Used in bio−mechanics to model trabecular bone from CT−scan data.
- Also being used by Nissan Motor Corp. to mesh complex automotive parts.

−> saves human time, but uses more computer time.

BASIC IDEAS OF VOXEL MESHING:

a) Develop a mathematical model to describe spatial location and shapes of objects in a model.

−> For textiles, yarns are modeled as a sequence of elliptical cylinders.

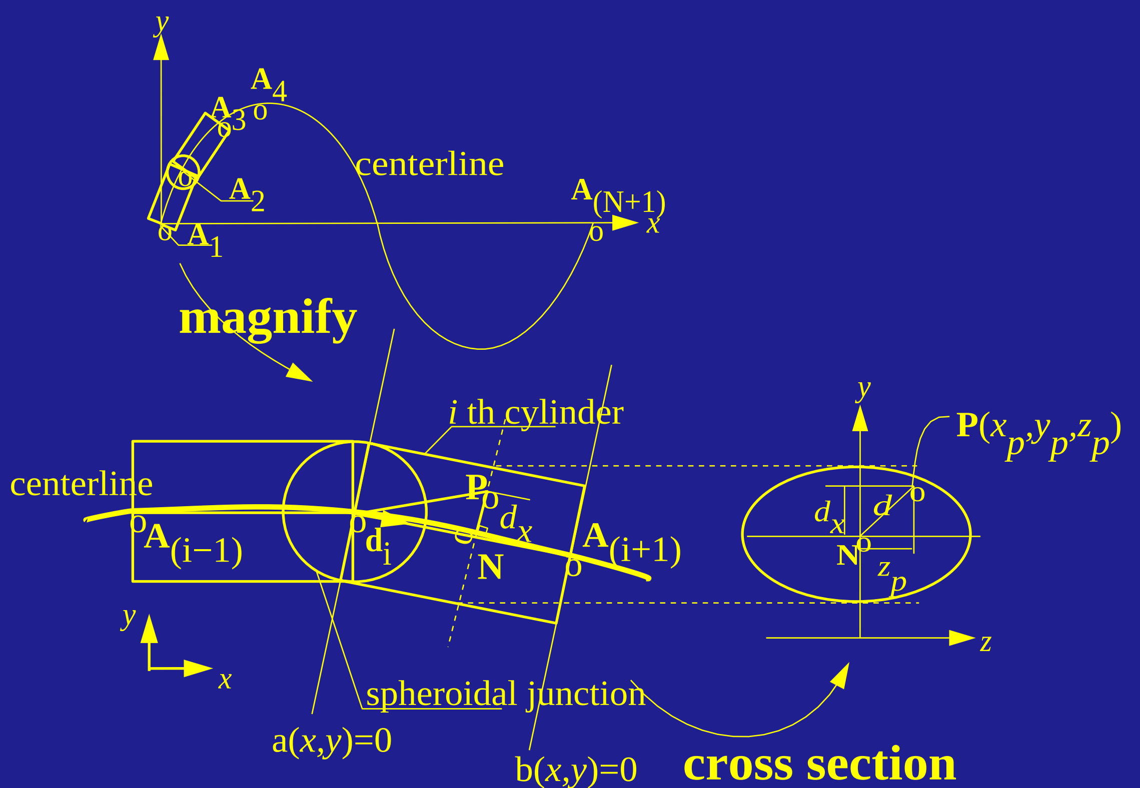
−> Based on spatial yarn model, any material point can be determined as either "inside" or "outside" of the yarn.

b) Construct a uniform mesh of volume elements (voxels):

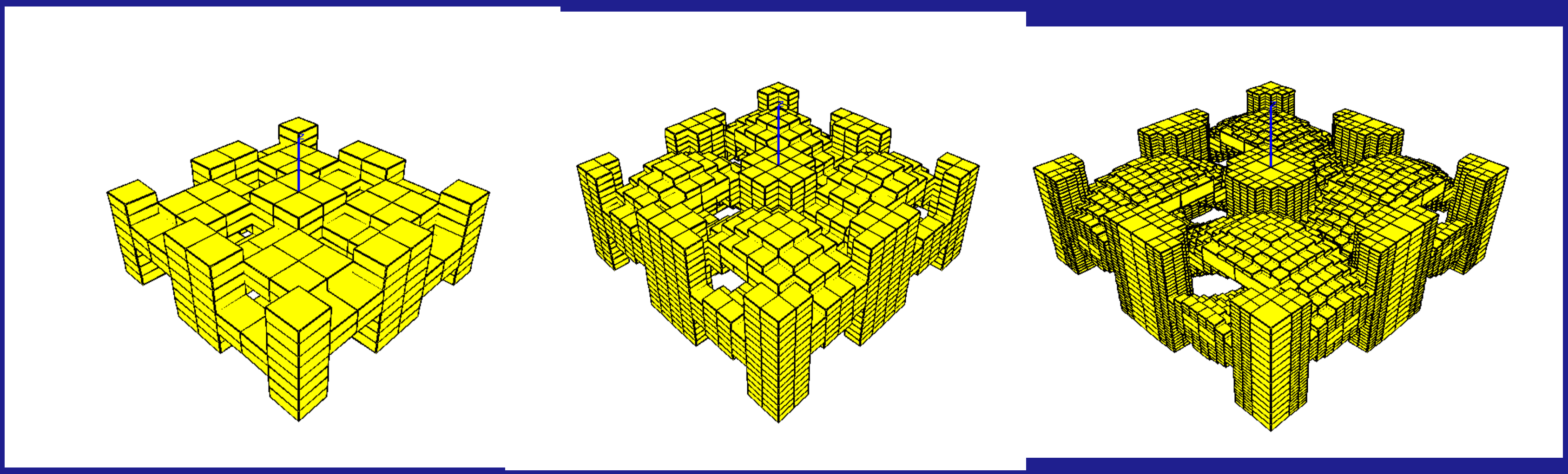
−> For each element, sample at a finite number of points ($\approx 10^3$) to determine the volume fraction of that element which is inside of a given yarn.

c) Impure voxels are treated with Voigt−Reuss type mixing rules.

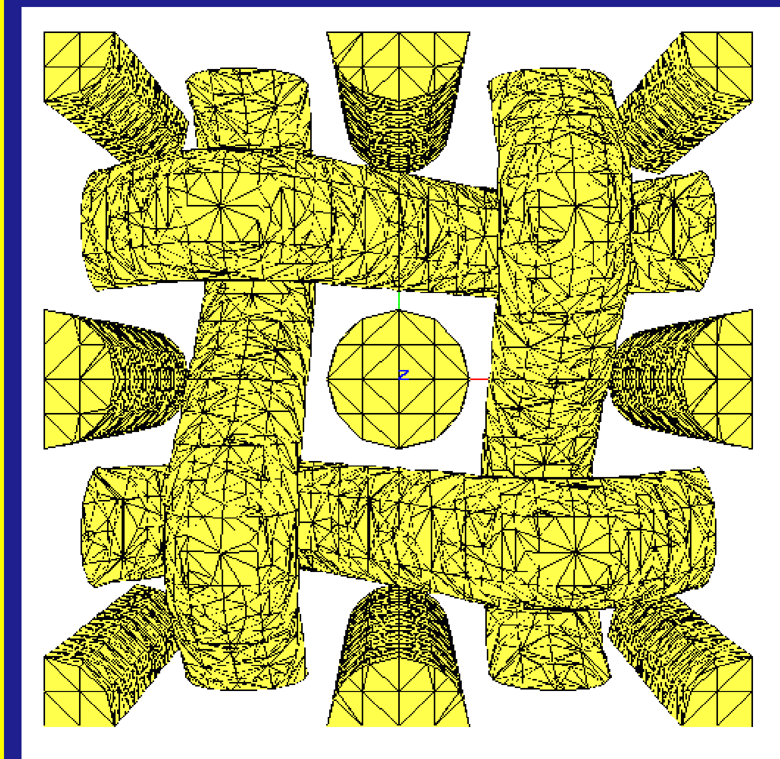
YARN APPROXIMATION AS DISCRETE CYLINDER SEQUENCE.

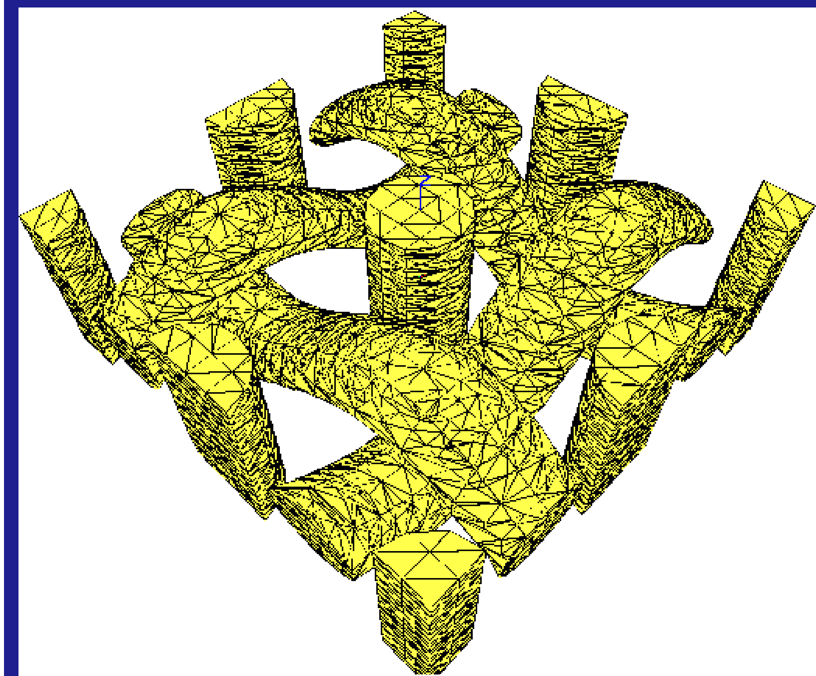


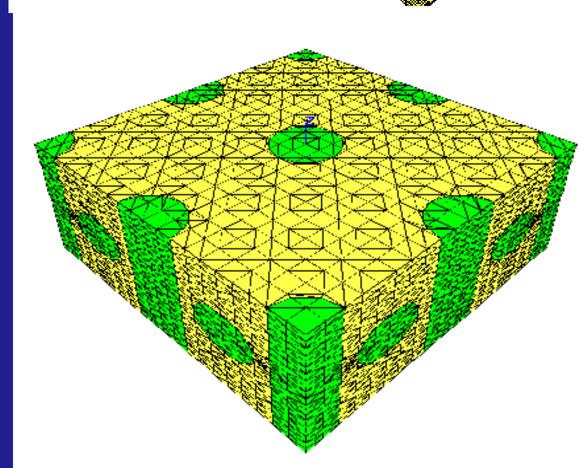
TEXTILE MESHES OF INCREASING REFINEMENT



Textile Models Created with tri−quadratic tetrahedrons.







SUMMARY:

- Moderate success. Many challenges.

- Major challenges in development of efficient and realistic models of textiles:

    a) Automated meshing techniques:

    - to capture material arrangements
    - adaptive refinement so that results are not mesh−dependent.

    b) Efficient computing (analysis problems are both large and nonlinear)

    c) Constitutive modeling: matrix; fibers; yarns; textiles;