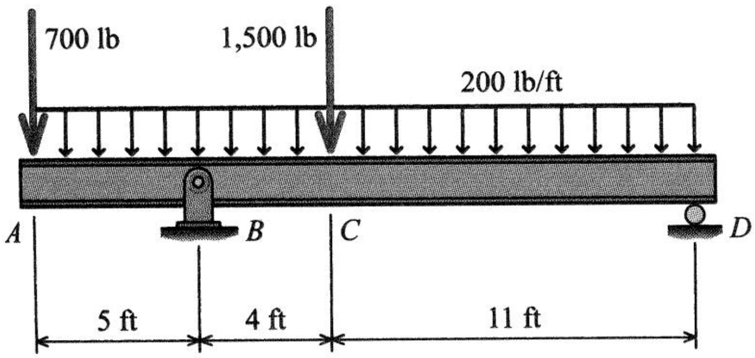
The University of Iowa
57:019:BBB Intro. To Mechanics of Deformable Bodies
Spring Semester 2011
Quiz #3

<u>Problem 1 (35 pts)</u>:

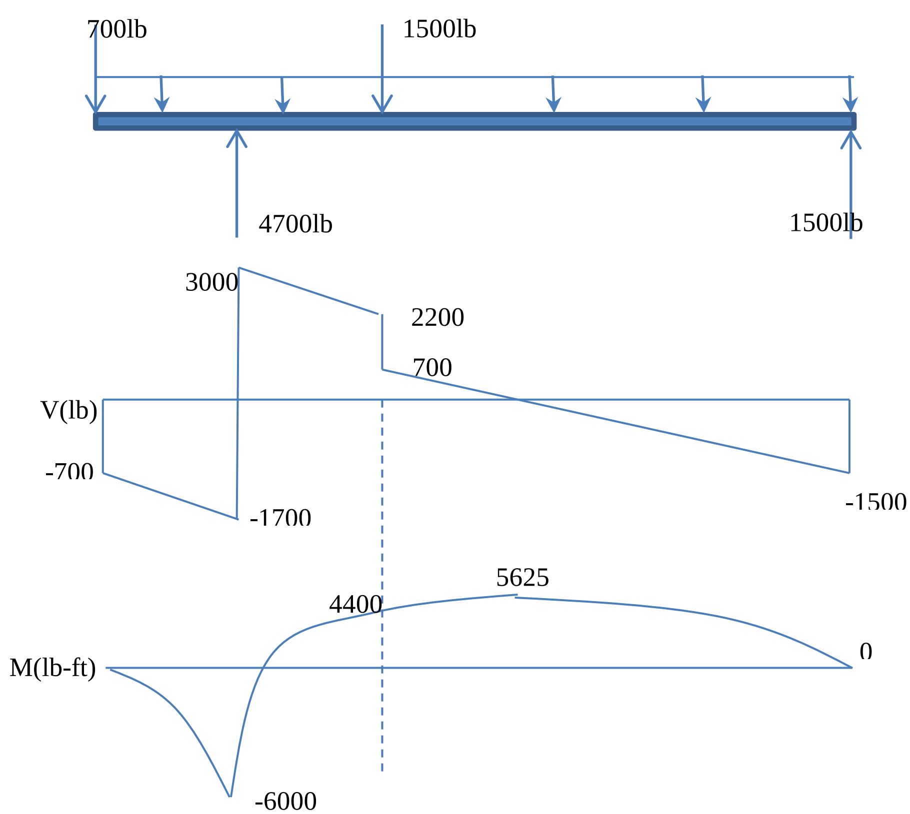


For the beam shown;

a) calculate the support reactions at B and D;
b) Calculate the distribution of shear V in the beam and illustrate with a diagram that shows magnitudes;
c) Calculate the bending moment distribution M in the beam and illustrate with a diagram that shows magnitudes.

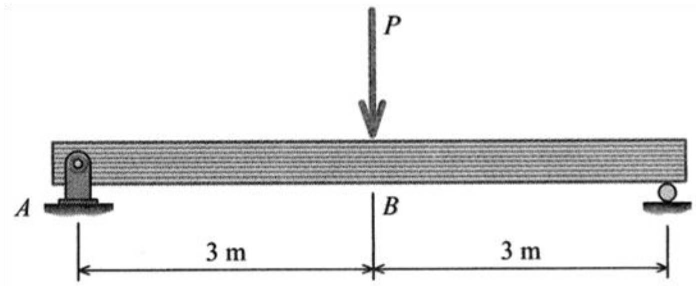
<u>Solution:</u>

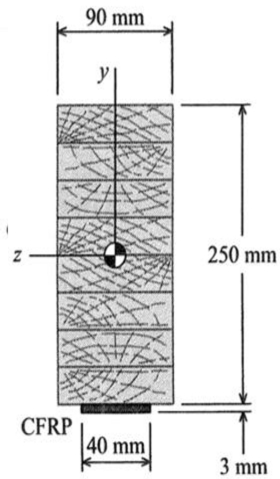
- From the equations of static equilibrium you can find that: $R_B = 4700lb$, and $R_D = 1500lb$



Problem 2 (30 pts):

A glue-laminated timber beam is reinforced by carbon fiber reinforced plastic (CFRP) bonded to its bottom surface. The elastic Young's modulus of the wood is 13 GPa and that of the CFRP is 130 GPa. The simply supported beam spans 6m and carries a concentrated load of P=4kN.
Determine the maximum bending stresses in both the timber and the CFRP.





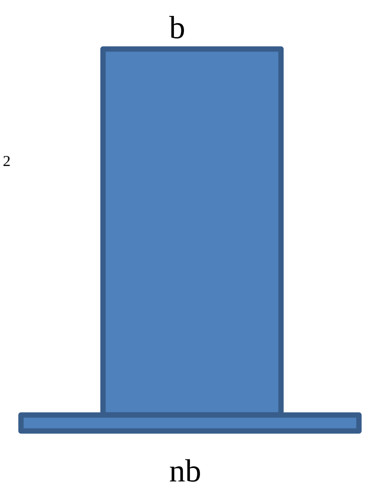
Solution:

- The maximum bending moment in the beam has a magnitude of $M_{\max} = PL/4 = 6kN \cdot m$ and occurs at the center of the beam.

- Transform the section into all wood. Transformation factor $n = E_{CFRP} / E_{wood} = 10$

- From bottom of the cross-section:

$$\bar{y} = \frac{(400)(3)(1.5) + (90)(250)(128)}{(400)(3) + (90)(250)} = 121.6mm$$



- $$I = \frac{(90)(250)^3}{12} + (90)(250)(128-121.6)^2 + \frac{(400)(3)^3}{12} + (1200)(121.6-1.5)^2$$
  $$= 1.354 \cdot 10^8 mm^4 = 1.354 \cdot 10^{-4} m^4$$

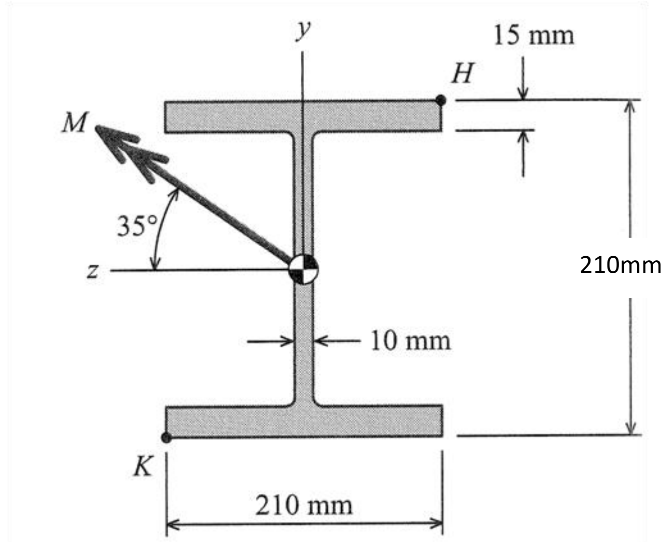
- $$(\sigma_{\max})_{wood} = \frac{Mc_{wood}}{I} = \frac{(6kN \cdot m)(.253m - .1216m)}{1.354 \cdot 10^{-4} m^4}$$
  $$\boxed{(\sigma_{\max})_{wood} = 5.82MPa}$$

- $$(\sigma_{\max})_{CFRP} = \frac{nMc_{CFRP}}{I} = \frac{(10)(6kN \cdot m)(.1216m)}{1.354 \cdot 10^{-4} m^4}$$
  $$\boxed{(\sigma_{\max})_{CFRP} = 53.8MPa}$$

Problem 3 (35 pts):

The moment acting on the cross-section of the wide flange beam has a magnitude of M=12kN-m and is oriented as shown. Determine:

a) The bending stress at point H;
b) The bending stress at point K; and
c) The orientation of the neutral axis relative to the z-axis. Show its location on a sketch of the cross-section.



Solution:

$$M_y = 12kN \cdot m \sin 35^\circ = 6.88kN \cdot m$$

$$M_z = 12kN \cdot m \cos 35^\circ = 9.83kN \cdot m$$

$$I_{yy} = \frac{(30)(210)^3}{12} + \frac{(180)(10)^3}{12} = 2.317 \cdot 10^{-5} m^4$$

$$I_{zz} = \frac{(210)(210)^3}{12} - \frac{(200)(180)^3}{12} = 6.49 \cdot 10^{-5} m^4$$

$$\sigma(y,z) = \frac{M_y z}{I_{yy}} - \frac{M_z y}{I_{zz}}$$

at H, y=0.105m; z=-0.105m

$$\sigma_H = \frac{6.88kN \cdot m(-0.105m)}{2.317 \cdot 10^{-5} m^4} - \frac{(9.83kN \cdot m)(0.105m)}{6.49 \cdot 10^{-5} m^4}$$

$$\boxed{\sigma_H = -47.1MPa}$$

at K, y=-0.105m; z=0.105m

$$\sigma_K = \frac{6.88kN \cdot m(+0.105m)}{2.317 \cdot 10^{-5} m^4} - \frac{(9.83kN \cdot m)(-0.105m)}{6.49 \cdot 10^{-5} m^4}$$

$$\boxed{\sigma_K = 47.1MPa}$$

Along the NA

$$\sigma(y,z) = 0 = \frac{M_y z}{I_{yy}} - \frac{M_z y}{I_{zz}}$$

$$z = \left(\frac{M_z}{M_y} \frac{I_{yy}}{I_{zz}} \right) y = \left(\frac{9.83}{6.88} \frac{2.317}{6.49} \right) = 0.51y$$

, $\boxed{z = 0.51y}$

$$\alpha = \tan^{-1}(.51) = 27^\circ$$

$$\beta = 90^\circ - \alpha = 63^\circ$$