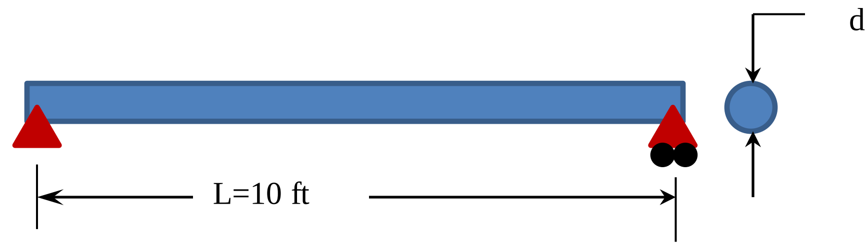
The University of Iowa
57:019:BBB Intro. To Mechanics of Deformable Bodies
Spring Semester 2011
Quiz #3 Makeup Solution

<u>Problem 1 (33 pts)</u>:

A solid steel rod of diameter *d* is simply supported as shown. The unit weight for steel is $\gamma = 490 lbs \cdot ft^{-3}$. Determine the required diameter *d* of the rod so that the bending stress in the beam does not exceed 4 ksi. **Hint:** treat the weight of the beam as a uniformly distributed load $w = \gamma A$ where A is the cross-sectional area of the beam. Recall that for a circular cross-section, $I = J/2$.



Solution:

Let the uniform load acting on the beam be w=γA, where A is the cross-sectional area of the beam, and γ is the unit weight. Here, $w = \frac{490 lbs}{ft^3} \cdot \pi c^2$

For a simply supported and uniformly loaded beam the end reactions will have magnitude wL/2. The maximum bending moment occurs at mid-span and has magnitude:

$$M_{\max} = \frac{wL^2}{8} = 490\pi c^2 * \frac{(10')^2}{8} = 19,242 \frac{lbs}{ft} c^2$$

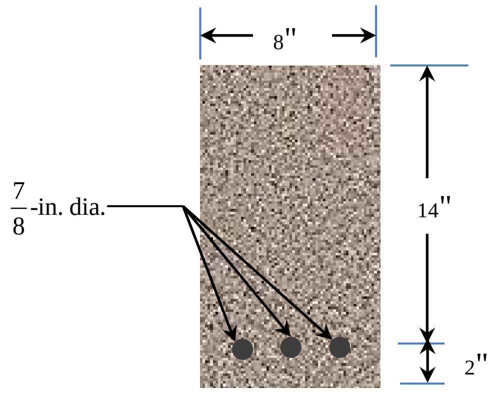
At the location of maximum bending moment, the maximum bending stress is given by the flexure formula as follows: $\sigma_{\max} = \frac{Mc}{I} = \left(19,242 \frac{lbs}{ft} c^2\right) * \frac{c}{\frac{\pi}{4} c^4} = 4ksi = 576ksf$

Solving for c yields: $c = 0.0425' = 0.514"$

There $\boxed{d_{reqd} = 2c = 1.03"}$

Problem 2 (33 pts):

A concrete beam is reinforced by three steel rods placed as shown. For the concrete E=3,000ksi, and for the steel E=29,000ksi. If the allowable stress in the concrete is 1.35ksi and that in the steel is 20ksi, what is the largest positive bending moment that can be applied to the beam section?



Solution:

$$n=\frac{E_{st}}{E_c}=\frac{29}{3}=9.67$$

$$A_{st}=3*\frac{\pi}{4}*\left(\frac{7"}{8}\right)^2=1.804in^2$$

$$nA_{st}=17.44in^2$$

To locate the NA, moment of area above should be equal to transformed steel area below:

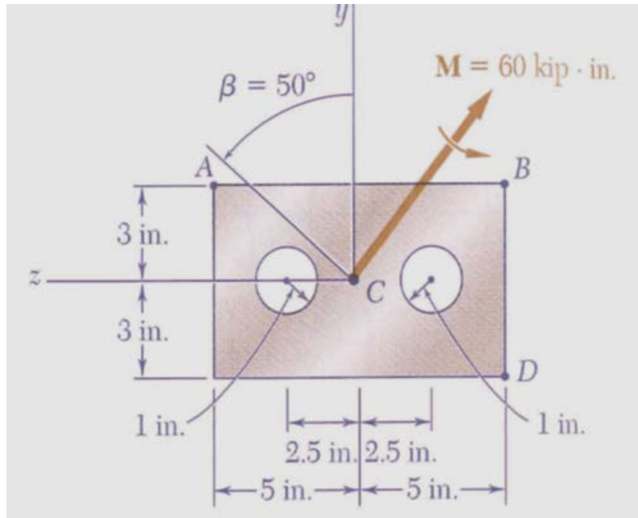
$$nA_{st}\left(d-h'\right)=\frac{bh'^2}{2}\Rightarrow h'=5.93"$$

$$I^*=\frac{b\left(h'\right)^3}{12}+\frac{b\left(h'\right)^3}{4}+nA_{st}\left(d-h'\right)^2=1692in^4$$

$$\left(\sigma_{all}\right)_{conc}=1.35ksi=\frac{\left(M_{allow}\right)_{conc}h'}{I^*}\Rightarrow\left(M_{allow}\right)_{conc}=\frac{\left(\sigma_{all}\right)_{conc}I^*}{h'}=385k\cdot in$$

$$\left(\sigma_{all}\right)_{st}=20ksi=\frac{n\left(M_{allow}\right)_{st}\left(d-h'\right)}{I^*}\Rightarrow\left(M_{allow}\right)_{st}=\frac{\left(\sigma_{all}\right)_{st}I^*}{n\left(d-h'\right)}=434k\cdot in$$

$$M_{allowable}=\min\{385,434\}=385k\cdot in$$



Problem 3 (34 pts):

The moment of 60 kip-in acts on the cross-section shown and makes an angle of 40° with respect to the section's y-axis. Calculate:

a) I_{zz} and I_{yy}
b) The bending stress at point A;
c) The bending stress at point D;
d) The orientation of the neutral axis on the cross-section. (illustrate with a sketch)

Solution:

a)

$$I_{zz} = \frac{10 \cdot 6^3}{12} - 2\frac{\pi}{4}(1")^4 = 178.4 in^4$$

$$I_{yy} = \frac{6 \cdot 10^3}{12} - 2\pi\left(2.5in\right)^2 - 2\frac{\pi}{4}(1")^4 = 459.2 in^4$$

b)

$$M_y = 60k \cdot in \cos 40^\circ = 45.96 k \cdot in$$

$$M_z = -60k \cdot in \sin 40^\circ = -38.57 k \cdot in$$

$$\sigma_A = \frac{M_y z_A}{I_{yy}} - \frac{M_z y_A}{I_{zz}} = \frac{(45.96k \cdot in)(5")}{459.17 in^4} - \frac{(-38.57k \bullet in)_z (3")}{178.4 in^4}$$

$$= 1.15 ksi$$

c)

$$\sigma_D = \frac{M_y z_D}{I_{yy}} - \frac{M_z y_D}{I_{zz}} = \frac{(45.96k \cdot in)(-5")}{459.17 in^4} - \frac{(-38.57k \bullet in)_z (-3")}{178.4 in^4}$$

$$= -1.15 ksi$$

d)

$$\sigma(y,z) = \frac{M_y}{I_{yy}} z - \frac{M_z}{I_{zz}} y = 0 \text{ (along the NA)}$$

z=-2.16y (equation for the NA)

z=tan(α)y where $\alpha = \tan^{-1}\left(-2.16\right) = -65.16^\circ$