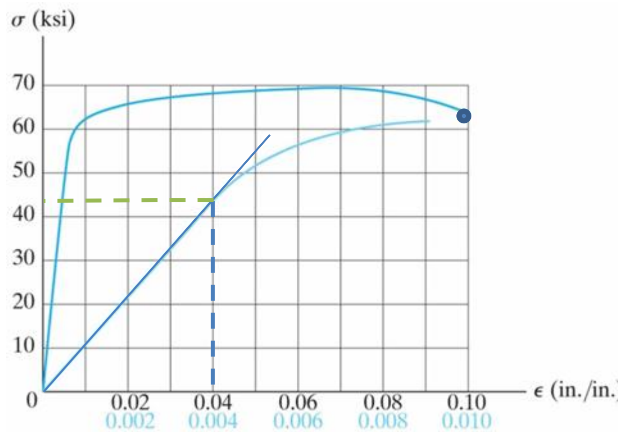
The University of Iowa
57:019:BBB Mechanics of Deformable Bodies
Spring Semester 2011
Quiz #2 Solution

Problem #1: (33 points). The stress-strain diagram of a cylindrical aluminum alloy test specimen is provided. The specimen had an original cross-sectional area of 0.020in.^{2} and an original gauge length of 2 in.



a. Determine the modulus of elasticity.
b. Determine the force in the structure at the breaking point
c. If the specimen were completely unloaded from a strain of 0.04, what would be the final elongation of the specimen?

Solution:

a. Modulus of elasticity: From the initial linear portion of the low strain portion of the curve:

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{44ksi}{0.004} = 11,000ksi$$

b. Force in the structure at breaking point:

Stress level at breaking point is approximately 64 ksi.

$$F = \sigma \cdot A = 64ksi \cdot 0.02in^2 = 1.28kip$$

c. At $\varepsilon = 0.04$, σ=68ksi. Elastic unloading from this point

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} = \frac{-68ksi}{11,000ksi} = -0.00618$$

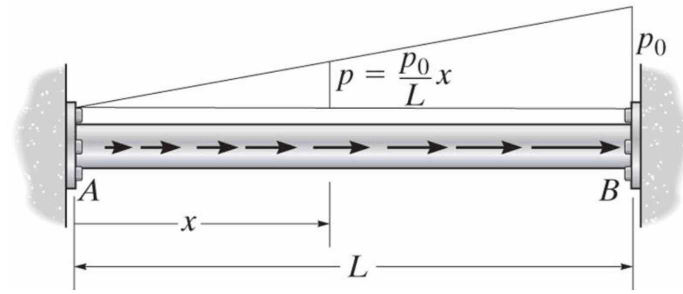
$$\varepsilon_f = 0.04 - 0.00618 = 0.0338$$

$$L_f = \left(1+\varepsilon_f\right)L_o = 1.0338 \cdot 2"$$

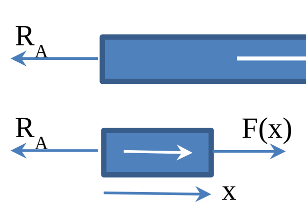
$$L_f = 2.0676"$$

$$\boxed{\text{final elongation} = \varepsilon_f L = (0.0338)(2") = 0.0676"}$$

Problem #2: (34 points). The rod AB has a diameter *d* and fits snugly between the rigid supports at *A* and *B* when it is unloaded. The modulus of elasticity is *E*. Determine the support reactions at *A* and *B* if the rod is subjected to a linearly distributed axial load as shown.



Solution:



$$F(x) = R_A - \frac{p_o x^2}{2L}$$

The magnitude of the distributed load acting on the rod:

$$P = \int_0^L p(x)dx = \int_0^L \frac{p_o x}{L}dx = \frac{p_o L}{2}$$

For static equilibrium of the rod: $R_A + R_B = P = \frac{p_o L}{2}$

Additional constraint:

$$\delta_{AB} = 0$$

$$= \int_0^L \frac{F(x)}{AE}dx = \frac{1}{AE}\int_0^L F(x)dx$$

$$= \frac{1}{AE}\int_0^L \left[R_A - \frac{p_o x^2}{2L}\right]dx$$

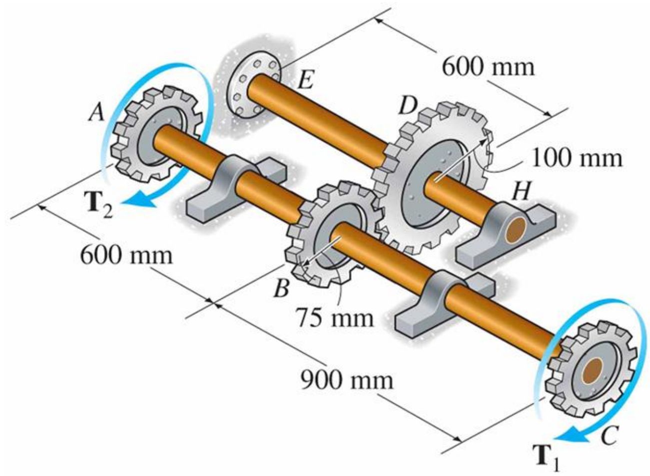
$$= \frac{1}{AE}\left[R_A L - \frac{p_o L^3}{6L}\right] = \frac{L}{AE}\left[R_A - \frac{p_o L}{6}\right]$$

$$R_A = \frac{p_o L}{6}; \quad R_B = \frac{p_o L}{2} \text{-} R_A = p_o L\left[\frac{1}{2} - \frac{1}{6}\right] = \frac{p_o L}{3}$$

So,

$$\boxed{R_A = \frac{p_o L}{6}; \quad R_B = \frac{p_o L}{3}}$$

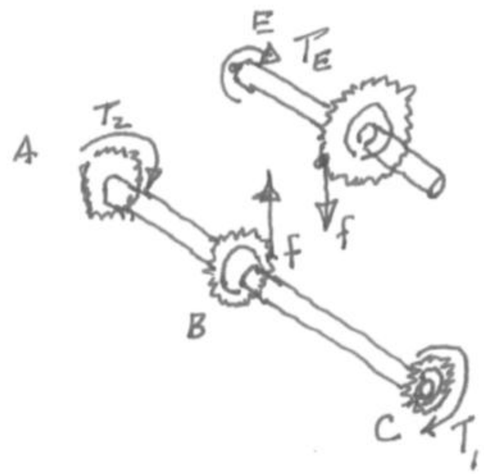
Problem #3: (33 points). Torques T_1=2kNm and T_2=4kNm are applied as shown. The shear modulus of the steel shafts is 75 GPa. The diameter of both shafts is 50mm.



a) Draw a free-body diagram of both shafts and gears;
b) Solve for the torque reaction at rigid support E;
c) Compute the maximum shear stress in the shafts;
d) Compute the angle of twist at gear A.

Solution:

a) Free body diagram:



b) From FBD of shaft ABC,

$$T_1 + T_2 = f \cdot r_B \Rightarrow f = \frac{T_1 + T_2}{R_B} = \frac{6kN \cdot m}{0.075m} = 80kN$$

From FBD of shaft DE:

$$T_E = f \cdot r_D = 80kN \cdot (0.10m) = 8kN \cdot m$$

c) Maximum shear stress in the shafts:

Both shafts have identical diameters, and the maximum torque occurs in shaft DE with a value of 8kNm. The maximum shear stress thus occurs in shaft DE.

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.025m)^4 = 6.136 \cdot 10^{-7} m^4$$

$$\tau_{\max} = \frac{T_{\max} c}{J} = \frac{(8kN \cdot m)(0.025m)}{6.136 \cdot 10^{-7} m^4} = 326,000kPa = 326MPa$$

d) Angle of twist at gear A:

$$\phi_D = \phi_E + \frac{T_{DE} L_{DE}}{JG} = \frac{(8000N \cdot m)(0.6m)}{(6.136 \cdot 10^{-7} m^4)(75 \cdot 10^9 N \cdot m^{-2})}$$

$$= 0.1043 \text{ rad}$$

From kinematic compatibility of gears B and D:

$$r_B \phi_B = -r_D \phi_D \quad \Rightarrow \quad \phi_B = \frac{-r_D}{r_B} \phi_D = -0.1391 \text{ rad}$$

$$\phi_A = \phi_B + \phi_{AB} = -0.1391 \text{ rad} + \frac{T_{AB} L_{AB}}{JG}$$

$$= -0.1391 \text{ rad} - \frac{(4000 N \cdot m)(0.6 m)}{(6.136 \cdot 10^{-7} m^4)(75 \cdot 10^9 N \cdot m^{-2})}$$

$$= -0.1912 \text{ rad}$$

$\boxed{\phi_A = -0.1912 \text{ rad} = -10.96^\circ}$ Twist angle resultant points to left, using right hand rule.