The University of Iowa  
57:019:BBB Mechanics of Deformable Bodies  
Spring Semester 2011  
Quiz #2 Solution

**Problem #1**: (33 points). The stress-strain diagram of a cylindrical aluminum alloy test specimen is provided. The specimen had an original cross-sectional area of 0.020in.\(^2\) and an original gauge length of 2 in.

a. Determine the modulus of elasticity.
b. Determine the force in the structure at the breaking point.
c. If the specimen were completely unloaded from a strain of 0.04, what would be the final elongation of the specimen?

**Solution**:

a. **Modulus of elasticity**: From the initial linear portion of the low strain portion of the curve:

\[
E = \frac{\Delta \sigma}{\Delta \varepsilon} = \frac{44\text{ksi}}{0.004} = 11,000\text{ksi}
\]

b. **Force in the structure at breaking point**:

Stress level at breaking point is approximately 64 ksi.

\[
F = \sigma \cdot A = 64\text{ksi} \cdot 0.02\text{in}^2 = 1.28\text{kip}
\]

c. **At \(\varepsilon = 0.04\), \(\sigma = 68\text{ksi}\)**. Elastic unloading from this point

\[
\Delta \varepsilon = \frac{\Delta \sigma}{E} = \frac{-68\text{ksi}}{11,000\text{ksi}} = -0.00618
\]

\[
\varepsilon_f = 0.04 - 0.00618 = 0.0338
\]

\[
L_f = (1 + \varepsilon_f) L_0 = 1.0338 \cdot 2"
\]

\[
L_f = 2.0676" \]

\[
\text{final elongation} = \varepsilon, L = (0.0338)(2") = 0.0676" \]
**Problem #2**: (34 points). The rod AB has a diameter $d$ and fits snugly between the rigid supports at $A$ and $B$ when it is unloaded. The modulus of elasticity is $E$. Determine the support reactions at $A$ and $B$ if the rod is subjected to a linearly distributed axial load as shown.

Solution:

The magnitude of the distributed load acting on the rod:

$$P = \int_0^L p(x)dx = \int_0^L \frac{p_0}{L}x \, dx = \frac{p_0L}{2}$$

For static equilibrium of the rod: $R_A + R_B = P = \frac{p_0L}{2}$

Additional constraint:

$$\delta_{AB} = 0$$

$$= \int_0^L \frac{F(x)}{AE} \, dx = \frac{1}{AE} \int_0^L F(x)dx$$

$$= \frac{1}{AE} \int_0^L \left[ R_A - \frac{p_0x^2}{2L} \right] \, dx$$

$$= \frac{1}{AE} \left[ R_AL - \frac{p_0L^3}{6L} \right] = \frac{L}{AE} \left[ R_A - \frac{p_0L}{6} \right]$$

$$R_A = \frac{p_0L}{6}; \quad R_B = \frac{p_0L}{2} - R_A = \frac{p_0L}{6} \left[ \frac{1}{2} - \frac{1}{6} \right] = \frac{p_0L}{3}$$

So,

$$R_A = \frac{p_0L}{6}; \quad R_B = \frac{p_0L}{3}$$
**Problem #3:** (33 points). Torques $T_1=2\text{kNm}$ and $T_2=4\text{kNm}$ are applied as shown. The shear modulus of the steel shafts is 75 GPa. The diameter of both shafts is 50mm.

a) Draw a free-body diagram of both shafts and gears;
b) Solve for the torque reaction at rigid support $E$;
c) Compute the maximum shear stress in the shafts;
d) Compute the angle of twist at gear A.

**Solution:**

a) Free body diagram:

b) From FBD of shaft ABC,

$$T_1 + T_2 = f \cdot r_B \implies f = \frac{T_1 + T_2}{R_B} = \frac{6\text{kN} \cdot \text{m}}{0.075\text{m}} = 80\text{kN}$$

From FBD of shaft DE:

$$T_E = f \cdot r_D = 80\text{kN} \cdot (0.10\text{m}) = 8\text{kN} \cdot \text{m}$$

c) Maximum shear stress in the shafts:

Both shafts have identical diameters, and the maximum torque occurs in shaft DE with a value of $8\text{kNm}$. The maximum shear stress thus occurs in shaft DE.

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.025\text{m})^4 = 6.136 \cdot 10^{-7} \text{m}^4$$

$$\tau_{\text{max}} = \frac{T_{\text{max}} c}{J} = \frac{(8\text{kN} \cdot \text{m})(0.025\text{m})}{6.136 \cdot 10^{-7} \text{m}^4} = 326,000\text{kPa} = 326\text{MPa}$$

d) Angle of twist at gear A:

$$\phi_D = \phi_E + \frac{T_{\text{DE}} L_{\text{DE}}}{JG} = \frac{(8000\text{N} \cdot \text{m})(0.6\text{m})}{(6.136 \cdot 10^{-7} \text{m}^4)(75 \cdot 10^9 \text{N} \cdot \text{m}^{-2})}$$

$$= 0.1043 \text{ rad}$$

From kinematic compatibility of gears B and D:

$$r_B \phi_B = -r_D \phi_D \implies \phi_B = -\frac{r_D}{r_B} \phi_D = -0.1391 \text{ rad}$$
$$\phi_A = \phi_R + \phi_{AB} = -0.1391 \text{ rad} + \frac{T_{AB} L_{AB}}{JG}$$

$$= -0.1391 \text{ rad} - \frac{(4000 N \cdot m)(0.6m)}{(6.136 \cdot 10^{-7} m^4)(75 \cdot 10^9 N \cdot m^{-2})}$$

$$= -0.1912 \text{ rad}$$

$$[\phi_A = -0.1912 \text{ rad} = -10.96^\circ]$$

Twist angle resultant points to left, using right hand rule.