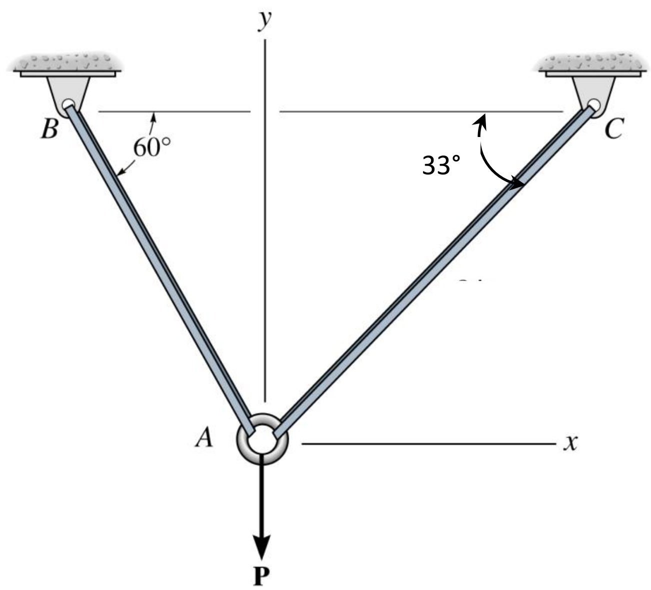
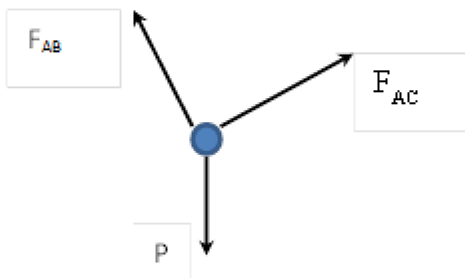
The University of Iowa
College of Engineering
57:019:BBB Mechanics of Deformable Bodies
Spring 2011
Quiz #1 Solution

Problem #1 (33 points): Members AB and AC both have cross-sectional areas of 1.00 in^2. What magnitude of force P can be applied to the ring while maintaining a factor of safety of 2.0 against yielding in members AB and AC? The yield stress for both members is 36 ksi.





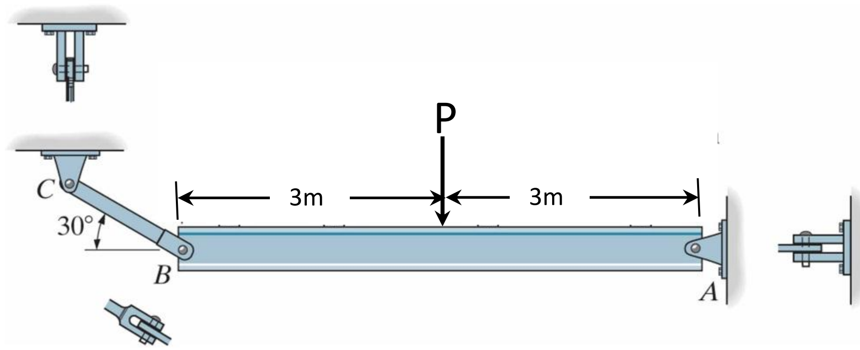
$$\sum F_x = 0 = -F_{AB}\cos 60^\circ + F_{AC}\cos 33^\circ$$
$$F_{AB} * 0.5 = F_{AC} * .8387$$
$$F_{AB} = 1.677 F_{AC}$$

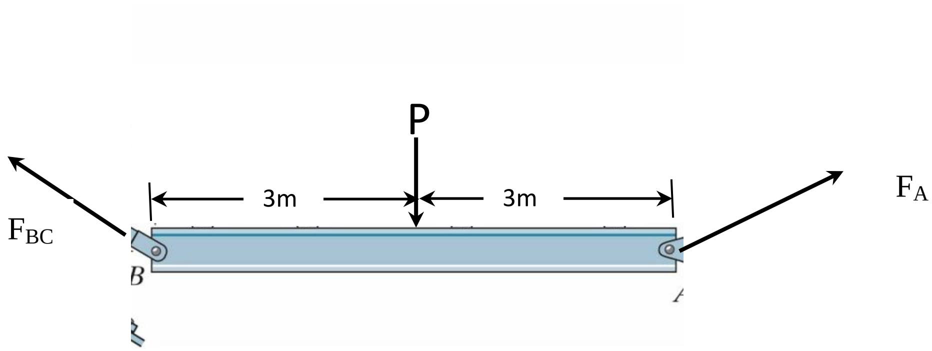
$$\sum F_y = 0 = F_{AB}\sin 60^\circ + F_{AC}\sin 33^\circ - P$$
$$F_{AB} * 0.866 + F_{AC} * .5446 = P$$
$$1.444 F_{AC} + 0.545 F_{AC} = P$$
$$F_{AC} = 0.5029P$$
$$F_{AB} = 0.8433P$$

Since both members have the same cross-sectional area, the stress in member AB will be larger because the internal force is larger.

$$(\sigma_{AB})_{allowable} = \frac{36ksi}{FS} = 18ksi = \frac{F_{AB}}{1in^2} = \frac{0.8433P}{1in^2}$$
$$P = \frac{18kip}{0.8433} = 21.3kip$$
$$\boxed{P = 21.3kip}$$

Problem #2 (33 points). The beam is supported by a pin at A and a short link BC. Determine the maximum magnitude P of the load the beam will support if the average shear stress in each pin is not to exceed 80MPa. All pins are in double shear and have a diameter of 18mm.





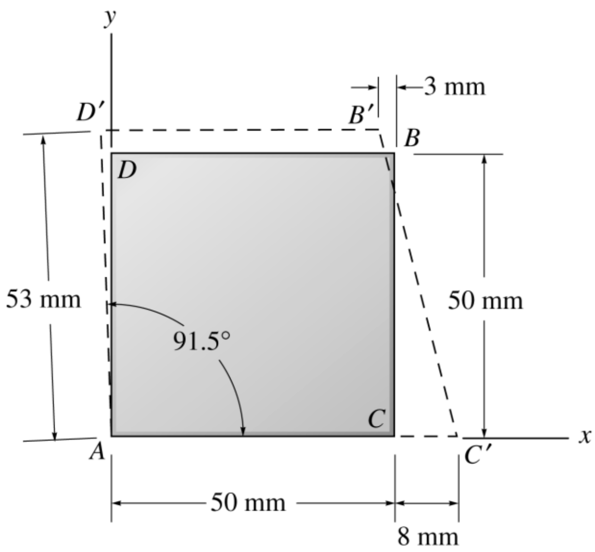
$$\sum M_A = 0 = -3m * P + 6m * F_{BC}\sin 30^\circ$$
$$F_{BC} = P$$
$$\sum F_x = 0 = -F_{BC}\cos 30^\circ + F_{Ax} \Rightarrow F_{Ax} = F_{BC}\cos 30^\circ$$
$$\sum F_y = 0 = F_{BC}\sin 30^\circ + F_{Ay} - P \Rightarrow F_{Ay} = 0.5F_{BC} = F_{BC}\sin 30^\circ$$
$$\therefore F_A = F_{BC} = P$$

Since all of the pins experience double-shear, the magnitude of the shear force is half of the pin force, or P/2.

$$\tau_{allowable} = 80N/mm^2 = \frac{V}{A} = \frac{P/2}{\pi d^2/4} = \frac{2P}{\pi d^2}$$
$$P_{allowable} = \frac{(80N \cdot mm^{-2})\pi(18mm)^2}{2} = 40{,}715N$$
$$\boxed{P_{allowable} = 40.7kN}$$

Problem #3 (34 points): The original square deforms into the position shown by the dashed lines. Sides D′B′ and AC′ remain horizontal. Calculate:

a. The shear strain at corner A
b. The shear strain at corner C
c. The normal strain along AC
d. The normal strain along fiber AB.



a. $\gamma_A = \frac{\pi}{2} - \theta_A = \frac{\pi}{2} - 91.5^\circ * \frac{\pi}{180^\circ} = \frac{\pi}{2}\left(1 - \frac{91.5}{90}\right)$

$\boxed{\gamma_A = -0.0262}$

$\gamma_C = \frac{\pi}{2} - \theta_C$: Here, θ_C is in radians

b. $\tan\theta_C = \frac{53mm\left(\sin 91.5^\circ\right)}{11mm} = 4.8165 \Rightarrow \theta_C = 78.271^\circ = 1.3661rad$

$\boxed{\gamma_C = 0.205}$

c. $\varepsilon_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{8}{50} = 0.16$

$\boxed{\varepsilon_{AC} = 0.16}$

$\varepsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}}$

$L_{AB} = \sqrt{2} \cdot 50mm = 70.7107$

d. $L_{AB'} = \sqrt{\left(47mm\right)^2 + \left(53\sin 91.5^\circ\right)^2} = 70.8242mm$

$\varepsilon_{AB} = \frac{70.8242 - 70.7107}{70.7107} = 0.00160$

$\boxed{\varepsilon_{AB} = .00160}$