The University of Iowa
College of Engineering
53:243/58:251 Computational Inelasticity
Fall Semester 2005
Prof. C.C. Swan

Assignment #3: Due: 14 October 2005

Consider the following elasto-plasticity model:

$$
\begin{aligned}
f(\boldsymbol{\sigma}) &= \|\mathbf{s}\| - F_e(I_1) \leq 0 \\
F_e(I_1) &= \alpha + \theta\left(1 - \exp[\beta I_1]\right) \\
I_1 &= tr(\boldsymbol{\sigma}) \\
\mathbf{s} &= dev(\boldsymbol{\sigma}) \\
\|\mathbf{s}\| &= (\mathbf{s} : \mathbf{s})^{1/2}
\end{aligned}
$$

In the above equations, α, β, θ are all non-negative material strength constants. The elastic response of the material model is assumed to be linear and isotropic and the plastic flow rule is associated.

For this model, please address the following:

a. Derive an expression for $\dot{\gamma}$.
b. Derive an expression for the elastoplastic (continuum) tangent operator.
c. With all of the material constant values noted above chosen non-negative, can this material model feature softening behaviour? (Please show why, or why not.)
d. Derive a complete and detailed backward Euler integration algorithm for this model.
e. For the special case when $|\beta I_1| << 1$, please show that:
    1. $F_e(I_1) = \alpha - \theta\beta I_1$
    2. For this special case show that no iterations are required in the plastic correction part of the stress update.
f. For the general case, derive an expression for the consistent tangent operator for this model. (That is, a tangent operator that is consistent with the Backward Euler integration algorithm employed). As a rough check on the consistent tangent expression, show that: $\lim_{\dot{\gamma} \to 0} \mathbf{C}^{consistent} = \mathbf{C}^{continuum}$
g. Assume that the above material model is valid for frictional materials like portland cement concrete.
    1. How would you go about estimating the three strength parameters α, β, θ?
    2. Briefly describe the physical effect associated with each of these three parameters.