Problem 1:

Consider a structure which is loaded not by tractions or body forces, but rather by non-vanishing applied displacements $\mathbf{u} = \mathbf{g}$ on $\Gamma_\text{g} \subset \Gamma$. Assuming that the structure is linear elastic, use adjoint sensitivity analysis to derive a complete expression for the design derivative for the compliance of the structure under the applied loading. That is, provide thorough details on computing

$$M(\mathbf{u}, \mathbf{b}) = -\frac{1}{2} \int_{\Gamma_\text{g}} \mathbf{n} \cdot \mathbf{\sigma} \cdot \mathbf{g} \, d\Gamma$$

Assume a Q4/Q4 formulation in which both nodal displacements and nodal design variables (which are volume fractions of the solid material) are interpolated using bilinear shape functions.

Also assume that a powerlaw mixing rule is used to relate local volume fraction to local stiffness.

Problem 2:

Assume that a plane-stress Q4/U continuum topology formulation of Bendsøe and Kikuchi is employed, wherein the element level design variables are: $(a_1, a_2, \theta)$. Present an algorithm for computation of $\partial \mathbf{\sigma} / \partial \mathbf{b}$ in each element.