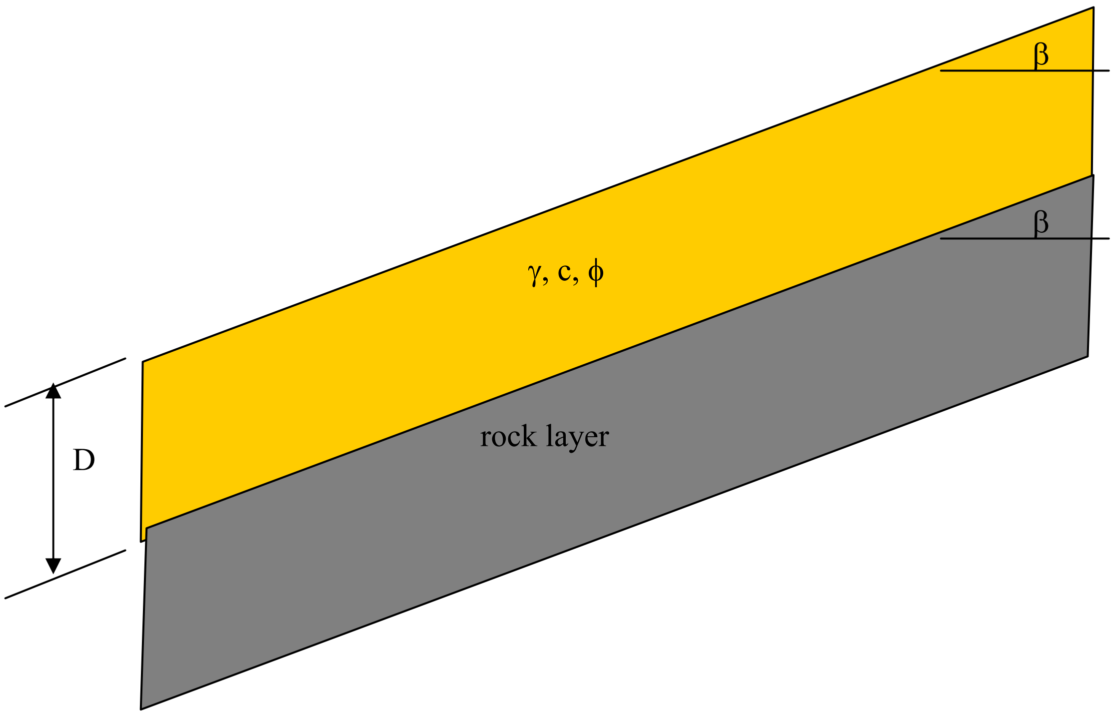
53:139 FOUNDATIONS OF STRUCTURES
College of Engineering
The University of Iowa
Spring Semester, 2009

ASSIGNMENT #2 **SOLUTION**

1. For the slope shown below, find the critical thickness D of the soil layer that yields shear failure along the soil-rock interface. Given β=20°; γ=17.3 kN/m^3; ϕ=15°; and c=12 kPa.



Solution: For an infinite slope without seepage occurring, we have:

$$FS = \frac{c}{\gamma D \sin\beta \cos\beta} + \frac{\tan\phi}{\tan\beta}$$

Setting FS=1 and solving for D yields:

$$D_{cr} = \frac{c}{\gamma \cos^2\beta(\tan\beta - \tan\phi)}$$

Plugging the numbers yields: $\boxed{D_{cr} = 8.18m}$

2. For the figure shown above with the same soil parameters, assume that the soil is saturated (γ_{sat}=19.5 kN/m^3) and that seepage is occurring parallel to the slope face. What is the factor of safety against shear failure along the interface?

Solution: Letting $D = D_{cr} = 8.18m$ we have for a saturated slope in which seepage is occurring:

$$FS = \frac{c}{\gamma_{sat} D \sin\beta \cos\beta} + \frac{\gamma_b \tan\phi}{\gamma_{sat} \tan\beta}$$

Plugging the numbers yields:

$\boxed{FS = 0.60}$ **Observe: the addition of moisture and seepage to this scenario reduces the stability.**

3. Soil of properties c=80 kPa; ϕ=25°; γ=18kN/m^3 comprises a steep slope of height H=20m and β=72°.
    a. What is the factor of safety along a planar mechanism passing through the toe at an inclination of 35° with respect to the horizontal?
    b. What is the slope system's critical factor of safety against shear failure?
    c. What angle does the critical failure mechanism make with respect to the horizontal?
    d. What height of the slope would yield a critical factor of safety of one against failure?

Solution:

a. For a slope of height H and a planar mechanism at inclination θ passing through the toe the expression developed in the class notes for the factor of safety is:

$FS = \dfrac{2c \sin\beta}{\gamma H \sin(\beta-\theta)\sin\theta} + \dfrac{\tan\phi}{\tan\theta}$. So plugging in all of the given values with θ=35° yields

$\boxed{FS = 1.89}$

b. Using the approximation that $\boxed{\theta_{cr} \cong \tfrac{1}{2}(\beta+\phi) = 48.5^\circ \Rightarrow FS = 1.83}$

c. $\boxed{\theta_{cr} \cong \tfrac{1}{2}(\beta+\phi) = 48.5^\circ}$

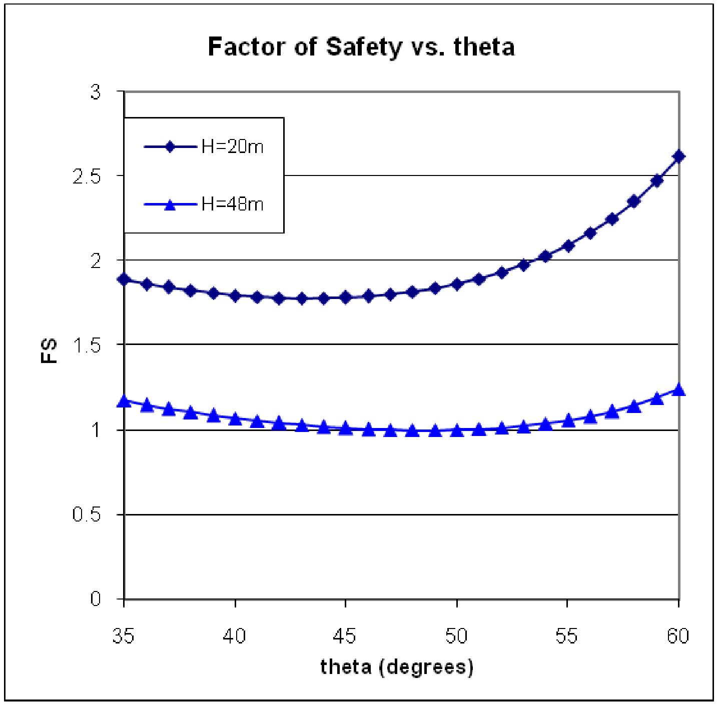
d. Setting FS=1 and letting $\theta = \theta_{cr} \cong \tfrac{1}{2}(\beta+\phi)$ and solving for H yields:

$H_{cr} = \dfrac{4c}{\gamma}\dfrac{\sin\beta\cos\phi}{1-\cos(\beta-\phi)}$. Plugging the numbers gives: $\boxed{H_{cr} = 48.2m}$

4. In Culmann's method, the critical failure mechanism passing through the toe of the slope is approximated by $\theta_{cr} = (\beta+\phi_d)/2$. For the soil and slope properties of Problem #3, write a program to compute and plot $FS(\theta)$ vs. θ. Compare and briefly discuss the values of θ_{cr} from your plot and from the approximate formula $\theta_{cr} = (\beta+\phi)/2$. Are they roughly the same or far apart? Is the approximation acceptable in this case?

Solution: Consider the spreadsheet results below in which the computed FS is computed vs. θ for both H=20m and also H=48.2m. For the case H=20m, the actual value of $\theta_{cr} = 43^\circ$ is should be compared to the approximate value $\theta_{cr} \cong (\beta+\phi_d)/2 = 48.5^\circ$. While the angle values are somewhat different, the associated values of FS are close (1.78 vs. 1.83). For the case H=48.2m, the actual value of $\theta_{cr} = 48.5^\circ$ is the same as $\theta_{cr} \cong (\beta+\phi_d)/2 = 48.5^\circ$. Thus the approximation seems acceptable.

c=80kPa	80	80
phi=25 degrees	25	0.436332313
gamma=18kN/m3	18	18
beta = 72 degrees	72	1.25663706
H=20m	20	20



theta(deg)	theta(rad)	H=20m FS	48.2m FS
35	0.610865	1.890487	1.17406
36	0.628319	1.865268	1.149473
37	0.645772	1.843341	1.126915
38	0.663225	1.824626	1.106298
39	0.680678	1.80907	1.087555
40	0.698132	1.796652	1.070632
41	0.715585	1.78738	1.055494
42	0.733038	1.781293	1.042122
43	0.750492	1.778461	1.030513
44	0.767945	1.77899	1.020682
45	0.785398	1.783023	1.012663
46	0.802851	1.790749	1.006508
47	0.820305	1.802404	1.002293
48	0.837758	1.818283	1.000122
49	0.855211	1.83875	1.000125
50	0.872665	1.86425	1.00247
51	0.890118	1.89533	1.007368
52	0.907571	1.932659	1.015083
53	0.925025	1.977061	1.025941
54	0.942478	2.029559	1.040355
55	0.959931	2.091428	1.058842
56	0.977384	2.164272	1.082057
57	0.994838	2.250139	1.110838
58	1.012291	2.35167	1.146273
59	1.029744	2.472335	1.189792
60	1.047198	2.61677	1.243309

5. A cut slope is to be excavated in a saturated clay soil with $c=c_u$=500 psf and $\phi_u=0°$ and γ_{sat} = 110 pcf. Answer the following questions using the Mass Method.
   a. If the slope angle is to be 56°, how deep can the slope be excavated?
   b. Where would the critical circular mechanism intersect the slope system?
   c. How deep could the same slope be excavated while maintaining a FS=2.5?
   d. If the slope angle were reduced to 45°, how deep could the slope be excavated?
   e. With a slope angle of 45°, identify where the circular mechanism will intersect the slope system.

Solution: Since there is no mention of D in the problem statement, assume $D \to \infty$

a.

The slope stability number for $\beta = 56^\circ$ and $D \to \infty$ can be found from Fig. 14.9 of Reference #1 as m=0.185.

$$\boxed{H_{cr} = \frac{c_u}{\gamma m} = \frac{500\,psf}{110\,pcf * 0.185} = 24.6\,ft}$$

b.

Since $\beta = 56^\circ$ and $D \to \infty$, this should be a toe circle. From Fig. 14.10 of Ref. 1, $\alpha = 32^\circ$ and $\theta = 77^\circ$. In this case, the scarp will intersect the top of the slope at a distance

$$\boxed{L = H\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = 24.6\,ft * \left(\cot 32^\circ - \cot 56^\circ\right) = 23\,ft.}$$ from the crown of the slope.

c. With a FS=2.5, the critical slope height would be:

$$\boxed{H_{cr} = \frac{c_u}{FS * \gamma m} = \frac{500\,psf}{2.5 * 110\,pcf * 0.185} = 9.84\,ft \to 10\,ft}$$

d. With $\beta = 45^\circ$ and $D \to \infty$, the slope stability number from Fig. 14.9 is m=0.18. Therefore,

$$\boxed{H_{cr} = \frac{c_u}{\gamma m} = \frac{500\,psf}{110\,pcf * 0.180} = 25.25\,ft}$$

e. With $\beta = 45^\circ$ and $D \to \infty$, the mechanism should still be a toe circle with $\alpha \cong 25^\circ$ and $\theta \cong 85^\circ$. Accordingly, the scarp would intersect the top of the slope at a distance L from the crown, where

$$\boxed{L = H\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) \cong 25.25\,ft * \left(\cot 25^\circ - \cot 45^\circ\right) \cong 29\,ft.}$$

6. A cut slope (β=40°) was excavated in a saturated clay soil (γ_{sat}=18.5 kN/m^3) and the slope experienced failure when depth of the excavation reached H=8.5m. Previous subsurface site exploration indicated the presence of a rock stratum 12 m beneath the original ground surface.
   a. Estimate the undrained cohesion of the saturated clay soil.
   b. What would be the expected nature of the critical circle?
   c. With reference to the top of the slope, at what distance did the surface of sliding intersect the bottom of the excavation?

Solution: From the problem statement, one deduces that D=(12m/8.5m)=1.41

a.

The slope fails, which indicates that it has a FS just less than unity. Thus the undrained cohesion of the soil can be estimated as follows: (note $m \cong 0.175$) from Fig. 14.9.

$$\boxed{c_u = \gamma m H_{cr} = \left(18.5kN/m^3\right)\cdot\left(0.175\right)\cdot\left(8.5m\right) = 27.5 \to 28kPa.}$$

b.

Using Fig. 14.9 once again, it appears that **the critical mechanism should be a midpoint circle**.

c.

Reading from Fig. 14.11 of Reference 1, with D=1.4 and β=40° gives $n \cong 0.8$. **Accordingly, the midpoint circle should intersect the base at a distance** $nH = 0.8 * 8.5m = 6.8m$ **from the toe of the slope.**

7. A slope of height 4m is cut in a saturated clay deposit in which the undrained cohesion increases linearly with depth as follows: c_u(kPa) = 5kPa +3z where z is the depth beneath the original ground surface. If β=27° and γ_{sat}=18.5kN/m^3, what is the factor of safety for the slope system?

Solution:

For this problem $c_u(z) = c_u(z=0) + a_0 z = 5kPa + 3kPa \cdot m^{-1} z$. Thus $a_0 = 3kPa \cdot m^{-1}$.
Using Koppula's formulas from Section 14.6 of Reference #1, the product of slope stability number and factor of safety is given by:

$$FS * m = \frac{c_u(z=0)}{\gamma H} = \frac{5kPa}{(18.5kNm^{-3})(4m)} = .0676$$

The quantity c_R represents the ratio of the increase in cohesion at the depth of the cut to that at the top of the cut:

$$c_R = \frac{a_0 H}{c_u(z=0)} = \frac{(3kPa \cdot m^{-1})(4m)}{(5kPa)} = 2.4$$

Reading Table 14.1 of Reference #1 with c_R=2.4 and β=27° and using linear interpolation gives:
m = 0.6*(.0529)+.4*(.0402)=.0478

Thus the factor of safety for the slope system can be computed as

$$\boxed{FS = \frac{FS * m}{m} = \frac{.0676}{.0478} = 1.4}$$

It is worth noting here that if we had used the Mass-Method (Fellenius-Taylor solution) with $D \to \infty$ and an average undrained cohesion estimated from a depth (2/3)H beneath the original ground surface, that is: $\bar{c}_u \cong 5kPa + 2.67m * 3kPa \cdot m^{-1} = 13kPa$, the slope stability number would be 0.18 (Fig. 14.9) and the resulting FS for the slope system would be:

$$\boxed{FS = \frac{\bar{c}_u}{\gamma H m} = \frac{13kPa}{(18.5kN \cdot m^{-1})(4m)(0.18)} = 1.1}$$

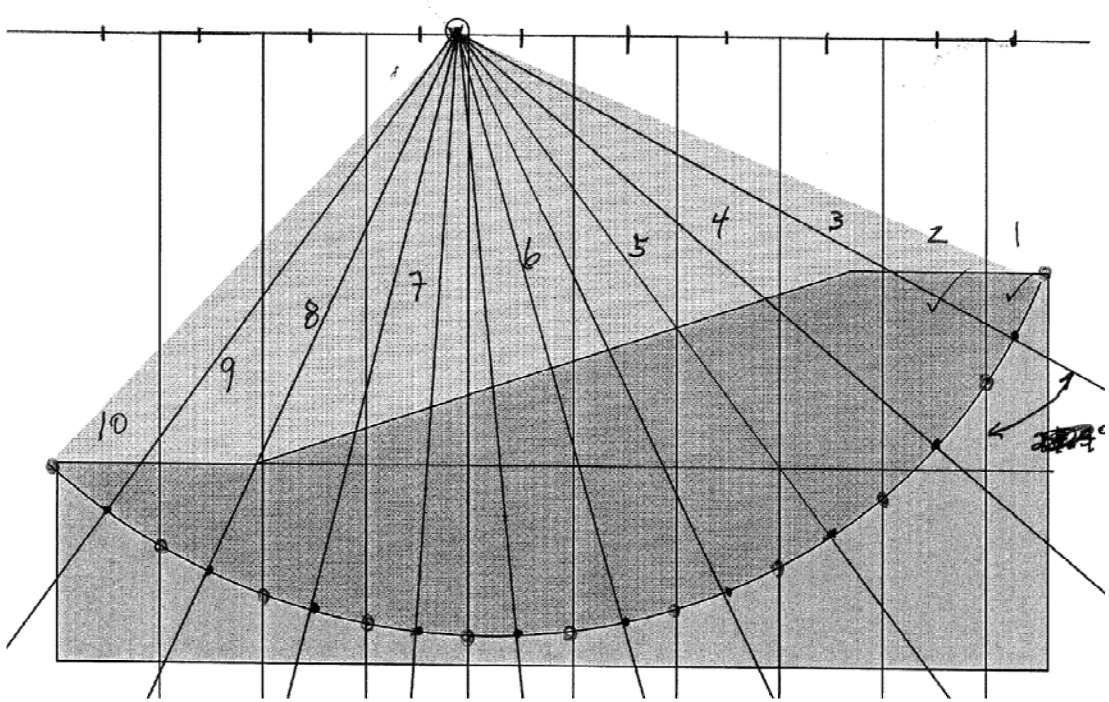
8. A sandy soil has a unit weight of 17kN/m^3 and a friction angle 35° makes a slope of height 30m and angle β=20°. For the mechanism shown in the Figure below, compute the factor of safety against shear failure: For each case, divide the slope system into 10 slices, each having an equal lateral dimension.

   a. Using the ordinary method of slices;
   b. Using Bishop's simplified method of slices.

Solution:

a. For the Ordinary Method of Slices: $$FS = \frac{\sum_{n=1}^{N}\left(c_n L_n + W_n \cos(\alpha_n)\tan(\phi_n)\right)}{\sum_{n=1}^{N} W_n \sin(\alpha_n)};$$

b. For Bishop's Method of Slices: $$FS = \frac{\sum_{n=1}^{N}\left(c_n b_n + W_n \tan(\phi_n)\right) * \frac{1}{m_{\alpha,n}}}{\sum_{n=1}^{N} W_n \sin(\alpha_n)}$$ where $m_{\alpha,n} = \cos(\alpha_n) + \frac{\tan(\phi_n)\sin(\alpha_n)}{(FS)_n}$


10
9
8
7
6
5
4
3
2
1

Bishop's Method of Slices

slice index	FS (assumed)	c	tan(phi)	height (m)	b(m)	W(kN)	alpha(deg)	alpha(rad)	L(m)	m_n	Numerator	Denominator
1	3.14	0	0.7	9.57	9.57	1557	62.6	1.093	20.80	0.658	1656	1382
2	3.14	0	0.7	26.8	15.96	7271	49.8	0.869	24.73	0.816	6240	5554
3	3.14	0	0.7	38.3	15.96	10392	36.9	0.644	19.96	0.934	7792	6239
4	3.14	0	0.7	42.8	15.96	11612	25.7	0.449	17.71	0.998	8147	5036
5	3.14	0	0.7	42.8	15.96	11612	16.2	0.283	16.62	1.022	7950	3240
6	3.14	0	0.7	38.9	15.96	10554	5.91	0.103	16.05	1.018	7260	1087
7	3.14	0	0.7	33.8	15.96	9171	3.16	0.055	15.98	1.011	6351	506
8	3.14	0	0.7	25.5	15.96	6919	-14.4	-0.251	16.48	0.913	5304	-1721
9	3.14	0	0.7	16.6	15.96	4504	-25.8	-0.450	17.73	0.803	3925	-1960
10	3.14	0	0.7	7.7	15.96	2089	-36.8	-0.642	19.93	0.667	2192	-1251
											56816	18111
							Trial #	FS_assumed	FS_output			
							1	1	2.95			
							2	2.95	3.14			
							3	3.13	3.14			

Ordinary Method of Slices

slice index	c	tan(phi)	height (m)	b(m)	W(kN)	alpha(deg)	alpha(rad)	L(m)	Numerator	Denominator
1	0	0.7	9.57	9.57	1557	62.6	1.093	20.80	502	1382
2	0	0.7	26.8	15.96	7271	49.8	0.869	24.73	3285	5554
3	0	0.7	38.3	15.96	10392	36.9	0.644	19.96	5817	6239
4	0	0.7	42.8	15.96	11612	25.7	0.449	17.71	7325	5036
5	0	0.7	42.8	15.96	11612	16.2	0.283	16.62	7806	3240
6	0	0.7	38.9	15.96	10554	5.91	0.103	16.05	7349	1087
7	0	0.7	33.8	15.96	9171	3.16	0.055	15.98	6410	506
8	0	0.7	25.5	15.96	6919	-14.4	-0.251	16.48	4691	-1721
9	0	0.7	16.6	15.96	4504	-25.8	-0.450	17.73	2838	-1960
10	0	0.7	7.7	15.96	2089	-36.8	-0.642	19.93	1171	-1251
									47193	18111
						FS	2.61			