

Period #3 Notes: Defects & Material Strength

A. THEORETICAL STRENGTHS OF MATERIALS

In period #2, we looked briefly at the different types of primary atomic bonding that form compounds, and the secondary bonding that occurs between the molecules. Based on the nature of the interatomic bonding that occurs in a material, one should be able to predict the elastic moduli of that material.

If one used the force potentials between atoms presented in slide 2.7, one should also be able to predict the theoretical strength of the material. When this has been done in the past, people usually come to the estimate that the tensile strength of a material should be approximately:

$$\sigma_{theoretical} \cong \frac{E}{20} \Leftrightarrow \frac{E}{5}$$

In reality, the actual tensile strength of most materials is far below these estimated values:

For example, consider a mild steel which has a Young's modulus of $E=207$ GPa (29 Mpsi). The corresponding theoretical strength of steel should thus be roughly:

$$\sigma_{theoretical} \cong \frac{E}{20} \Leftrightarrow \frac{E}{5} \Rightarrow 10 \text{ GPa} - 40 \text{ GPa} \text{ (1.5 Mpsi - 6 Mpsi)}$$

In reality, for varying grades of structural steel, the tensile strength typically falls in the range of:

$$0.70 \text{ GPa} \leq \sigma_{ult}^{steel} \leq 2.1 \text{ GPa} \text{ (50 ksi - 150 ksi)}$$

These actual strengths are well below the theoretical strengths.

Another example is silicon glass. This material has a Young's modulus of 70 GPa, and a corresponding theoretical strength range of:

$$\sigma_{theoretical}^{glass} \in (3.5 \text{ GPa} - 14 \text{ GPa}) \text{ or about } (500 \text{ ksi} - 2000 \text{ ksi})$$

From our everyday experience, though, we don't think of silicon glass as such a strong material since we all know how easy is it to break a drinking glass, or a glass window. In fact, we tend to think of silicon glass as a very fragile material.

So, why is there this discrepancy between the theoretical strengths of materials based on the atomic bonds, and the actual strengths of materials?

The answer, in a word, is **defects**. Defects are imperfections in the material structure that usually occur at scales ranging from a few nanometers to a few millimeters. We will discuss here two basic types of defects:

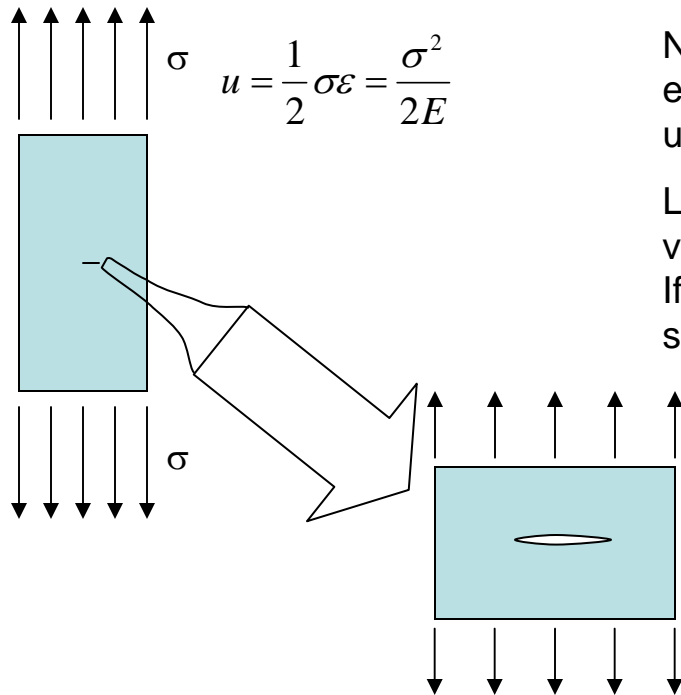
1. micro-voids or cracks in materials; and
2. dis-locations in the lattice structure of crystalline materials.

We will discuss micro-voids or cracks first. Dislocations will be discussed later in the context of steel.

B. GRIFFITH'S MICRO-CRACK MODEL

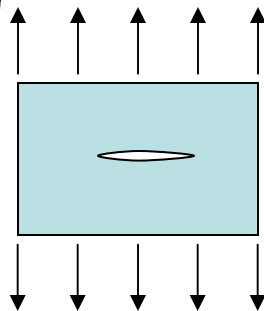
The purpose of Griffith's model is to explain how voids or crack-like defects in materials make the material weaker. It uses a number of simplifying approximations, but the end result is quite helpful.

Let's begin by assuming that we have a sheet-like specimen of material in a state of pure tension as shown. If the Young's modulus of the material is E , and the state of axial stress is σ , then the corresponding axial strain in the material will be: $\epsilon = \sigma/E$. The strain energy density in the material is:

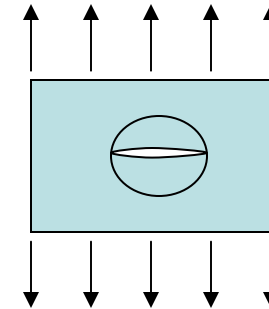

$$u = \frac{1}{2} \sigma \epsilon = \frac{\sigma^2}{2E}$$

Note that the units of strain energy density are simply energy per unit volume (FL/L^3)= (F/L^2) which are the same units as those of stress.

Let's now assume that in this material, there is a very, very small flaw that we can only see if we look very hard. If we magnify the region around this flaw, it will look something like the following:



Let's give the length of the crack a dimension "2a". Assuming the material specimen has a unit thickness in the out-of-plane direction, the total surface area of this crack is "4a". The elevated surface energy associated with this crack is $S=4a*G_c$. Here, G_c is the specific surface energy of the material with units of (FL/L²) or (F/L). (The idea of surface energy was briefly mentioned in slide 2.8.)



And because the stress cannot be carried through the crack, let's assume that the circular region of radius "a" around the crack has no stress in it due to the crack. The lost strain-energy due to this stress-free region is:

$$U_{lost} = u * V = \frac{\sigma^2}{2E} \pi a^2$$

Now, if we want to know whether or not this crack will grow, we ask whether the strain energy released due to growth of the crack will be sufficient to match the increased surface energy associated with the crack.

Mathematically, if the differential strain energy lost exceeds the increased surface energy, the crack will grow. Otherwise, it won't. This gives a useful criterion for growth of the fracture:

$$dU_{lost} > dS$$

$$\frac{\sigma^2}{2E} 2\pi a * da > 4G_c * da$$

$$\sigma > \sqrt{\frac{4EG_c}{\pi a}} = \sigma_{fract}$$

Essentially this result tells us that if the stress level exceeds σ_{fract} , the crack will grow. As the crack grows (as “a” increases) the stress level required for further growth actually decreases.

Observations:

1. Materials with low surface energies G_c will have low fracture stresses;
2. The larger the existing defect size “a” in the material, the lower will be the fracture stress.

The fracture stress of a material is sometimes written:
$$\sigma_{fract} = \sqrt{\frac{4EG_c}{\pi a}} = \frac{2K_{Ic}}{\sqrt{\pi a}}$$

The quantity K_{Ic} is the “mode-1 fracture toughness” of the material and it typically has the following units:

Units of K_{Ic} : MPa \sqrt{m} or ksi \sqrt{in}

A table of surface energies fracture toughness values for different materials is shown below:

Table 3.1. Surface energy and fracture toughness of materials (from Illston, 1994).

Material	G_c (kJ/m ²)	K_{Ic} (MPa m ^{1/2})
Pure ductile metals	100-1000	100-350
High strength steels	15-120	50-150
Mild steel	100	150
Polyethylene	6-8	1
Common timbers [£]	0.5-2.0	0.5-1.0
Silicon glass	0.01	0.8

£ parallel to grain

Example 3.1: Consider silicon glass which has a very low fracture toughness relative to the other materials in Table 3.1. We'll explore the fracture stress or fracture strength of this material with changing flaw size of radius "a".

From Table 3.1, $K_{Ic}=0.8 \text{ MPa m}^{1/2}$ for glass. Therefore:

$$\sigma_{fract} = \frac{2K_{Ic}}{\sqrt{\pi a}} = \frac{2*(0.8 \text{ MPa}\sqrt{m})}{\sqrt{\pi a}} = \frac{0.903 \text{ MPa}}{\sqrt{a(m)}}$$

Table 3.2. Fracture stress vs. flaw size in silicon glass.

flaw size a(m)	fracture stress (MPa)
10^{-8}m	9030
10^{-7}m	2860
10^{-6}m	903
10^{-5}m	286
10^{-4}m	90.3
10^{-3}m	28.6

Observe: as the flaw size in glass gets smaller, the computed fracture stress approaches the theoretical strength of glass (c.f. slide 3.2).

If glass can be produced so that it is free of voids and micro-cracks, it can actually be stronger than steel. Since glass has a mass density roughly 1/3 that of steel, it is an attractive structural material.

Glass fibers used in fiber-glass composites, are actually produced with very small diameters on the order of 1-10 μm since this helps to eliminate voids and defects.

Aside from usage as a fibrous reinforcing material in structural composites, glass is an important glazing materials for usage in windows of buildings, homes, and automobiles. The development and usage of safety glasses is a current research topic of high interest in hurricane-prone regions. There are two general classes of window glass commonly available: (1) annealed glass; and (2) tempered glass (see Foundations of Materials Science and Eng., 3rd Ed. By W.F. Smith for more info.)

- **Annealed glass** is what we consider to be “normal” glass. As the hot glass comes out of the melting furnace, it undergoes very slow cooling so that all of the internal residual thermal stresses are relieved. When fractures grow in annealed glass, it does not necessarily shatter, and the large broken glass fragments can cause serious and potentially fatal injuries.
- **Tempered glass** is sometimes considered to be “safety” glass. As the hot glass comes out of the melting furnace, it is cooled relatively rapidly, so that the surfaces of the glass plate solidify quickly, while the inner layer is still molten. As the inner molten glass solidifies, it goes into a state of tension, whereas the glass on the surface of the plate goes into a state of compression. The state of compression in the surface layers of tempered glass make it less resistant to mode-1 fracturing. When tempered glass does fracture, however, it does so in a “crazing” pattern that creates huge amounts of surface area and very small pieces. As such tempered glass has a high breaking strength than annealed glass, and is safer.

In many classes of structural materials, voids or micro-cracks greatly reduce the integrity of material.

While this point has been discussed here in the context of glass, the same principle also applies to portland cement concrete. Specifically, the strength of the hydrated cement paste (hcp) phase that binds the aggregate together, is greatly affected by the presence of relatively large voids called “capillary pores.” Stronger portland cement concrete is achieved by minimizing the capillary pores in the hcp phase.