Period #10: Multi−dimensional Fluid Flow in Soils (II)

A. Review

- Our objective is to solve multi−dimensional fluid flow problems in soils.
- Last time, mass conservation and Darcy's Law were used to derive the so−called *Laplace Equation* which governs seepage in homogeneous, isotropic soil deposits.

$$\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2 + \partial^2 h/\partial z^2 = 0$$

B. Possible Methods for Solving the Laplace Equation.

1) Analytical, closed form or series solutions of the PDE.
   - quite mathematical, and not very general.

2) Numerical solution methods
   - typically, the *finite element method* or the *finite difference method*.
   - very powerful and easy to apply
   - can deal with heterogeneity, anisotropy, 2D, 3D
   - Will use *finite element method* in Lab 6.

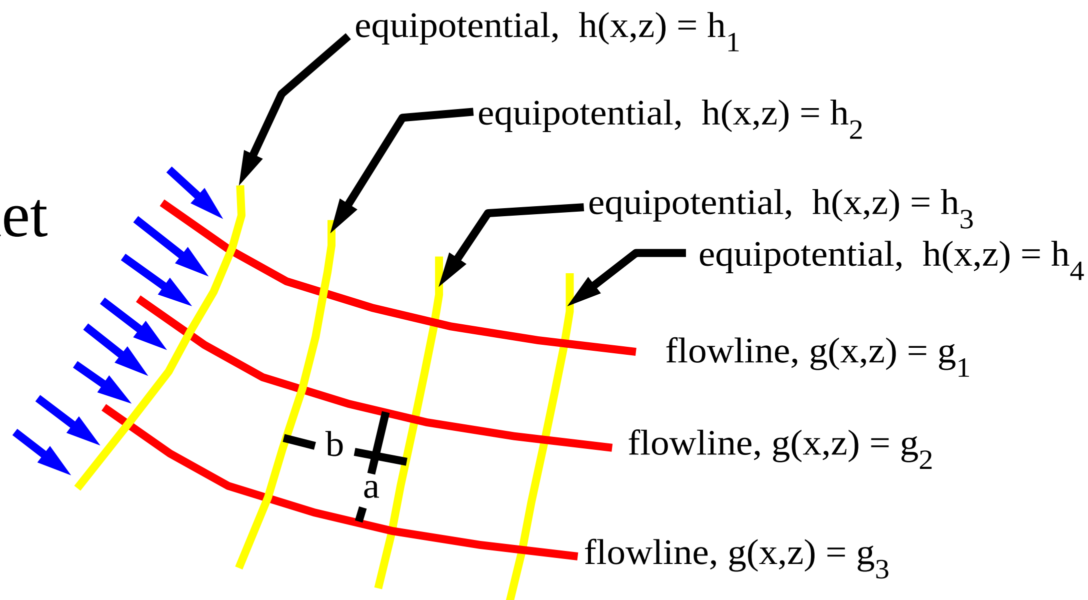
3) Graphical Techniques − *Flow−net Methods*
   - commonly used in engineering practice to solve 2D flow problems.
   - the ideas behind this method are now explained.

C. Flow−net Methods

- straightforward graphical method to solve 2D seepage problems.
- underlying idea:
  - solutions of Laplace Equation consist of two families of orthogonal curves in the (x,z) plane. These families of curves make a flow net.

    - equipotentials: $h(x,z) = c$: family of curves along which head is constant
    - flow lines : $g(x,z) = d$: family of curves across which flow does not occur
    - h and g curves must intersect at right angles wherever they cross.
    - two h curves cannot intersect each other; two g curves cannot intersect.

Partial Flow−net



- Consider the flow rate Δq though a given rectangle formed by two h−curves and two g−curves.
  - since seepage is occuring parallel to the g−curves, can use 1−D form of Darcy's Law

$$\begin{aligned}\Delta q &= k\, i\, a \\ &= k\, (\Delta h/b)\, a \\ &= k\, \Delta h\, (a/b)\end{aligned}$$

- In practice, a net of equipotentials (h−curves) and flow lines (g−curves) are drawn on the flow domain such that:

  a) The soil domain is drawn to scale;
  b) The boundary conditions are clearly identified (for example, are the boundaries of the flow domain equipotentials or flow lines?);
  c) The cells formed by intersecting families of curves are all approximately square with ratios $(a/b) \sim 1$.
  d) The equipotentials and flow−lines are orthogonal, wherever they intersect.

- Drawing good flow nets that satisfy these criteria is not always easy. Usually it takes a fair amount of trial and error (and a pencil with a good eraser !).

- If flow nets can be drawn satisfying these requirements, then:
  1) the flow in each channel will contain an equal flow. (A channel is the region between two flow lines or g− curves.)
  2) the head drop between all adjacent equipotentials or h−curves is the same.

- Good flow nets provide **a good deal of information**.
- Therefore, it is important to learn:
  - how to draw good flow nets
  - how to use flow nets

D. Using Flow Nets

- A good flow net can be used to compute such things as:
  - total flow rates;
  - fluid pressure distributions;
  - local flow velocities;
  - local hydraulic gradients;
  - etc.

1) Calculating the total flow rate q in a given problem

$$\begin{aligned}\text{flow rate} = q &= (\text{\# of flow channels}) \cdot (\text{flow rate in each channel}) \\ &= n_f \cdot \Delta q = n_f\, k\, \Delta h (a/b) = n_f\, k\, \Delta h \\ &= n_f\, k\, [\Delta H / n_d] = k \Delta H [n_f / n_d]\end{aligned}$$

where: n_f is the number of flow channels in the flow net;
n_d is the number of head drops in the flow net (or the number of equipotentials − 1);
ΔH is the total head loss in the flow problem;
k is the soil permeability.

Observation:

- People sometimes think that drawing very fine flow nets with many h and g lines will give them more accurate results.
- Since it is the ratio of n_f and n_d that determines the accuracy of the results, good results can often be achieved with coarse, but well−drawn flow nets.

2. Using the Flow net to compute fluid pressures in the flow domain.

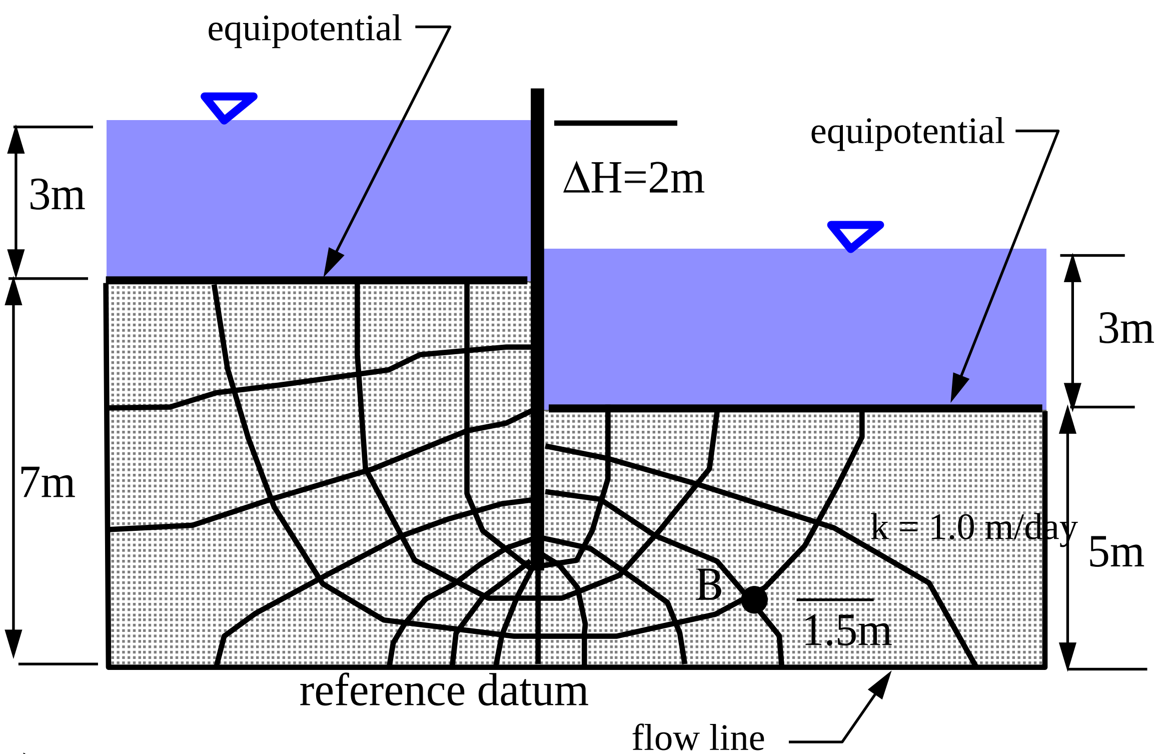
- The equipotentials of a good flow net give the piezometric head distribution throughout the flow domain.

- Once the head value at a given point is known, the pressure can be computed using the definite of head as follows:

Since $h = h_z + p_w/\gamma_w$, and h and h_z are known, then pressure can easily be computed as: $p_w = \gamma_w(h - h_z)$.

3. Example Problem (This flow net is *definitely* not perfect, but is *acceptable*.)

For the flow net shown:
a) compute q
b) compute (p_w) at B



Solution:
First: note that
$n_f = 4$; $n_d = 12$;
$\Delta h = 2\text{m}/n_d = 2\text{m}/12$
$= 0.167\text{m}$

a) $q = k\,\Delta H \cdot (4/12)$
$= (1.0\text{m/day})*2\text{m}*(4/12)$
$= 0.67\text{m}^2/\text{day}$

b) $h_B = 10\text{m} - 10\Delta h$
$= 8.33\text{m}$

$(p_w)_B = \gamma_w (h - h_z)_B$
$= 9.81\text{kN/m}^3(8.33\text{m} - 1.5\text{m})$
$= 67\text{kN/m}^2.$

4. Additional Example Problem.

E. Procedures for Using Flow Nets with Anisotropic Soils

(This material covered in class).