The University of Iowa
Dept. of Civil & Environmental Engineering
53:030 SOIL MECHANICS
Midterm Exam #2, Fall Semester 2005

<u>Question #1</u>: (20 points)

A sand with a minimum void ratio of 0.45 and a maximum void ratio of 0.97 has a relative density D_r= 40 %. The average specific gravity of the minerals in the soil is 2.68, and γ_w = 9.81 kN·m^{-3}

a. Compute γ_{dry} and γ_{sat} for the sand in its present state with D_r = 40%.
b. Under vibratory loading, the sand is densified to D_r= 75%. How much will a 3m thick stratum of this sand settle under this densification?

<u>Question #2</u>: (40 points)

Figure 1a shows a homogeneous, saturated sandy soil deposit with a horizontal ground surface. Before the strip load is applied, the stresses at point A are as follows: vertical stress σ_v=100 kPa; horizontal stress σ_h=75kPa; pore water pressure p_w= 50kPa.

a. Compute the maximum shear stress at point A before the surface pressure is applied.
b. Using the information provided in Figure 1, estimate (compute) the maximum shear stress at point A after the uniform strip load is applied.
c. Estimate (compute) the intensity of the strip surface pressure q required to cause shear failure in the soil at point A. Assume that the sand features a drained behavior, such that any excess pore pressures due to the applied load are dissipated quickly.
d. What are the orientations of the failure planes at point A when shear failure is reached by increasing the surface pressure?

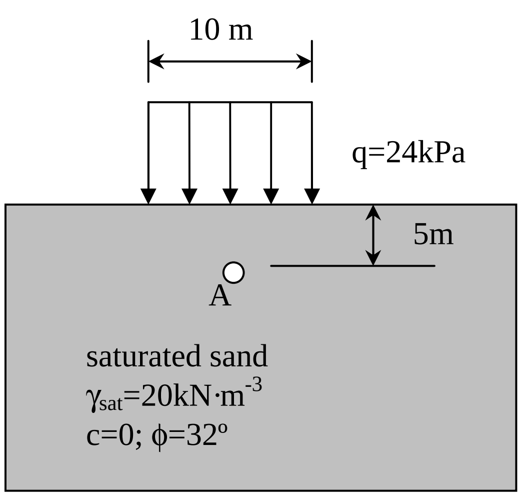


Fig. 1a.

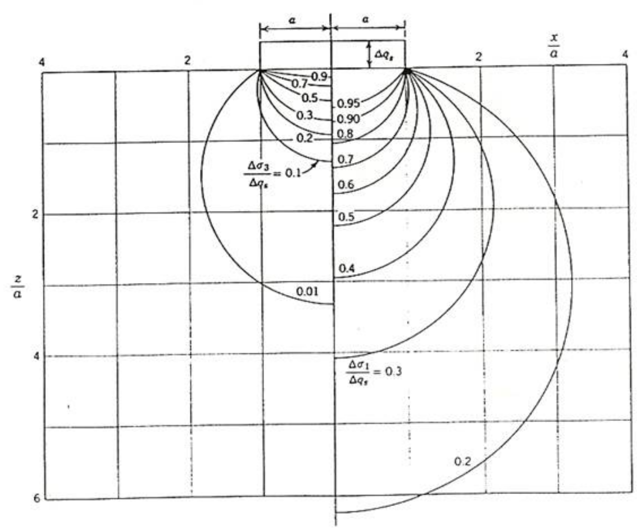


Fig. 1b.

<u>Question #3</u>: (40 points)

A wastewater treatment aeration tank of diameter 40m and gross weight 286.5 MN is to be constructed on the site shown below in Figure 2a. To construct the tank, 6m of the top dense sand layer will be excavated, and the tank will be built as shown in Figure 2b. For the values provided in Figure 2:

a. Compute the increased average vertical stress in the silty clay layer directly beneath the center of the tank.
b. Calculate the ultimate consolidation settlements that will occur at the center of the tank due to compression of the silty clay layer.
c. How long will it take for 90% of this consolidation settlement to occur? (Use one dimensional consolidation theory to answer this question.) The nondimensional time constant for 90% consolidation is $T_{90} = 0.848$.

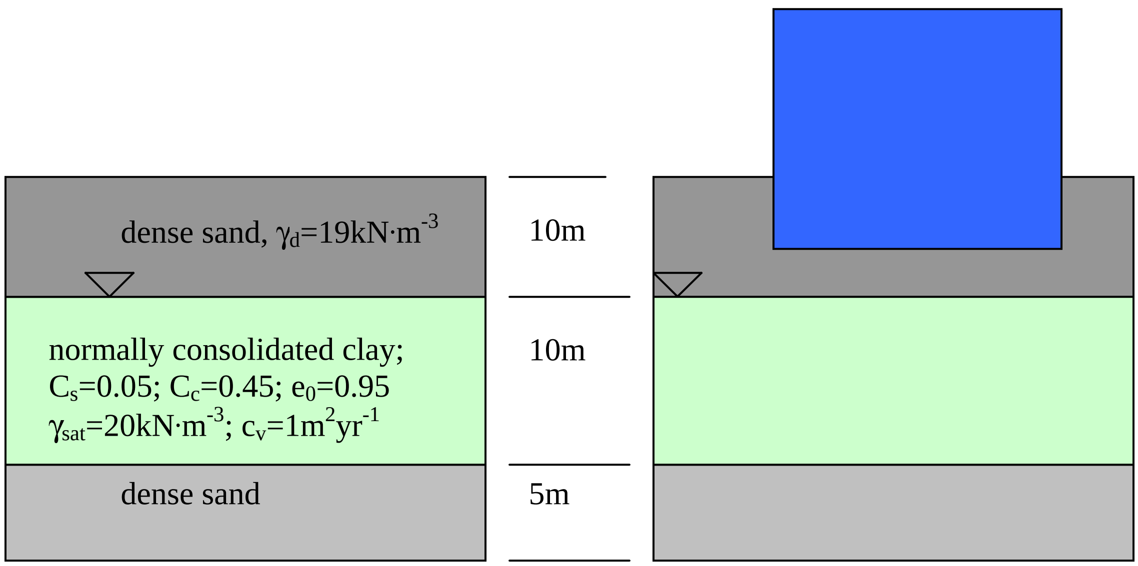


Figure 2.

Note: The vertical stress increase directly beneath the center of a circular, uniformly loaded area is given by the following relation in which q is the magnitude of the uniform load; R is the radius of the circular area; and z is the depth of interest beneath the loaded area:

$$\Delta\sigma_v = q\left[1 - \frac{1}{\left[\left(\frac{R}{z}\right)^2 + 1\right]^{3/2}}\right]$$

Bonus Question (10 points!!)

Explain in detail how one can go about estimating the permeability of a fine-grained soil from a 1-dimensional consolidation test.

Solution of 53:030 Soil Mechanics Midterm Exam #2, Fall Semester, 2005.

Question #1: (20 points)

a) 10 points

$$D_r = 100\% * \frac{e_{\max} - e}{e_{\max} - e_{\min}} \Rightarrow e = e_{\max} - D_r * (e_{\max} - e_{\min})$$

For D_r=0.40, $e = 0.97 - 0.40 * (.97 - .45) = 0.762$

$$\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.7 * 9.81 kN \cdot m^{-3}}{1+0.762} = 14.92 kN \cdot m^{-3}$$

$$\gamma_{sat} = \frac{\gamma_w (G_s + e)}{1+e} = \frac{9.81 kN \cdot m^{-3} * (2.7 + 0.762)}{1+0.762} = 19.16 kN \cdot m^{-3}$$

b) 10 points

For D_r=0.75, $e = 0.97 - 0.75 * (.97 - .45) = 0.580$

The resulting settlement of the 3m thick sand layer is:

$$\Delta H = \frac{H_o}{1+e_o} \Delta e = \frac{3m}{1+0.762} * (.580 - .762) = -0.31m$$

Question #2: (40 points)

a) 10 points

$$\sigma_1' = \sigma_v' = 100kPa - 50kPa = 50kPa$$
$$\sigma_3' = \sigma_h' = 75kPa - 50kPa = 25kPa$$

The resulting effective stress Mohr's Circle has a center at
$\sigma_c = \frac{1}{2}(50+25) = 37.5kPa$, and a radius of $r = \frac{1}{2}(\sigma_1' - \sigma_3') = 12.5kPa$
$\tau_{\max} = r = 12.5kPa.$

b) 10 points

From the chart, at the point 5m directly beneath the center of the strip load, the increase of major principal stress is: $\Delta\sigma_1 = 0.82 * q$ while the minor principal stress increase is: $\Delta\sigma_3 = 0.18 * q$. Since the soil behavior is said to be fully drained, these changes in stress are due to changes in effective stress rather than changes in the pore water pressure. Thus:

$$\sigma_1' = \sigma_v' = 50kPa + 0.82 * q = 69.68kPa$$
$$\sigma_3' = \sigma_h' = 25kPa + 0.18 * q = 29.32kPa$$
$$\tau_{\max} = r = \tfrac{1}{2}(\sigma_1' - \sigma_3') = \tfrac{1}{2}(69.68 - 29.32)kPa = 20.18kPa$$

c) 10 points

At shear failure, the effective stress Mohr's Circle is tangent to the failure envelope. Accordingly, the operative relationship between the major and minor effective principal stresses is: (recalling that the cohesion c vanishes)

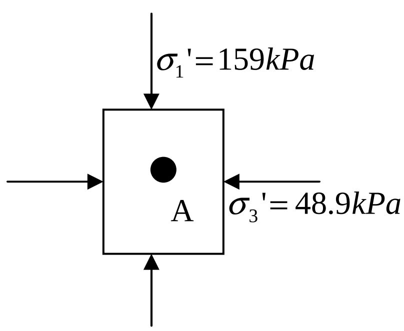
$$\sigma_1' = \sigma_3' \tan^2\left(45^\circ + \tfrac{\phi}{2}\right) = \sigma_3' \tan^2\left(61^\circ\right) = 3.25\sigma_3'$$

Noting that $\sigma_1' = 50kPa + 0.82 * q$ and that $\sigma_3' = 25kPa + 0.18 * q$ it follows that

$$\sigma_1' = 3.25\sigma_3'$$
$$50kPa + 0.82 * q = 3.25 * (25kPa + 0.18 * q)$$

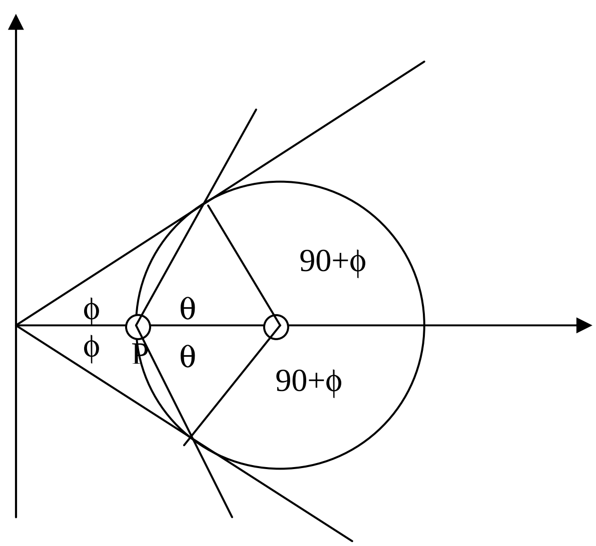
Solving for q gives: $q = 133kPa$

d) 10 points



Orientation of the planes on which shear failure occurs at A:

$$\theta = \pm\left(45^\circ + \frac{\phi}{2}\right) = \pm 61^\circ \text{ wrt the horizontal}$$



Question #3 (40 points)

a) 14 points

Just beneath the tank, the original vertical stress was 6m*19kN/m^3=114kPa.

After the tank is built and filled, the vertical stress at the same level is

$$q = W/A = 286.5MN/\pi(20m)^2 = 228kPa$$

The net bearing stress, is the final stress just beneath the tank minus the original vertical stress at this level. The net bearing stress gives rise to stress increases in the soft clay layer beneath. So, $q_{net} = q - 6m(\gamma_d)_{sand} = 228 - 114 = 114kPa.$

At the top of the clay layer, z = 4m. The vertical stress increase due to the tank:

$$(\Delta\sigma_v)_{top} = q_{net} * \left[1 - \frac{1}{\left[\left(\frac{R}{z}\right)^2 + 1\right]^{3/2}}\right] = 114kPa * \left[1 - \frac{1}{\left[\left(\frac{20}{4}\right)^2 + 1\right]^{3/2}}\right] = 113.1kPa$$

At the middle, z=9m.

$$(\Delta\sigma_v)_{mid} = q_{net} * \left[1 - \frac{1}{\left[\left(\frac{R}{z}\right)^2 + 1\right]^{3/2}}\right] = 114kPa * \left[1 - \frac{1}{\left[\left(\frac{20}{9}\right)^2 + 1\right]^{3/2}}\right] = 106.1kPa$$

And at the bottom, z=14m.

$$(\Delta\sigma_v)_{bot} = q_{net} * \left[1 - \frac{1}{\left[\left(\frac{R}{z}\right)^2 + 1\right]^{3/2}}\right] = 114kPa * \left[1 - \frac{1}{\left[\left(\frac{20}{14}\right)^2 + 1\right]^{3/2}}\right] = 92.5kPa$$

Taking the weighted average of these stress increases

$$(\Delta\sigma_v)_{ave} = \frac{(\Delta\sigma_v)_{top} + 4(\Delta\sigma_v)_{mid} + (\Delta\sigma_v)_{bot}}{6} = \frac{113.1 + 4(106.1) + 92.5}{6}$$

$$= 105\text{kPa}$$

b) 13 points

Since the soil is normally consolidated,

$$S_c = \frac{H_o}{1+e_o}\Delta e = \frac{H_o}{1+e_o} C_c \log\left(\frac{(\sigma_v)_f'}{(\sigma_v)_o'}\right)$$

At the center of the clay layer,

$$(\sigma_v)_o' = 10m \cdot (\gamma_d)_{sand} + 5m(\gamma_b)_{clay} = 10 * 19 + 5 * (20 - 9.81) = 241kPa$$

$$(\sigma_v)_f' = (\sigma_o)_f' + (\Delta\sigma_v)_{ave} = 241kPa + 105kPa = 346kPa$$

So:

$$S_c = \frac{10m}{1+0.95} * 0.45\log\left(\frac{346}{241}\right) = 0.362m$$

c) 13 points

1. $$T_{90} = 0.848 = \frac{t_{90} * c_v}{(H_{dr})^2} \Rightarrow \ t_{90} = \frac{0.848 * (H_{dr})^2}{c_v} = \frac{0.848 * (5m)^2}{1m^2 yr^{-1}} = 21.2 yr$$

Bonus Question (10 points)

How to measure the permeability from the 1-D consolidation test?

This can be done following the same procedure used in processing of the Lab 9 experimental data:

1) From the displacement versus time curve, note the time t_{50} required for 50% consolidation to occur.
2) Using the known drainage distance, estimate the coefficient of consolidation over the interval as:
$$c_v = \frac{0.196(H_{dr})^2}{t_{50}}$$
3) From consolidation theory, $c_v = \dfrac{k(1+e_o)}{\gamma_w a_v}$, so re-arranging gives an expression for the soil permeability k. The soil compressibility coefficient a_v over the interval needs to be estimated as $a_v = \Delta e / \Delta \sigma_v'$.