Viscoelastic Damping Characteristics of Indium-Tin/SiC Particulate Composites

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43rd AIAA SDM Conference
Denver, CO
24 April 2002

Overview

- Objectives: Materials that feature both high stiffness and high viscoelastic damping (G tan δ)

  What composite material structure can provide both properties?

2. Experimental Approach:
- Based on past experience, indium-tin has well-characterized stiffness/damping.
- Fabricate and test composites with "high" volume fractions of SiC particulate reinforcement.

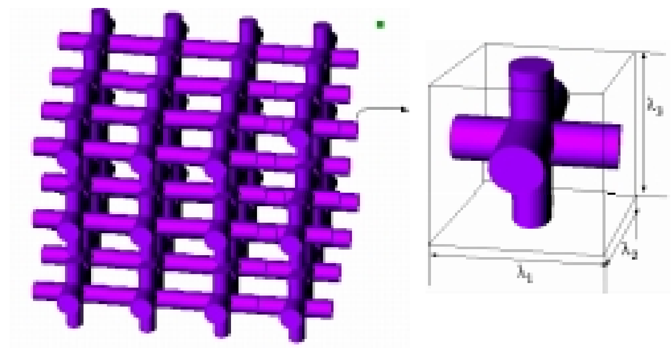
- Modeling Approach:
- Unit cell analysis of particulate composites at high reinforcement volume fractions.
- Correspondence principle to predict effective stiffness and damping.

Essence of Unit-Cell Homogenization
(for heterogeneous, periodic media)

- On a given length scale at which the material is heterogeneous (micro scale), apply an average stress or average deformation to a detailed model (unit cell)
- For each loading, compute detailed, equilibrium microscale stress and deformations fields.
- Take the spatial average of the “microscale” stress and deformation fields, to get their “macroscopic” correspondent.
- Develop/calibrate a constitutive model that adequately relates the macroscale stresses and deformations.
- When performing analysis of the system on the “macroscale” use the “homogenized” constitutive model to represent the medium.

Micro-/Macro-scale Notation

- Periodic medium and unit cell



- Microscale stress/deformation

$$\boldsymbol{\sigma}(\mathbf{X}) = \boldsymbol{\Sigma} + \boldsymbol{\sigma}^*(\mathbf{X});$$

$$\mathbf{F}(\mathbf{X}) = \boldsymbol{\Phi} + \mathbf{F}^*(\mathbf{X});$$

$$< \boldsymbol{\sigma}^*(\mathbf{X}) > = \mathbf{0};$$

$$< \mathbf{F}^*(\mathbf{X}) > = \mathbf{0};$$

$$\mathbf{F}(\mathbf{X} + n\lambda_i \mathbf{e}_i) = \mathbf{F}(\mathbf{X}); \text{ periodicity of microscale deformation}$$

$$\boldsymbol{\sigma}(\mathbf{X} + n\lambda_i \mathbf{e}_i) = \boldsymbol{\sigma}(\mathbf{X}); \text{ periodicity of microscale stress}$$

- Averaging stress/deformation to find macroscale correspondents

$$\boldsymbol{\Sigma} = \langle \boldsymbol{\sigma} \rangle = \frac{1}{V}\int_{\Omega_s} \boldsymbol{\sigma}\, d\Omega_s;$$

$$\boldsymbol{\Phi} = \langle \mathbf{F} \rangle = \frac{1}{V}\int_{\Omega_s} \mathbf{F}\, d\Omega_s;$$

PROCESS: Deformation-Controlled Loading of Unit-Cell

- Specify an average state of deformation $\mathbf{\Phi}$ for the unit cell.
- Apply a consistent "homogeneous" displacement field $\mathbf{u} = \mathbf{\Phi}\cdot\mathbf{X}$ to unit cell.
- To achieve stress-field equilibrium on microscale, solve for the additive, periodic, heterogeneous displacement field $\mathbf{u}^*(\mathbf{X})$.
- Resulting equilibrium displacement field: $\mathbf{u}(\mathbf{X}) = \mathbf{\Phi}\cdot\mathbf{X} + \mathbf{u}^*(\mathbf{X})$
- For each macroscopic state of deformation $\mathbf{\Phi}$, compute the corresponding macroscopic state of stress $\mathbf{\Sigma}$.
- Consider the $\mathbf{\Sigma}$ versus $\mathbf{\Phi}$ behavior of the unit cell model.
- Provide and calibrate a macro-scale constitutive model $\mathbf{\Sigma} = \mathbf{\Sigma}(\mathbf{\Phi})$.

Symmetric, Conjugate, Macro Stress/Strain Measures

- Using conjugate macroscopic stress/strain measures ensures energy conservation between micro- and macro-scales.

- Nemat-Nassar (2000) demonstated/used conjugacy between macroscale deformation gradient Φ and the macroscale nominal stress <**P**>.

$$\left\langle \mathbf{P} : \dot{\mathbf{F}} \right\rangle = \left\langle \mathbf{P} \right\rangle : \left\langle \dot{\mathbf{\Phi}} \right\rangle$$

- It is preferred to develop constitutive models in terms of symmetric, macroscopic stress and deformation measures. Here, we use:

$$\hat{\mathbf{\Sigma}} = \left\langle \mathbf{P} \right\rangle \mathbf{\Phi}^{-\mathrm{T}};$$

$$\hat{\mathbf{E}} = \tfrac{1}{2}\left[\mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi} - \mathbf{I}\right];$$

- These symmetric measures satisfy the following conjugacy relationship:

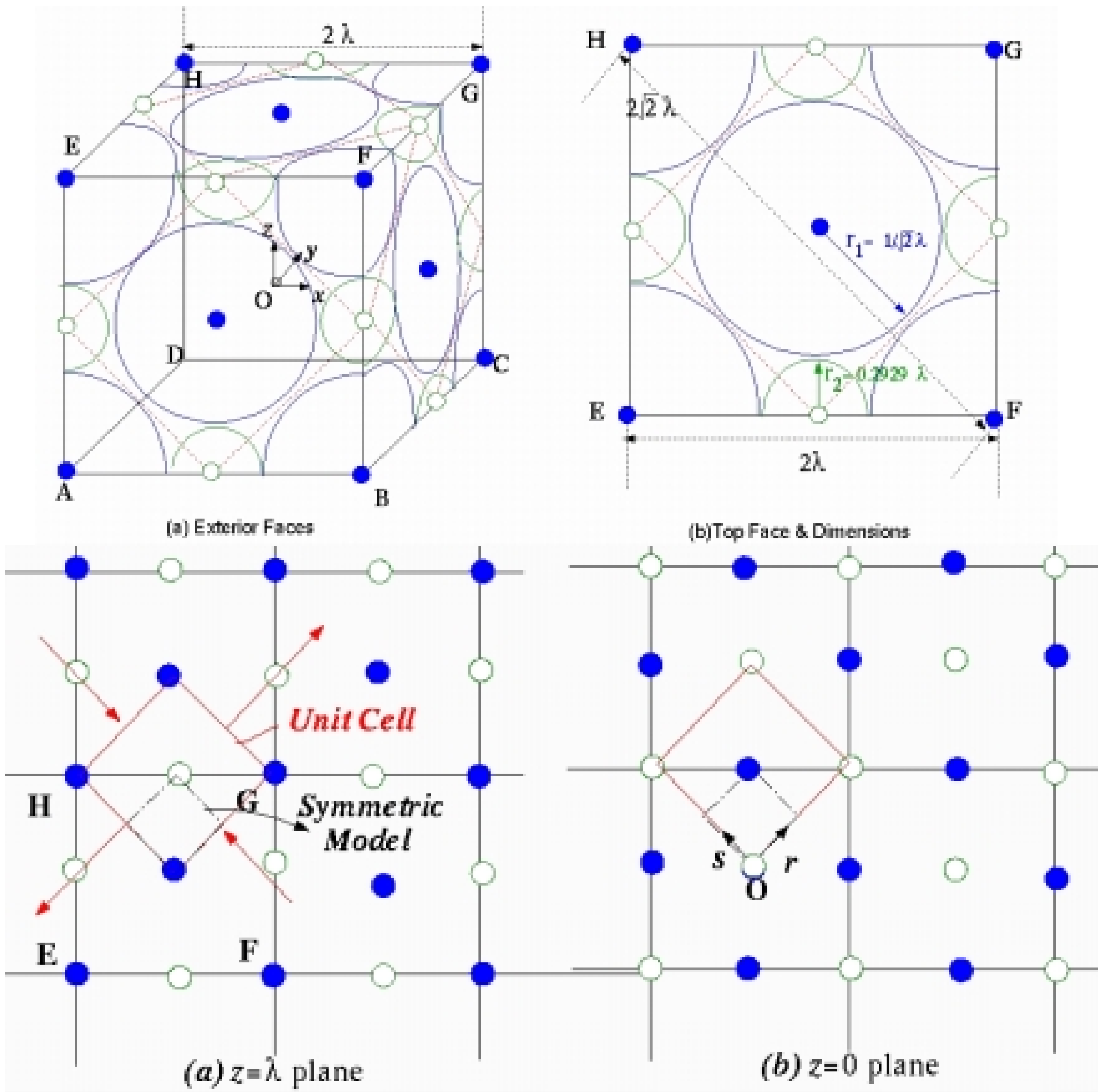
$$\hat{\mathbf{\Sigma}} : \dot{\hat{\mathbf{E}}} = \left\langle \mathbf{P} : \dot{\mathbf{F}} \right\rangle = \left\langle \mathbf{S} : \dot{\mathbf{E}} \right\rangle$$

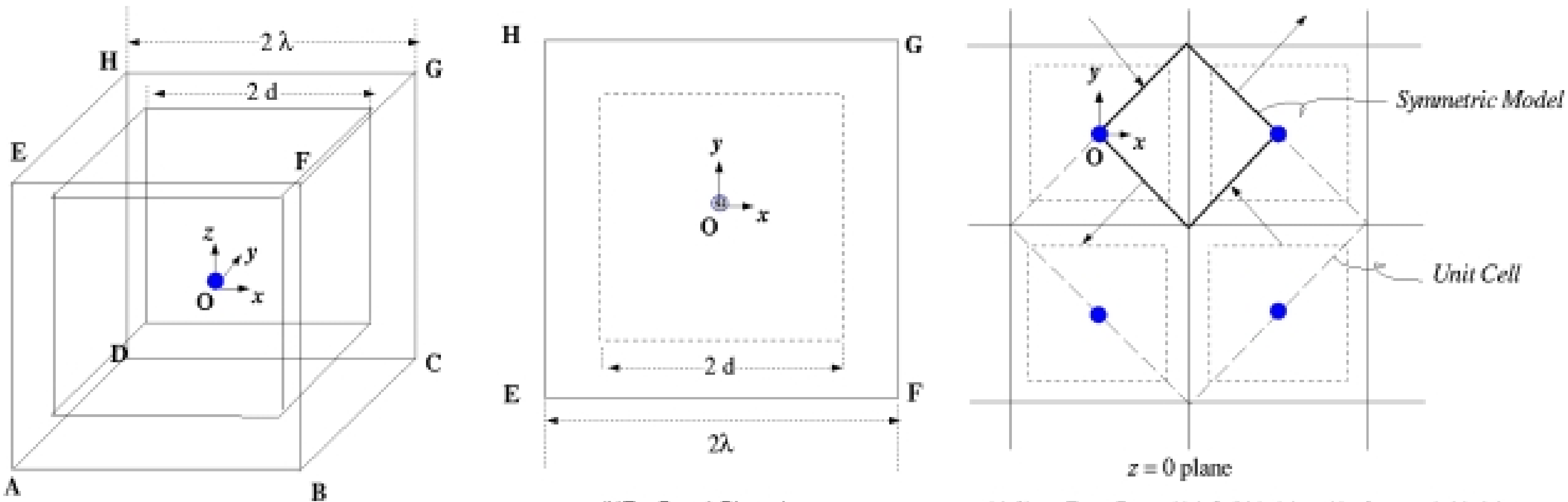
Elastic moduli of composite constituents

	Elastic Constants		
	E	G	ν
SiC	400 GPa	175 GPa	0.14
InSn	20.2 GPa	7.5 GPa	0.35

To realize high G tan δ, must achieve high volume fractions of particulates

- consider multiple sizes of spherical particles
- consider cubical particles
- past experience with Sn matrix shows that it does not “wet” SiC


2 λ
H
G
E
F
z
y
O
x
D
C
A
B
(a) Exterior Faces
H
G
2√2 λ
r1= 1/√2 λ
r2=0.2929 λ
E
F
2λ
(b)Top Face & Dimensions
Unit Cell
H
G
Symmetric Model
E
F
(a) z=λ plane
s
r
O
(b) z=0 plane

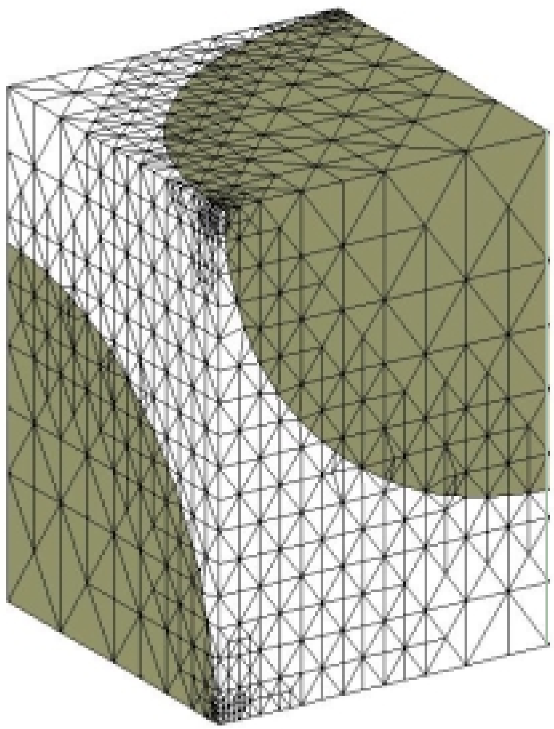


(a) Exterior Faces

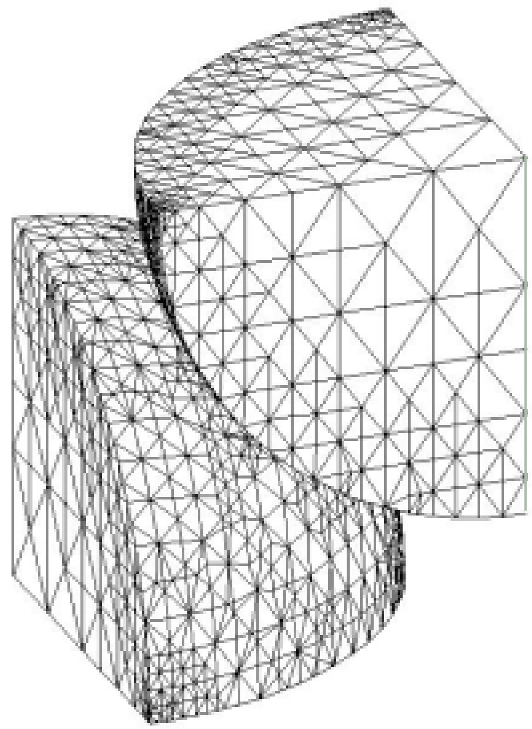
(b)Top Face & Dimensions

(c) Shear Test : Proper Unit Cell Model and its Symmetric Model

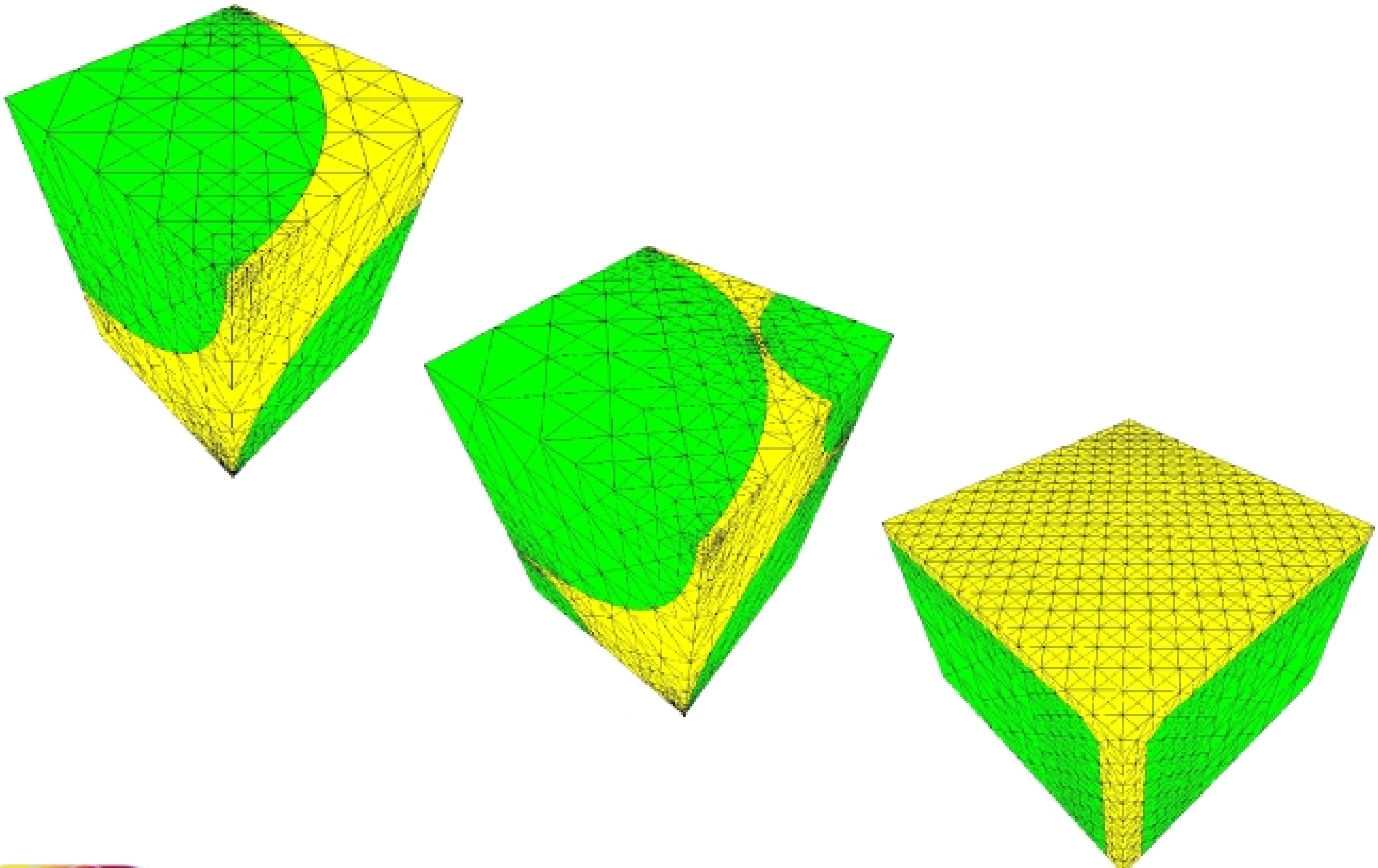
Typical Unit-Cell Mesh for Particulate Composite



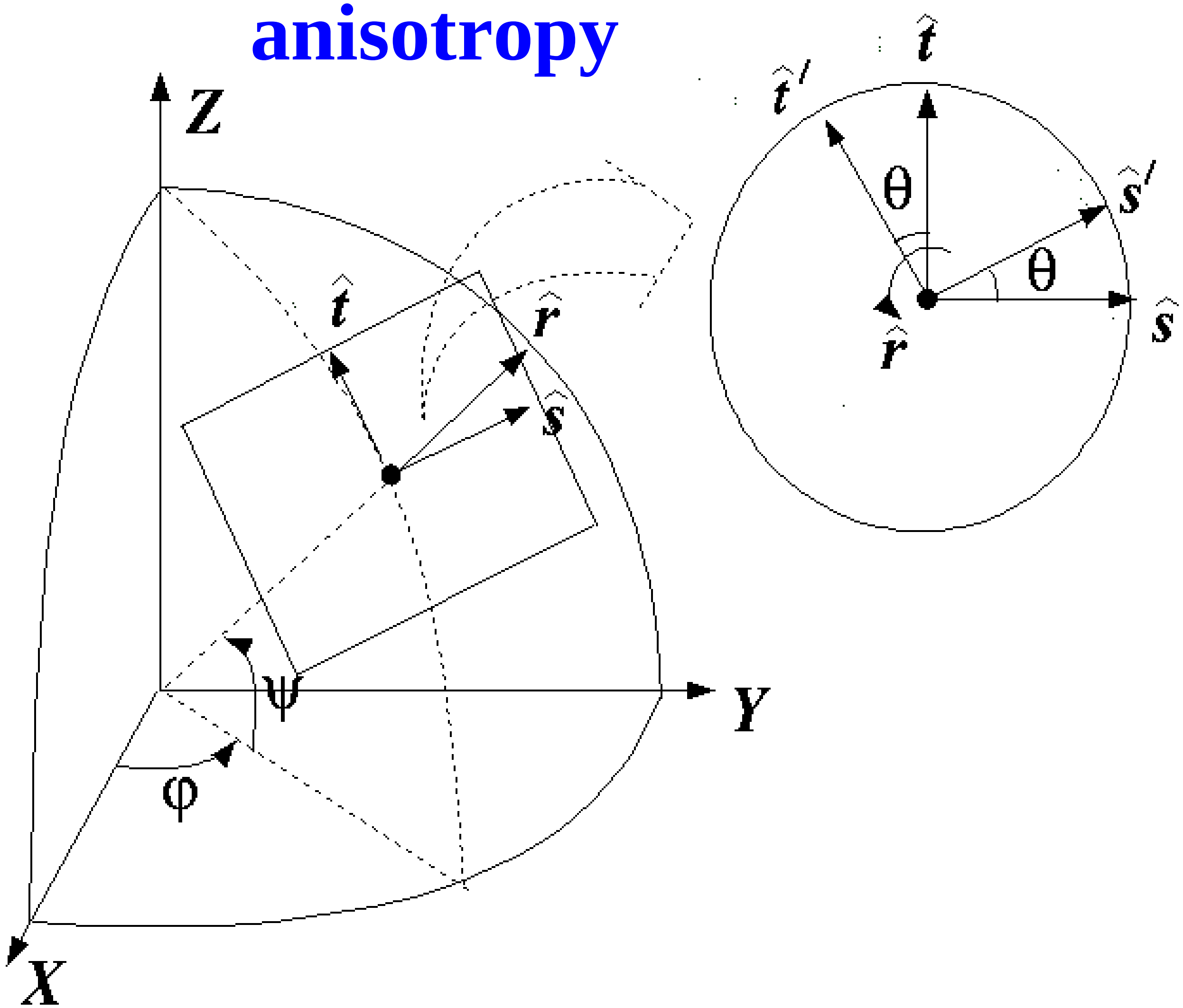
(a) Unit-cell



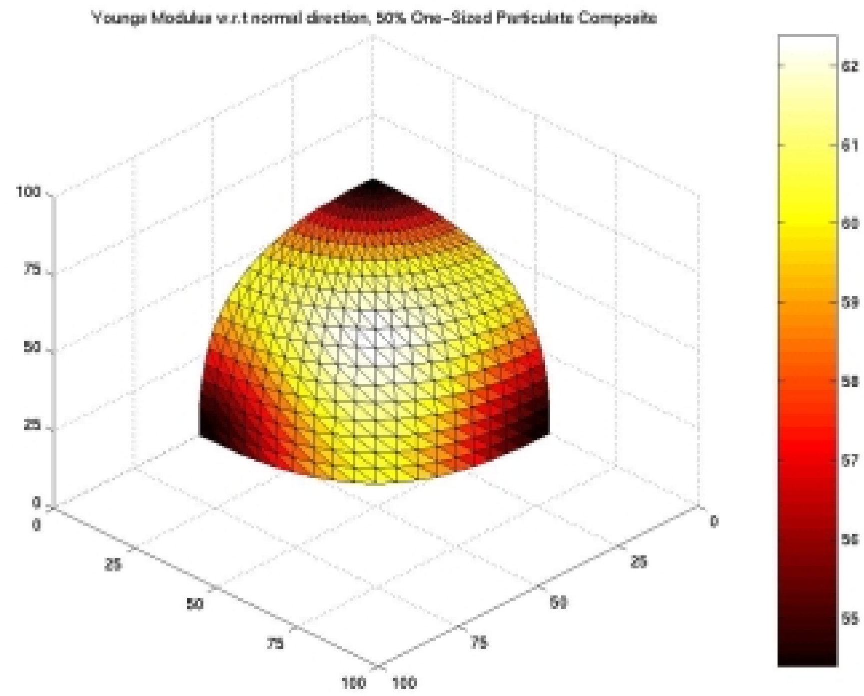
(b) Reinforcement only

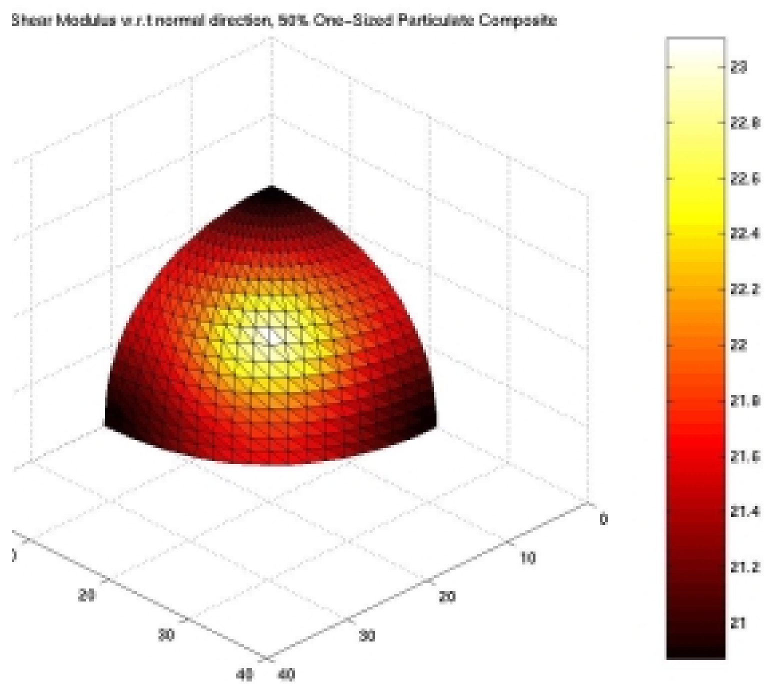


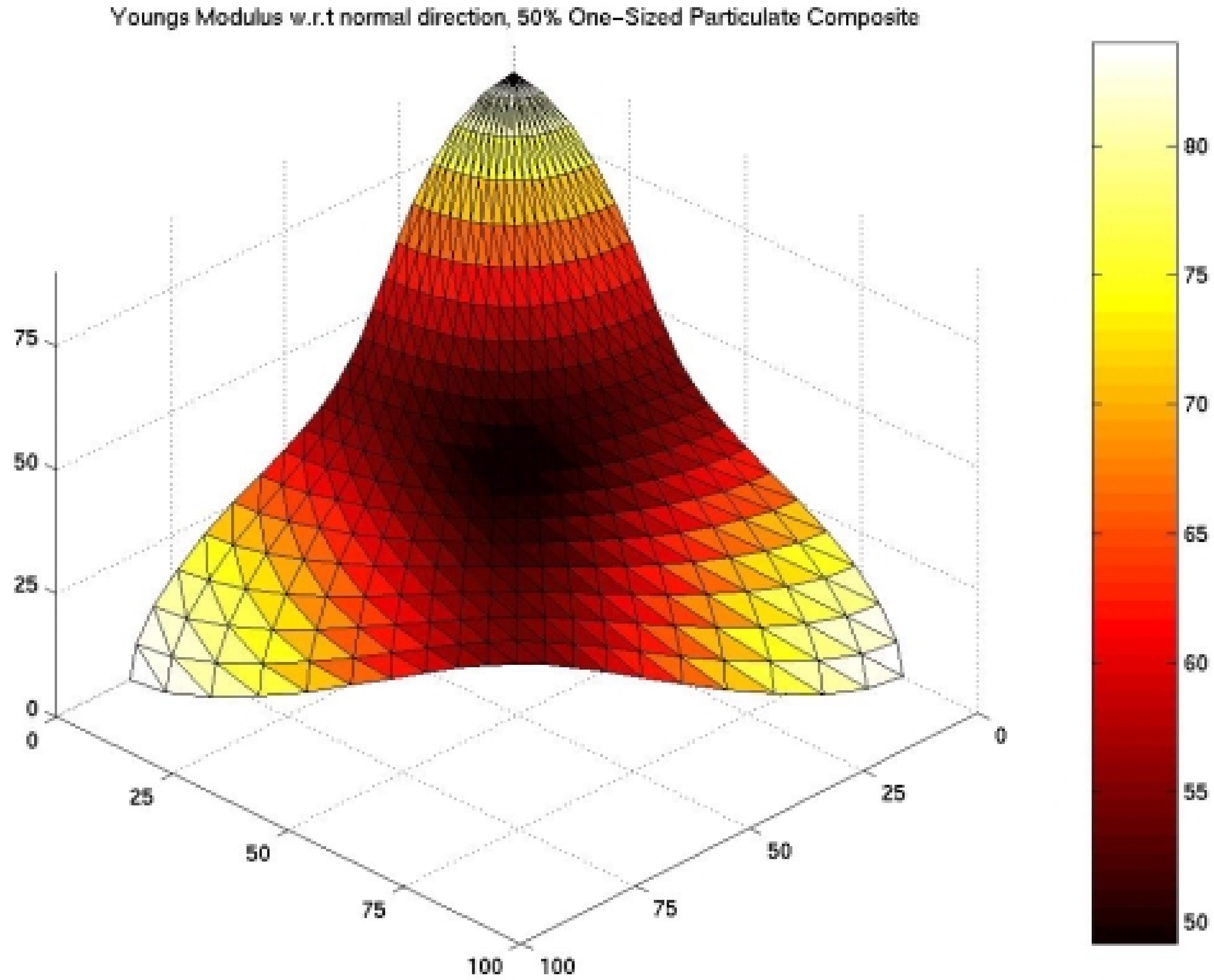
Framework for discussing elastic anisotropy

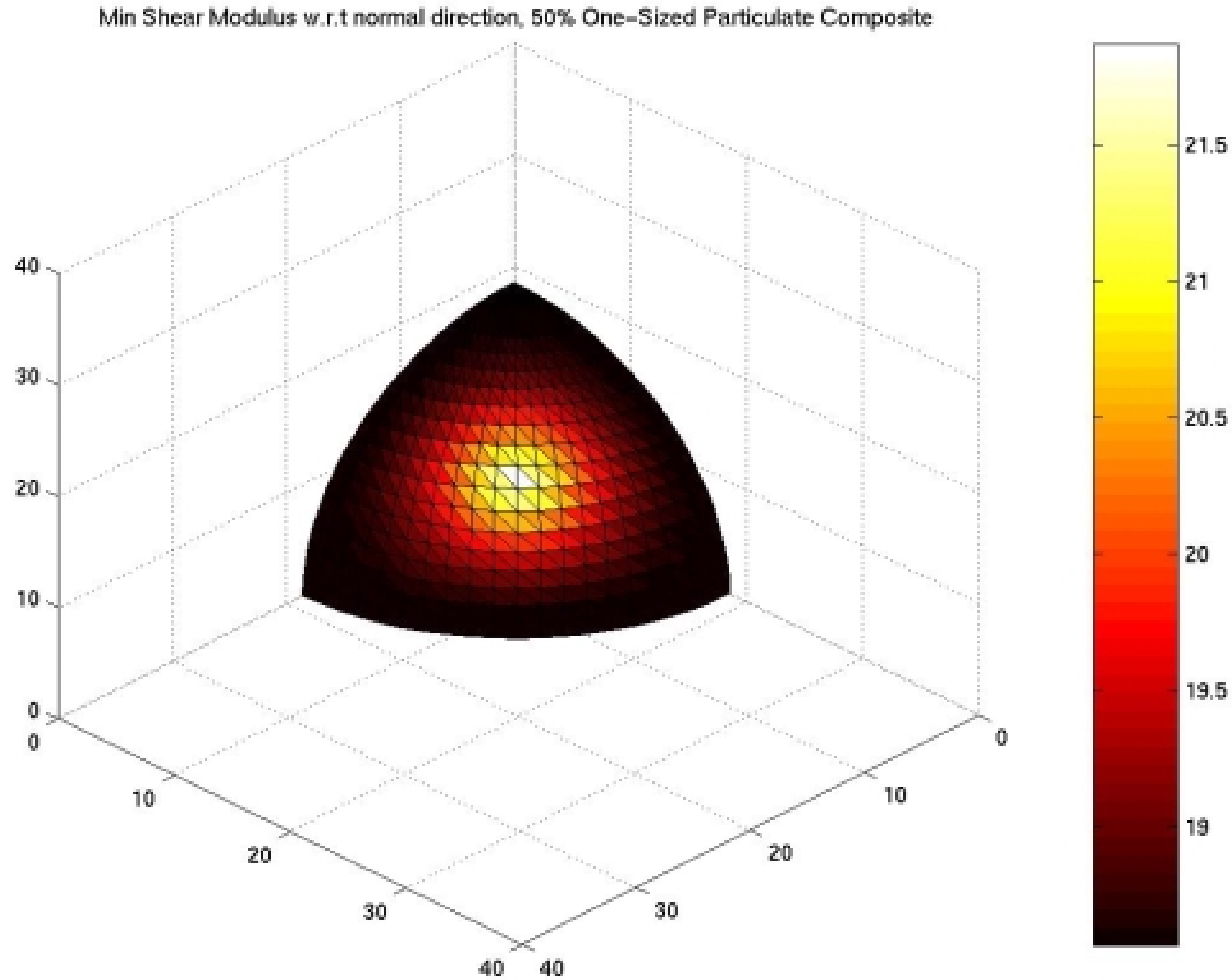


Anisotropy of E and G for spherical reinforcement, 50% volume fraction.






Youngs Modulus w.r.t normal direction, 50% One-Sized Particulate Composite
0
25
50
75
0
25
50
75
100
100
75
50
25
0
90
75
70
65
60
55
50


Min Shear Modulus w.r.t normal direction, 50% One-Sized Particulate Composite
40
30
20
10
0
0
10
20
30
40
40
30
20
10
0
21.5
21
20.5
20
19.5
19

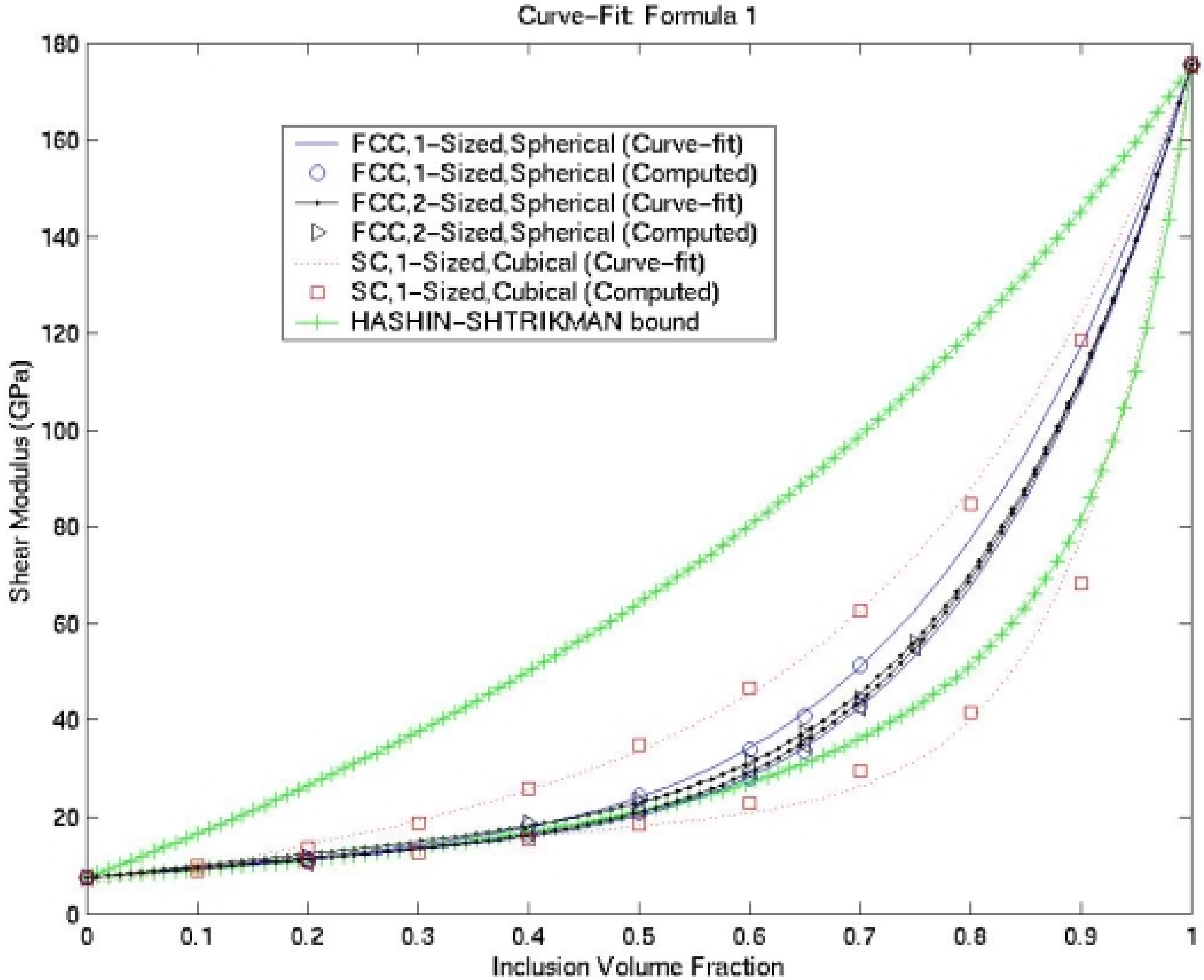
Variation of homogenized elastic constants with orientation for different particulate-reinforced composites with 50% SiC particle volume fraction.

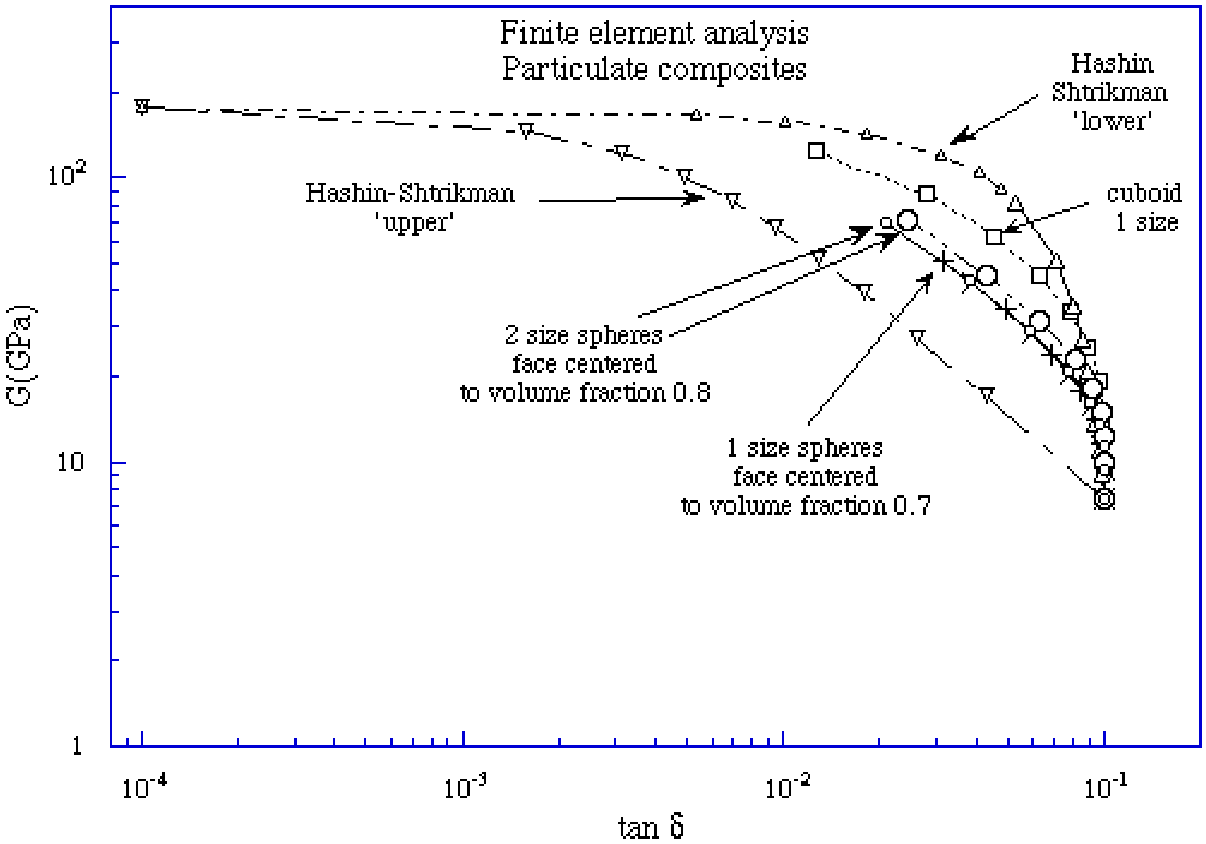
Arrangement/ Particles	Young's modulus		Shear modulus	
	min	max	min	max
FCC/one-sized spherical particl es	54.4	62.4	21.0	24.4
FCC/Two-sized particles	56.1	59.8	21.7	23.4
BCC/One-sized cubical particl es	49.2	84.0	18.6	35.0

$$G = G_0(1 + A_1(\phi - \phi^{A_2}) - \phi^{A_3}) + G_1\phi^{A_4}$$

Coefficients used to fit shear modulus versus volume fraction results.

Type of composite	A_1	A_2	A_3	A_4
FCC Single-sized spheres, Upper limit	2.698	4.527	4.527	4.527
FCC Single-sized spheres, Lower limit	2.452	5.340	5.340	5.340
FCC Two-sized spheres, Upper limit	3.185	5.339	5.339	5.339
FCC Two-sized spheres, Lower limit	2.522	5.223	5.223	5.223
BCC One-sized cubicles, Upper limit	7.172	4.584	0.364	4.584
BCC One-sized cubicles, Lower limit	2.7890	9.9849	9.9849	9.9849


Curve-Fit Formula 1
FCC,1-Sized,Spherical (Curve-fit)
FCC,1-Sized,Spherical (Computed)
FCC,2-Sized,Spherical (Curve-fit)
FCC,2-Sized,Spherical (Computed)
SC,1-Sized,Cubical (Curve-fit)
SC,1-Sized,Cubical (Computed)
HASHIN-SHTRIKMAN bound
Shear Modulus (GPa)
Inclusion Volume Fraction
0
20
40
60
80
100
120
140
160
180
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Finite element analysis
Particulate composites
Hashin
Shtrikman
'lower'
cuboid
1 size
Hashin-Shtrikman
'upper'
2 size spheres
face centered
to volume fraction 0.8
1 size spheres
face centered
to volume fraction 0.7
G (GPa)
tan δ

Summary of Results

- **With polymer matrix composites best performance is $G \tan \delta \sim 0.23$ GPa.**
- **With cubical SiC inclusions in InSn matrix, best $G \tan \delta \sim 2.7$ GPa**
- **With single-sized spherical SiC inclusions in InSn matrix, best $G \tan \delta \sim 1.6$ GPa**
- **With two-sized spherical SiC inclusions in InSn matrix, best $G \tan \delta \sim 1.7$ GPa**