5.3. Find $I_0$ using linearity, assumption $I_0 = 1\text{ mA}$

![Circuit Diagram]

**Solution:** If we assume $I_0 = 1\text{ mA}$, set node 0 as reference.

$$U_2 = I_0 \cdot (4k+2k) = 1\text{ mA} \cdot 6k = 6\text{ V}$$

Then:

$$I_2 = \frac{U_2}{4k} = \frac{6\text{ V}}{4k} = 1.5\text{ mA}$$

Use KCL @ Node 2: $I_2 = I_1 + I_0 = 1\text{ mA} + 1.5\text{ mA} = 2.5\text{ mA}$

For $I_2 = \frac{U_1 - U_2}{4k} = \frac{U_1 - 6}{4k} = 2.5\text{ mA}$

$\Rightarrow U_1 = 2.5\text{ mA} \cdot 4k + 6 = 16\text{ V}$

$$I_3 = \frac{U_1}{12k+4k} = \frac{16\text{ V}}{16k} = 1\text{ mA}.$$  

Use KCL @ Node 0: $I_s = I_3 + I_2 = 1\text{ mA} + 2.5\text{ mA} = 3.5\text{ mA}$

Therefore, the assumption $I_0 = 1\text{ mA}$ produced a source current of 3.5 mA. Since the actual source current is 4 mA, the output current is $1\text{ mA} \left(\frac{4}{3.5}\right) = \frac{8}{7}\text{ mA}$

$$I_0 = \frac{8}{7}\text{ mA}$$
Find \( I_0 \), using superposition.

\[ 12k\Omega \]

\[ I_0 \quad 12k\Omega \quad 4mA \]

\[ 12k\Omega \quad 12k\Omega \]

\[ 12k\Omega \quad 12k\Omega \]

\[ + \quad 6V \quad - \]

\[ 12k\Omega \]

\[ 12k\Omega \]

**Solution:**

1. Turn off the 6V voltage source, replace it with a short circuit.

\[ 12k\Omega \]

\[ I_0 \quad 12k\Omega \quad 4mA \]

\[ 12k\Omega \quad 12k\Omega \]

\[ 12k\Omega \quad 12k\Omega \]

\[ + \quad 6V \quad - \]

\[ 12k\Omega \]

\[ 12k\Omega \]

\[ I_1 \]

\[ 12k\Omega \]

\[ I_2 \]

\[ 12k\Omega \]

\[ 12k\Omega \]

\[ I_0 \]

\[ 4mA \]

\[ I_1 + I_2 = 4mA \quad (KCL) \quad (1) \]

\[ \frac{I_1}{I_2} = \frac{12k}{12k + (12k + 12k)} \quad \text{(current divider)} \quad (2) \]

\[ \Rightarrow \frac{I_1}{I_2} = \frac{2}{3} \]

\[ I_0 = \frac{12k}{12k + 12k} I_1 \quad \text{(current divider)} \quad (3) \]

From (1), (2), (3) \[ I_1 = 1.6mA \quad I_0' = 0.8mA \]
(2) Turn off the 4mA current source, replace it with an open circuit.

Use Node Analysis. Set Node 0 as reference. Use KCL @ Node 1.

\[ I_{0}'' + I_{2} + I_{1} = 0 \]  \( \text{(4)} \)

\[ I_{0}'' = \frac{V_{1} - 6}{12k} \]  \( \text{(5)} \)

\[ I_{1} = \frac{V_{1}}{12k} \]  \( \text{(6)} \)

\[ I_{2} = \frac{V_{1}}{12k + 12k} \]  \( \text{(7)} \)

Substitute (5), (6), (7) to (4)

\[ \frac{V_{1}}{12k} + \frac{V_{1}}{24k} + \frac{V_{1}}{12k} = 0 \Rightarrow V_{1} = 2.4 \text{ V} \]

\[ I_{0}'' = \frac{V_{1} - 6}{12k} = \frac{2.4 - 6}{12k} = -0.3 \text{ mA} \]

\[ I_{0}'' = -0.3 \text{ mA} \]

\[ I_{0} = I_{0}'' + I_{0}''' = 0.8 \text{ mA} - 0.3 \text{ mA} = 0.5 \text{ mA} \]

\[ I_{0} = 0.5 \text{ mA} \]
5.23 Use Thevenin's theorem to find $V_0$. 

![Circuit Diagram]

(1) Cut off $4k\Omega$ to make a two-terminal network.

![Circuit Diagram]

(2) According to Thevenin's theorem, network above is equivalent to

![Circuit Diagram]

(3) Find $V_{th}$, when open-circuit the two terminals.

\[ V_{th} = V_{oc} \]

\[ V_{oc} = V_{AD} = V_{AB} + V_{CD} \]

Using voltage divider to find $V_{AB}$

\[ V_{AB} = \frac{6k\Omega}{3k\Omega + 6k\Omega} \cdot 12V = \frac{2}{3} \cdot 12V = 8V. \]

Using Ohm's law on the right part of the network.

\[ V_{CD} = 2mA \cdot 2k\Omega = 4V \]

So \[ V_{th} = V_{oc} = V_{AD} = 8V + 4V = 12V. \]

(4) Find $R_{th}$, $R_{th}$ is the equivalent resistance when set zero to the independent sources.

(See next page.)
When set zero to voltage source then it turns into short circuit.
When set zero to current source then it turns into open circuit.

3kΩ and 6kΩ are in a parallel combination.

\[ R_{eq} = \left( \frac{3k\Omega \cdot 6k\Omega}{16k\Omega} \right) + 2k\Omega \\
= 2k\Omega + 2k\Omega \\
= 4k\Omega \]

5. Put 4kΩ back to get the complete circuit.

Using voltage divider

\[ V_o = \frac{4k\Omega}{4k\Omega + 4k\Omega} \cdot 12V \]

\[ V_o = 6V. \]
5.48 Find $V_o$ using Norton's theorem.

1. Cut off $3k\Omega$ to make a two-terminal network.

2. According to Norton's theorem, network above is equivalent to:

3. Find $I_N$ when short-circuit the two terminals.

Using loop analysis to solve $I_{sc}$

1) Choose loop as follows:
2) Using KVL on each Loop

Loop 1: \(-3V + 2k (I_1 - I_3) + 1k (I_1 - I_2) = 0\)
Loop 2: \(I_2 = 1mA\)
Loop 3: \(1k (I_2 - I_3) + 2k (I_3 - I_1) = 0\)

Simplify them:

\[
\begin{cases}
2k I_2 - 1k I_2 - 2k I_3 = 3V \\
I_2 = 1mA \\
-2k I_1 - 1k I_2 + 3k I_3 = 0
\end{cases}
\]

the solution is:

\[
I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \frac{14}{5} mA \\ 1mA \\ \frac{11}{5} mA \end{bmatrix}
\]

so \(I_{sc} = I_3 = \frac{11}{5} mA\)

4) Find \(R_{th}\). \(R_{th}\) is the equivalent resistance when set zero to the independent sources

\[
R_N = R_{eq} = \left( \frac{1k \Omega}{2k \Omega} \right) + 1k \Omega = \frac{5}{3} k\Omega
\]

5) Put 3k\(\Omega\) back to get the complete circuit.

\[
V_0 = \frac{11}{5} mA \cdot \left( \frac{5}{3} k\Omega || 3k\Omega \right) = \frac{33}{14} V
\]
Current source 2mA → Voltage source $V_i$

$V_i = 2mA \times 6k\Omega = 12V$

Combine $12V$ and $6V$ → $V_2 = 12V - 6V = 6V$

Voltage source $6V$ → Current source $I_1$

$I_1 = \frac{6V}{6k\Omega} = 1mA$

Voltage source $3V$ → Current source $I_2$

$I_2 = \frac{3V}{6k\Omega} = 0.5mA$

$1mA = I_1$
Combine \(6k\Omega \times 6k\Omega = 6k\Omega / 6k\Omega = 3k\Omega\)

Combine \(I_1\) \& \(I_2\): \(I_1 - I_2 = 0.5\text{mA}\)

Current division:

\[
I_0 = -0.5\text{mA} \times \left[ \frac{3k\Omega}{3k\Omega + 2k\Omega} \right] = -0.3\text{mA}
\]

\[
I_0 = -0.3\text{mA}
\]
5.105 Find $R_L$ for maximum power transfer and the maximum power.

![Circuit Diagram]

**Step 1:** take out $R_L$

- Simply the network using Thévenin's theorem, find $V_{th}$ & $R_{th}$

**Step 2:** combine the simplified network with $R_L$, $R_L = R_{th}$.

$P_L = \left(\frac{V_{th}}{R_L + R_{th}}\right)^2 \cdot R_L$

---

**Step 1:**

- $30V$ to $a$, $12\Omega$

\[ I = \frac{30}{12 + 4 + 4} = 1.5A \]

\[ V_{ab} = I \cdot (4\Omega + 4\Omega) = 12V \]

\[ V_x = I \cdot 4\Omega = 6V \]

We know $V_{ab} = V_{th} - 4V_x$

\[ V_{th} = V_{ab} + 4V_x = 12 + 4 \times 6 = 36V \]

**Step 2:**

- Find $R_{th}$

\[ V_{ab} = -4V_x \Rightarrow I_x = \frac{V_{ab}}{4 + 4} = \frac{-4V_x}{8} = -\frac{V_x}{2} \]

\[ I_x \text{ is current on } R_1 \Rightarrow I_x = \frac{V_x}{4} \]

\[ \Rightarrow -\frac{V_x}{2} = \frac{V_x}{4} \Rightarrow V_x = 0V \]

\[ \Rightarrow I_x = \frac{V_x}{4} = 0 \Rightarrow 4\Omega \text{ & } 4\Omega \text{ are open circuit} \]

\[ V_x = 0 \Rightarrow \text{Source } 4V_x \text{ is short circuit} \]
We can simplify network like this:

\[ 30V \quad \text{Isc} \quad a \]

\[ b \]

\[ I_{sc} = \frac{30V}{12\Omega} = 2.5A \]

\[ R_{th} = \frac{V_{th}}{I_{sc}} = \frac{36}{2.5} = 14.4 \Omega \]

**Step 2: Combine RL with simplified network**

\[ V_{th} \quad \text{R}_{th} \quad \text{R}_L \]

\[ P_L = R_{th} = 14.4 \Omega \]

\[ P_L = \left( \frac{V_{th}}{R_L + R_{th}} \right)^2 \cdot R_L = \left( \frac{36}{14.4 + 14.4} \right)^2 \times 14.4 \]

\[ P_L = 22.5 \text{W} \]