Risk-Based Considerations in Developing Strategies to Ensure Pipeline Integrity—Part I: Theory

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As pipelines age, a flaw population that varies initially along the pipeline can advance in size and number. Analysis of the serviceability of pipelines based on either in-line inspection or hydrotesting can lead to overly conservative decisions or an excessive risk of failure when the random nature of this population and the pipeline's properties are represented by a "typical" flaw and "average" properties. It follows that decisions on serviceability should reflect the random nature of the variables involved or be justified by demonstrating that the uncertainty in these parameters does not adversely affect cost and safety. This is the first in series of two papers generated from a recent study on risk-based analysis for developing strategies to ensure pipeline integrity. In this paper (Part I—Theory), a new probabilistic methodology is developed to conduct fracture evaluations of pipelines subjected to ductile flaw growth in service. The study is made under the assumption that continuing serviceability is based on the use of hydrotesting. The analysis involves time-dependent elastic-plastic fracture mechanics for the underlying deterministic model, and Monte Carlo simulation for structural reliability analysis. Using these models, pipe fracture evaluations will be conducted in the light of a hydrotest-based approach to ensure pipeline integrity. They will be discussed in the companion paper, Part II—Applications (Rahman and Leis, 1994).

1 Introduction

Because of stress-corrosion cracking (SCC) that occurs occasionally on the external surface of pipelines, defects present since construction can advance in size and new defects can be introduced to the initial population and also grow during service. Historically, safe operation of thousands of miles of natural gas transmission pipelines underscores the merits of hydrotesting as a means to verify the integrity of the line as constructed and to demonstrate continuing serviceability through the use of periodic retesting programs. Hydrotesting is currently the only viable means to detect and control certain types of defects, such as SCC. Hydrotesting will remain the sole means to ensure safe serviceability for the many lines that cannot pass in-line inspection tools. Therefore, hydrotesting is and will continue as a key tool in the management arsenal to ensure safe long-term service for the gas transmission pipeline network.

The evolution of the defect population with service means that the defect population at any instant in time of the pipeline is a random variable. The mechanical properties and the toughness of the pipe steel are similarly random variables along the pipeline as is the pressure loading during service. However, during hydrotesting the pressure history is closely controlled and varies only in a well-defined manner as a function of the pipeline's elevation. The variability in properties and defect population can confound decisions as to which test pressure provides the optimum balance between the number and size of defects that will be removed in the test versus the interval between hydrotests and the likelihood of an in-service failure. This variability also can complicate serviceability decisions based on in-line inspection results, which introduces the added uncertainty in the measurement of the flaw population.

Regardless of whether serviceability is based on in-line inspection or hydrotesting, the random nature of the flaw population and the properties can lead to overly conservative decisions or an excessive risk of failure when the random nature of this population and the loadings are represented by a "typical" flaw and "average" properties. It follows that the statistical variability in properties and defect population can confound the ability of methods like in-line inspection and hydrotesting to ensure safe operation when they are based on a simple characterization of the flaw population. Therefore, decisions on serviceability should reflect the random nature of the variables involved or be justified by demonstrating that the uncertainty in these parameters does not adversely affect cost and safety (Leis and Rahman, 1993).

This is the first in series of two papers generated from a recent study on risk-based analysis for developing strategies to ensure pipeline integrity. In this paper (Part I—Theory), a new probabilistic methodology is developed to conduct fracture evaluations of pipelines subjected to ductile flaw growth

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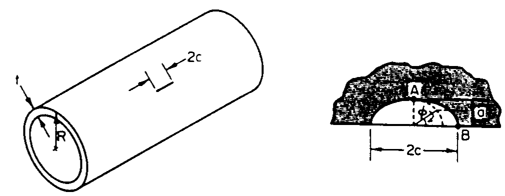
in service. The study is made under the assumption that continuing serviceability is based on the use of hydrotesting. The method of analysis involves 1) time-dependent elastic-plastic fracture mechanics for the underlying deterministic models, and 2) direct Monte Carlo simulation for conducting structural reliability analysis. The paper begins with a brief description of primary creep crack growth damage models and then formulates the deterministic J-integral equations for axial part-through-wall cracked pipes under internal pressure. Thereafter, the foregoing deterministic model is placed in a probabilistic framework to evaluate structural integrity of pipelines subjected to ductile flaw growth in service. Using these models, pipe fracture evaluations will be conducted in the light of a hydrotest-based approach to ensure pipeline integrity. They will be discussed in the companion paper (Rahman and Leis, 1994).

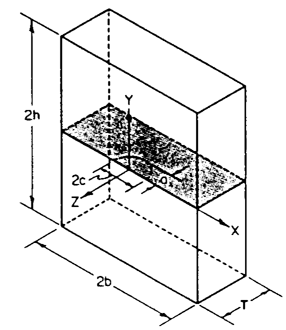
2 Analysis of Axial Flaws in Pipe

Pre-service and in-service inspections of typical gas transmission pipelines indicate that axial part-through-wall (PTW) cracks can be distributed along the length of a line (Leis and Rahman, 1993; Duffy et al, 1968). Cracks that would limit the initial serviceability or safety are removed from the crack population prior to service. Cracks that remain are small and in the absence of in-service growth could remain in the line at maximum operating pressure without further concern. However, in cases where these cracks can grow because the in-service loading and environment causes SCC, it is very important to be able to quantify the effects of that crack growth on the integrity of the pipeline. Because axial cracks are most responsive to the pressure loading of a pipeline, the following analysis focuses on fracture-mechanics controlled behavior of axial PTW cracks. Cases where the flaw depth approaches either zero or the wall thickness is characterized by ductile response of the pipe steel rather than by fracture mechanics are discussed by Leis et al. (1991). It follows that predictions of the population of axial PTW flaws using fracture mechanics are valid only when the failure of the pipe is controlled by fracture mechanics considerations. Similarly, probabilistic analyses based on fracture mechanics will be relevant only when failure is characterized by fracture mechanics. Predictions in the present paper thus are relevant between the aforementioned limits in the flaw size. This formulation could be easily extended for other cases such as moving lines, river crossings, and subsidence where significant bending and tension can promote the growth of circumferentially oriented cracks. It could similarly be adapted to other facets of pipeline operation, such as the safety and serviceability of compressor stations (e.g., engines and compressors). The ensuing sections develop and validate the analytical basis for the assessment of the significance of the random nature of the flaw population and the properties. Readers interested only in the results should proceed directly to the Results and Discussion section presented in the companion paper (Rahman and Leis, 1994).

2.1 Primary Creep Crack Growth. Consider a pipe, which has mean radius, R, and wall thickness, T, containing an axial semi-elliptical PTW crack with an initial length, $2c$, and maximum depth, a. Figure 1(a) shows the geometric parameters for this axial PTW crack in a pipe. The pipe is subjected to an internal pressure, p, which is held for some period of time, t, as occurs in a typical hydrotest. During the loading, a plastic zone develops around the crack tip that can be described by the Hutchinson-Rice-Rosengren (HRR) field (Hutchinson, 1982). The size of this zone depends on the magnitude of the load. As the hold period begins and primary creep deformation develops, the crack-tip stress field may decrease or increase depending on the material properties. For the type of material (line-pipe steel) dealt with here, the stresses at the crack tip



(a) Surface-cracked pipe



(b) Flat plate analog

Fig. 1 Geometric parameters for axial part-through-wall flaws in a pipe

increase, and effectively the plastic zone grows in size. For the type of material considered by Riedel (1986), crack-tip stresses relax as a creep zone grows within the plastic zone. They are discussed in the forthcoming.

For $t>0$, creep straining begins as the external load is held constant. The simplified power-law constitutive response in the rate form ($t>0$) can be represented by (Brust and Leis, 1992)

$$\dot{\epsilon}_{ij}=\dot{\epsilon}^{e}_{ij}+\dot{\epsilon}^{p}_{ij}+\dot{\epsilon}^{c}_{ij} \tag{1}$$

where

$$\dot{\epsilon}^{e}_{ij}=\frac{1+\nu}{E}\,\dot{\sigma}_{ij}-\frac{\nu}{E}\,\dot{\sigma}_{kk}\delta_{ij} \tag{2}$$

$$\dot{\epsilon}^{p}_{ij}=\frac{3}{2}\,B_0 n_p \sigma_e^{n_p-2}\dot{\sigma}_e S_{ij} \tag{3}$$

$$\dot{\epsilon}^{c}_{ij}=\frac{3}{2}\,B_1 \sigma_e^{N_c-1}(\epsilon^c_e)^{-q} S_{ij} \tag{4}$$

in which ϵ_{ij} is total strain with the associated superscripts e, p, and c representing its elastic, plastic, and creep components, σ_{ij} is total stress, $S_{ij}=\sigma_{ij}-(1/3)\sigma_{kk}$ is deviatoric stress, σ_e and ϵ_e are equivalent stress and strain, δ_{ij} is kronecker delta, E is elastic modulus, ν is Poisson's ratio, N_c, q, and B_1 are material constants for strain-hardening creep law, and B_0 and n_p are power-law-hardening plasticity constants. The overdot in Eqs. (1)-(4) represents material time derivative. For a time-hardening creep law, the creep strains can be represented by

$$\dot{\epsilon}^{c}_{ij}=\frac{3}{2}\,m\gamma\sigma_e^{n_c-1}t^{m-1}S_{ij} \tag{5}$$

The relations between the time and strain-hardening material constants are (Brust and Leis, 1992)

$$N_c = \frac{n_c}{m}$$

$$q = \frac{1-m}{m}$$

$$B_1 = m\gamma^{1/m} \quad (6)$$

where n_c, γ, and m are also material constants. For the situation of concern here, little difference between strain and time-hardening solutions will occur if $n_p > n_c$; whereas for the materials satisfying $n_c > n_p$ and considered by Riedel (1986), a significant difference in results may exist between time and strain-hardening solutions.

If $n_c > n_p$, which is the situation examined extensively in the literature, the $\dot{\epsilon}_{ij}^c$ term of Eq. (1) dominates very near the crack tip, and a creep zone develops at the crack tip which grows with time. The crack-tip stress state thus relaxes, and the crack-tip stress state becomes another HRR type of field growing within the plastic HRR field (see, for instance, Riedel (1986)). At time $t=0$, the strength of the stress singularity is

$$\sigma \propto r^{-\frac{1}{n_p+1}} \quad (7)$$

where r is the radial distance (coordinate) from the crack tip. After $t>0$, the strength of stress singularity approaches

$$\sigma \propto r^{-\frac{1}{n_c+1}}. \quad (8)$$

Clearly, for $n_c > n_p$, as is well known, the near-tip stress state decreases, although a relevant crack-driving parameter, such as the J-integral, will continue to increase.

If $n_p > n_c$, which is the requirement for the present analysis procedure, a plastic zone develops at the crack tip upon loading at time $t=0$. After the hold period begins, the stress state very near the crack tip increases rather than decreases, and the effective plastic zone increases in size. That this is so may be seen by observing that the $\dot{\epsilon}_{ij}^p$ term of Eq. (1) dominates the other terms. Hence, the time-independent plastic HRR field dominates the near-field stress and strain states. Thus, rather than switching from a "plastic" HRR field to a "creep" HRR field which occurs for $n_c > n_p$ and results in stress relaxation, here for $n_p > n_c$, the stress state remains "plastic" HRR field with stress singularity defined by Eq. (7) for $t>0$, and the stress state increases since the strength of the HRR field J increases.

For typical line-pipe steels from gas transmission pipelines under hydrotest, the power-law-hardening parameter for strength (plasticity), n_p is usually greater than that for creep parameter, n_c. Hence, the rest of the paper will focus on primary creep damage based on the condition, $n_p > n_c$. As discussed in the foregoing, the continued "loading" or "stressing" at the crack tip under conditions of a hydrotest is caused by the primary creep straining (Leis et al., 1991; Burst and Leis, 1992). Since the loading which occurs does not constitute a drastic change in loading path (proportionality) at each material point, it is assumed that the material does not exhibit any memory under these circumstances. In consequence, the stress state that is achieved after loading and subsequent hold at any given instant of time is not greatly different from that obtained by assuming time-invariant response but with material properties representative of the current time (Leis et al., 1991). Hence, the J-integral, $J(t)$, at any instant following creep deformation can be obtained by adding the time-invariant elastic component, J_e, and the time-variant plastic component, $J_p(t)$. Because of the functional dependence on time, time-dependent material properties obtained from isochronous (constant time) stress-strain curves can be used to calculate $J_p(t)$. When the stress-strain curve is represented by a suitable mathematical model, such as the Ramberg-Osgood model with time-varying parameters, simple closed-form equations can be developed for J_e and $J_p(t)$, as follows.

3 Deterministic Formulation

3.1 Material Properties. In conducting nonlinear fracture-mechanics analyses, several analytic idealizations are considered. For example, it is assumed that the constitutive law characterizing the steel's stress-strain response can be represented by the time-dependent Ramberg-Osgood model

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K(t)}\right)^{n(t)} \quad (9)$$

or the normalized version

$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha(t)\left(\frac{\sigma}{\sigma_0}\right)^{n(t)} \quad (10)$$

where σ_0 is an arbitrary reference stress usually assumed to be yield stress, E is the elastic modulus, $\epsilon_0 = \sigma_0/E$ is the associated reference strain, and $K(t)$, $\alpha(t)$, $n(t)$ are time-variant, strain-hardening parameters usually chosen from a best fit of test data. Note that the Eqs. (9) and (10) are equivalent if $\alpha(t) = E\sigma_0^{n(t)-1}/K(t)^{n(t)}$, which provides a means to calculate $\alpha(t)$ [or $K(t)$] when $K(t)$ [or $\alpha(t)$] and $n(t)$ are known for a given material. For typical gas transmission pipe materials, the time-dependent Ramberg-Osgood parameters also admit a multiplicative decomposition of the form (Leis et al., 1991)

$$\alpha(t) = \alpha_0 f_1(t)$$

$$n(t) = n_0 f_2(t)$$

$$K(t) = K_0 f_3(t) \quad (11)$$

where α_0, n_0, and K_0 are the initial values (i.e., at time, $t=0$) and $f_1(t)$, $f_2(t)$, and $f_3(t)$ are the time-varying functions, which can be obtained from the isochronous test data (Leis et al., 1991). Table 1 shows typical functional values of f_1 and f_2 derived from Leis et al. (1991) for X65 line-pipe steel. (Note: f_3 is the dependent function and can be obtained when E, σ_0, α_0, n_0, K_0, f_1, and f_2 are known.) Also, the J-resistance from the compact tension (CT) specimens is deemed to be adequately characterized by linear equation of the form (Leis et al., 1991)

$$J_R(\Delta a) = J_{Ic} + C\Delta a \quad (12)$$

in which Δa is the extension of crack length during crack growth, J_{Ic} is the fracture toughness at crack initiation, and $C = dJ_R/da$ is the slope parameter from best fit of experimental data. In the absence of CT specimen data, these toughness parameters J_{Ic} (k/in) and C (k/in^2) can also be obtained from empirical correlation with full-size Charpy plateau energy, CVP (ft-lb) and flow stress, σ_f (ksi) as (Leis et al., 1991)

Table 1 Deterministic functions $f_1(t)$ and $f_2(t)$ characteristic of X65 line steel

Time, t (s)	$f_1(t)$	$f_2(t)$
0	1.00	1.00
6	2.26	0.85
16	2.36	0.83
30	2.59	0.82
45	2.72	0.81
60	2.79	0.80
100	2.90	0.79
500	3.21	0.78
1500	3.41	0.77
3600	3.58	0.76
6000	3.68	0.76
10000	3.79	0.75
100000	4.30	0.74

$$J_{Ic} = 14.92 \times 10^{-5} \sigma_f \text{CVP}$$

$$C = \frac{dJ_R}{da} = \frac{108.2\sigma_f \text{CVP}}{E} \quad (13)$$

where $\sigma_f = (\sigma_y + \sigma_u)/2$ is the average of yield stress, σ_y and ultimate stress, σ_u.

3.2 Time-Dependent Elastic-Plastic Fracture. Because line-pipe steels experience only limited inelastic action at crack tips, the primary creep model for evaluating $J_p(t)$ for PTW surface cracks is based on the modification of elastic solutions. For semi-elliptical surface cracks subjected to tensile loading (due to membrane hoop stress), detailed compilations of finite-element solutions are readily available in the literature if elastic conditions prevail and the cracking in the pipeline can be represented by a flat-plate analog (Fig. 1(*b*) (Newman, 1979; Raju and Newman, 1986; Newman and Raju, 1981). Adaptation to the thin-walled gas transmission pipelines ($R/T \geq 40$) with flaw geometries of interest usually involves $a/T \geq 0.5$ and $c/a >> 5$, for which the effects of local bending and increased tension on stress intensity factor cannot be ignored. Hence, existing elastic solutions based on flat-plate analog may not be adequate for pipe geometries with deep long flaws. Further details on this subject and comparisons between new and existing solutions are available in Leis et al. (1991).

The primary creep model adopted in this paper consists of estimating $J_p(t)$ by using a time-dependent Irwin estimate of the plastic zone inserted into the elastic solution. Hence, the method relies on the small-scale yielding assumption. However, reasonably accurate predictions are reported even when this assumption is not strictly satisfied (Leis et al., 1991). Detailed derivations of J_e and $J_p(t)$ are available in Leis et al. (1991). For brevity, only the final expressions are given in this paper.

Elastic Component. The time-invariant elastic component, J_e for axial PTW cracked pipe is given by (Leis et al., 1991)

$$J_e = \left(\frac{pR}{T}\right)^2 \frac{\pi a}{Q} \frac{F(a/c,\, a/T,\, R/T,\, \phi)^2}{E'} \quad (14)$$

where

$$E' = \begin{cases} E & \text{for plane stress } (\phi = 0, \text{ point B of Fig. } 1(a)) \\ \dfrac{E}{1-\nu^2} & \text{for plane strain } (\phi = \dfrac{\pi}{2}, \text{ point A of Fig. } 1(a)) \end{cases} \quad (15)$$

in which E is the elastic modulus, ν is Poisson's ratio, Q is the square of the complete elliptical integral of the second kind, which can be accurately approximated by (Newman and Raju, 1981)

$$Q = \begin{cases} 1 + 1.464(a/c)^{1.65} & \text{when } a/c \leq 1 \\ 1 + 1.464(a/c)^{-1.65} & \text{when } a/c \geq 1 \end{cases} \quad (16)$$

The angle ϕ characterizes the location along the elliptical crack front where J_e is evaluated (see Fig. 1(*a*)), and F is the geometry function generically depending on a a/T, a/c, R/T, and ϕ.

Plastic Component. The time-variant plastic component, $J_p(t)$, for axial PTW cracked pipe is given by (Leis et al., 1991)

$$J_p(t) = \left(\frac{pR}{T}\right)^2 \frac{\pi r_p(t)}{Q} \frac{F(a/c,\, a/T,\, R/T,\, \phi)^2}{E'} \quad (17)$$

where $r_p(t)$ is the time-dependent plastic zone size ahead of crack tip, which can be estimated by the analogy to the approach suggested by Kujawski and Ellyn (1986) as

$$r_p(t) = \frac{2\hat{n}}{\hat{n}+1} \left(\frac{pR}{T}\right)^2 \frac{\pi a}{Q} F^2 \frac{1}{\beta \pi \sigma_y^2} \quad (18)$$

where

$$\beta = \begin{cases} 2 \text{ for plane stress } (\phi = 0, \text{ point B of Fig. } 1(a)) \\ 6 \text{ for plane strain } (\phi = \dfrac{\pi}{2} \text{ point A of Fig. } 1(a)) \end{cases} \quad (19)$$

$$\hat{n} = \frac{W_y^e + W_y^p}{W_y^e + \dfrac{W_y^p}{n(t)}} \quad (20)$$

with

$$W_y^e = \frac{\sigma_y^2}{2E} \quad \text{and} \quad W_y^p = \frac{n(t)}{1+n(t)} \sigma_y \alpha(t) \epsilon_0 \left(\frac{\sigma_y}{\sigma_0}\right)^{n(t)} \quad (21)$$

as the partitioned elastic and plastic energy densities, respectively, (Leis et al., 1991). Note that the time dependency in $r_p(t)$ and, hence, in $J_p(t)$, are achieved via time-varying Ramberg-Osgood parameters, $\alpha(t)$ and $n(t)$. The value of total $J(t)$ is then found by adding Eqs. (14) and (17) [i.e., $J(t) = J_e + J_p(t)$].

Both J_e and $J_p(t)$ in Eqs. (14) and (17) depend on the F-function. For $R/T = 40$ or larger, which is typical for gas pipelines, the F-function does not vary significantly with R/T, and hence, F can be conservatively assumed to be independent of R/T (Stonesifer et al., 1991). Thus, the value of F depends only on a/T, a/c, and ϕ for $R/T = 40$ or larger and is the key to estimating J_e and $J_p(t)$. The F-function used here was developed by Stonesifer et al. (1991) through the finite-element alternating method applied to axially surface-cracked pipes. Results from the alternating method with $R/T = 40$, for selected a/T and a/c conditions, are presented for the sake of illustrations here in Table 2 at $\phi = \pi/2$ (Leis et al., 1991; Stonesifer et al., 1991). These values of F characterize the behavior at $\phi = \pi/2$, which is the deepest part of the flaw and the location that controls the survival of pipe with part-though-wall flaws. All of the results reported in this paper are based on J-integral evaluations at the deepest point (point A of Fig 1(*a*)). For other values of a/T and a/c, F can be linearly interpolated between the tabulated values.

4 Probabilistic Formulation

4.1 Structural Reliability—Crack Growth Analysis. Structural-reliability analysis requires a mathematical model derived from the principles of mechanics and experimental data that relate various input random parameters for a specific performance criterion of interest. For a pipeline under pressure loading, p, the maximum load-carrying of the pipe limited by the presence of cracks, denoted as p_{max}, is obtained from the solution of two nonlinear equations based on J-tearing theory given by (Leis and Rahman, 1993; Leis et al., 1991)

Table 2 Values of flaw geometry function F at $\phi = \pi/2$

a/c	a/T		
	0.25	0.50	0.75
0.0	1.48	2.64	6.42
0.04	1.44	2.29	4.62
0.10	1.30	1.84	2.74
0.15	1.24	1.62	2.20
0.20	1.20	1.52	1.92
0.30	1.17	1.38	1.65
0.40	1.14	1.30	1.49
0.67	1.09	1.17	1.27
1.00	1.04	1.08	1.12
2.00	0.50	0.50	0.50

$$J(p_{max},a^*) = J_R(a-a^*)$$

$$\frac{\partial J}{\partial a}(p_{max},a^*) = \frac{dJ_R}{da}(a-a^*) \tag{22}$$

In this equation, a^* is the crack depth when internal pressure reaches p_{max} for a crack driving force J and a toughness J_R as defined earlier. It is based on the fact that fracture instability can occur after some amount of stable crack growth (i.e., from a to a^*) in ductile materials with an attendant higher load at fracture. The crack growth is assumed to occur in a geometrically self-similar manner.[1] This implies that the aspect ratio, $2c/a$ or a/c will remain constant throughout the crack growth, although it is random correlated with random crack size, $2c$. Standard techniques such as the Newton-Raphson method (Press et al., 1990) can be used to solve Eq. (22) for p_{max}.

Fracture-mechanics variables, which are inherently random, are 1) initial crack size, e.g., crack depth and length, and 2) material characteristics e. g., stress-strain properties and toughness properties of the pipe. Service conditions (e.g., stress levels, cyclic rate temperature, pressure, environment), particularly during a hydrotest and pipe geometry (e.g., pipe radius and thickness) for gas transmission pipes can be accurately calculated, and, hence, they will be assumed to be deterministic. Based on the formulation presented in this paper, the random variables are: a/T, $2c/a$, K_0, n_0, σ_y, σ_u, and CVP. Further details on the statistical characterization of these random inputs are discussed in the companion paper (Rahman and Leis, 1994).

In general, the solution of p_{max} can be represented by

$$p_{max} = h(a/T, 2c/a, K_0, n_0, \sigma_y, \sigma_u, \text{CVP}) \tag{23}$$

where h is a generic (implicit) response function of various input variables (only the random arguments are shown). The h-function can be evaluated when a relevant crack driving force from deterministic fracture (e.g., the J-integral from Eqs. (14) and (17)) and an appropriate fracture criterion (e.g., Eqs. (22)) are known. Suppose that the design or performance criteria requires p_{max} to be always greater than applied pressure p. This requirement cannot be satisfied with certainty because both flaw geometry and material properties are uncertain. Hence, the performance of the pipe should be evaluated by the reliability, P_S or its complement, the probability of failure, $P_F(P_S = 1 - P_F)$ defined as

$$P_F = Pr[g(\mathbf{X}) < 0] = \int_{g(\mathbf{x})<0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{24}$$

where

$$g(\mathbf{X}) = p_{max} - p = h(a/T, 2c/a, K_0, n_0, \sigma_y, \sigma_u, \text{CVP}) - p \tag{25}$$

is the performance function, $\mathbf{X} = \{a/T,\ 2c/a\ K_0,\ n_0,\ \sigma_y,\ \sigma_u,\ \text{CVP}\}$ is an input random vector characterizing uncertainty in the system parameters, and $f_{\mathbf{X}}(\mathbf{x})$ is known joint probability density function of $\mathbf{X}$. $f_{\mathbf{X}}(\mathbf{x})$ can be obtained following multiplication of conditional and/or marginal probability density functions of the component random variables. The probabilistic characteristics of these components are described in Rahman and Leis (1994). In general, the multi-dimensional integral in Eq. (24) cannot be determined analytically. As an alternative, numerical integration can be performed; however, this becomes impractical and computationally prohibitive when the dimension of $\mathbf{X}$ is greater than two. In the present case, we have seven dimensions.

Several approximate methods exist for conducting the multidimensional integration in Eq. (24). Some of them are first and second-order reliability methods (Madsen et al., 1986; Rahman, 1991; Rackwitz and Fiessler, 1978), importance sampling (Rahman, 1991; Melchers, 1984; Harbitz, 1986), directional simulation (Bjerager, 1988; Ditlevson, 1986), and Monte Carlo simulation (Rubinstein, 1981). In this paper, the computation of the failure probability, P_F, is based on direct Monte Carlo simulation (MCS). Work is currently underway to evaluate failure probability by more advanced methods of structural reliability theory. These advanced methods have been recently applied successfully for probabilistic pipe fracture evaluation of nuclear piping for leak-rate detection (Rahman et al., 1994; 1993; 1992).

[1] Self-similar growth is assumed, even though the actual behavior may cause the a/c ratio to increase slightly. This increase can be ignored because the F-function (Eqs. (14) and (17)) is very weakly dependent on such differences for most pipeline flaw geometries.

4.2 Monte Carlo Simulation. Consider a generic N-dimensional random vector $\mathbf{X}$, which characterizes certainty in the load and system parameters with the known joint distribution function $F_{\mathbf{X}}(\mathbf{x})$. Suppose $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(L)}$ are L realizations of input random vector $\mathbf{X}$, which can be generated independently. Methods of generating samples of $\mathbf{X}$ are available in Rubinstein (1981). Let $g^{(1)}, g^{(2)}, \ldots, g^{(L)}$ be the output samples of $g(\mathbf{X})$ corresponding to input $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(L)}$ that can be obtained by conducting repeated deterministic evaluation of the performance function in Eq. (25). Define L_f as the number of trials (analyses) which are associated with negative values of the performance function. Then, the estimate $P_{F,\,MCS}$ of the actual probability of failure, P_F, by simulation is given by

$$P_{F,\,MCS} = \frac{L_f}{L} \tag{26}$$

which approaches the exact failure probability, P_F, when L approaches infinity. When L is finite, a statistical estimate on the probability estimator may be needed. In general, the required sample size must be at least $10/\text{Min}(P_F,\ P_S)$ where $\text{Min}(P_F,\ P_S)$ is the minimum of P_F and P_S for a 30-percent coefficient of variation of the estimator (Rubinstein, 1981).

5 Summary and Conclusions

A probabilistic methodology was developed to determine structural integrity of axial part-through-wall cracked pipes subject to internal pressure. The study was made under the assumption that continuing serviceability is based on the use of hydrotesting. The method of analysis involved 1) time-dependent elastic-plastic fracture mechanics for the underlying deterministic models, and 2) direct Monte Carlo simulation for conducting structural reliability analysis. In the deterministic model, the J-tearing theory was extended to the time domain to account for primary creep damage in a pipeline. In the probabilistic model, the pipeline integrity was formulated in terms of failure probability, which was defined as the probability that the applied pressure in a given pipeline during service or a potential hydrotest exceeds its load-carrying capacity.

The companion paper (Rahman and Leis, 1994) provides numerical predictions in the light of a hydrotest-based approach to ensure pipeline integrity. In that paper, results from both deterministic and probabilistic models presented here were compared with the experimental data and showed that good correlations exist between the predictive and the test results.

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