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A computer model for probabilistic leak-rate analysis of nuclear piping and piping welds*

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This paper describes the development of a computer code entitled PSQUIRT for probabilistic evaluations of leak rate in nuclear piping. It is based on (1) the Henry–Fauske model of two-phase flow for thermal-hydraulic analysis and (2) an estimation model for elastic–plastic fracture-mechanics analysis. In both analyses, uncertainties arise due to the incomplete knowledge of the crack-morphology variables and statistical scatter of the pipe material properties. The relevant parameters required to conduct these analyses were modeled as random variables. Henceforth, the above thermal-hydraulic and fracture-mechanics models were put in a probabilistic format to allow statistical variability of input and determination of their effects in pipe fracture and leak-before-break (LBB) evaluations. A standard Monte Carlo simulation technique was used to perform the probabilistic analysis. Numerical examples are presented to illustrate the capabilities of the PSQUIRT code. Probabilistic analyses were performed by PSQUIRT for a stainless steel and a carbon steel pipe. Histograms were developed for leakage rate and flaw size in these pipes for LBB applications. The results suggest that the variability of leak rate can be significant due to statistical scatter of crack-morphology parameters. Using these histograms, the subsequent fracture stability of a leaking crack, actual or hypothetical, can be evaluated by either a deterministic or a probabilistic method. © 1997 Elsevier Science Ltd. All rights reserved.

1 INTRODUCTION

The developments of leak-rate estimation models were initiated in response to intergranular stress corrosion cracking (IGSCC) in boiling water reactor piping. Further interest in this area was stimulated by investigations into the application of a leak-before-break (LBB) philosophy to piping integrity analysis. Adoption of an LBB methodology requires reliable leak detection systems and verified leak-rate estimation techniques. Accurate leak-rate predictions require two important analyses. First, the fracture-mechanics analysis must be conducted so that the crack-opening area (COA) for a cracked pipe with known geometry, material properties, and applied loads can be determined. Second, given a pipe with correctly determined crack-opening characteristics, thermal-hydraulic analysis must be performed to predict fluid flow rate through cracks in a pipe. Currently, there are several leak-rate estimation models (and the resulting computer codes) available in the literature that are purely deterministic.[1,2] Using these models, only deterministic leak-rate analysis can be performed for LBB applications. However, in both fracture-mechanics and thermal-hydraulic analyses, uncertainties arise due to the incomplete knowledge of the crack-morphology variables and the material properties of the pipe. The examination of actual data for the key input variables in both analyses suggest that these variables, which exhibit distinct statistical scatter, should be modeled as random variables or processes. Therefore, decisions regarding pipe fracture evaluations should be based on the probability theory

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reflecting random nature of the input parameters or be justified by demonstrating that the uncertainty in these parameters do not adversely affect cost and safety.

This paper describes the development of a computer code entitled PSQUIRT (probabilistic seepage quantification of upsets in reactor tubes) for conducting probabilistic leak-rate analysis of nuclear piping typically used in LBB applications. It is based on (1) the Henry–Fauske model of two-phase flow for thermal-hydraulic analysis and (2) the GE/EPRI estimation model for elastic–plastic fracture-mechanics (EPFM) analysis. The key input parameters required to conduct these analyses were modeled as random variables. Standard statistical analyses were performed to determine the probabilistic characteristics of these variables. Henceforth, the above thermal-hydraulic and fracture-mechanics models were put in a probabilistic format to allow statistical variability of the input and determination of their effects in pipe fracture and LBB evaluations. A standard Monte Carlo technique was used to perform the probabilistic analysis.

Numerical examples are presented to illustrate the usefulness and the capabilities of the PSQUIRT code. Results from both deterministic and probabilistic analyses are presented and compared with the experimental data when available.

2 THE PSQUIRT COMPUTER PROGRAM

PSQUIRT is essentially a probabilistic extension of the SQUIRT code for deterministic leak-rate estimation that was previously developed by Paul *et al.*[1] in conjunction with the IPIRG-1[3] and short cracks in piping and piping welds[4] programs. PSQUIRT is a combination of several independent programs titled SCRAMP, SQUIRT5 or SQUIRT6, and FDACS, which were developed for conducting pre-processing of input, thermal-hydraulic and fracture-mechanics analyses, and post-processing of output, respectively. The input, in general, comprises statistical and/or deterministic description of crack-morphology parameters (e.g. surface roughness, number of turns in leakage path, entrance loss coefficients, and straightness of flow path, etc.), pipe material properties (stress–strain and J–R curves), and applied loads. A typical output is the estimated leak rate for flow through a crack when the crack size is known. In addition, PSQUIRT can compute the crack length and center-crack-opening displacement (COD) in a pipe when the pipe loads and leakage rates are specified. This usually involves numerical iteration between the thermal-hydraulic and fracture-mechanics parts of the code to solve for the unknown crack size. An additional aspect considered in the PSQUIRT code is the effect of crack-opening on the crack morphology variables. Several interface routines were developed to update the crack-morphology variables as a function of center-crack-opening displacement. This updating procedure was automated and can be continued as many times as needed during the iteration from which the leak-rate and leakage flaw size can be determined.

The following sections focus on the technical aspects of the PSQUIRT code. A brief outline of underlying deterministic models for both fracture-mechanics and thermal-hydraulics are provided. The PSQUIRT modules, that were developed for various aspects of the codes, are also described. Due to space limitations, only brief descriptions are given. Explicit technical details can be found in Paul *et al.*[1] and the references cited therein.

3 FRACTURE-MECHANICS ANALYSIS BY PSQUIRT

Generally, leak-rate calculations are performed for one of two purposes. First, given a flaw size, pipe dimensions, material properties, and loading, it is desired to know the fluid leak-rate through the crack. The aim is to estimate whether the given flaw size would result in a reliably detectable leak rate. Second, given a leak rate, it is desired to know what the crack-opening area must be. Then, knowing the COA, pipe dimensions, material properties, and loading, the aim is to estimate the flaw size, which is subsequently used to determine the pipe's load-carrying capacity. For either of the two purposes, it is desirable to have a mathematical model which is sufficiently accurate and relatively simple and inexpensive to use. For example, a detailed finite-element method (FEM), while generally accurate, would have very limited use because it would be too expensive and time consuming to be used routinely in probabilistic analyses. What is needed is a relatively simple equation (or a set of equations) to estimate flaw size and COA. In the PSQUIRT code, a simple estimation model proposed by Kumar *et al.*[5,6] and Brust *et al.*[7], was used to determine the COA.

3.1 Crack-opening-area (COA) analysis

Consider a through-wall-cracked (TWC) pipe under combined bending and tension in Fig. 1, which has mean radius, R_m, wall thickness, t, and crack angle, 2θ, with the crack circumferentially located in the pipe. Kumar *et al.* in pioneering work sponsored by EPRI,[5,6] developed a method, which enables one to generalize selected FEM solutions to be applicable to a wide range of flaw and pipe sizes and materials. This generalization is possible because of the key

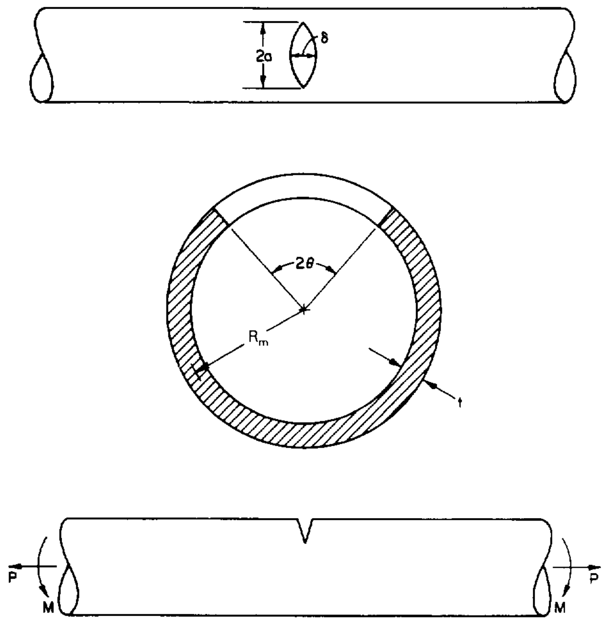


Fig. 1. Schematic of a through-wall-cracked pipe under combined bending and tension.

assumption in their approach that the nonlinear stress–strain behavior can be represented by the Ramberg–Osgood equation given by

$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha\left(\frac{\sigma}{\sigma_0}\right)^n \tag{1}$$

in which E is the elastic modulus, σ_0 is the reference stress, $\epsilon_0 = \sigma_0/E$ is the reference strain, n is a hardening exponent, and α is a material constant. Invoking the Ilyushin's theorem,[8] the plastic component of the center-crack-opening displacement, δ_p for a TWC pipe subjected to remote bending M can be expressed as[5–7]

$$\delta_p = \alpha\epsilon_0 a h_2(a/b, n, R_m/t)\left(\frac{M}{M_0}\right)^n \tag{2}$$

where M_0 is a reference moment representing the limit moment of a TWC pipe if σ_0 is the collapse stress, $a = R_m\theta$, and $b = \pi R_m$. For the case of a TWC pipe subjected to pure tension, M and M_0 would be replaced by the axial force P and the corresponding reference load P_0, respectively. In eqn (2), h_2 is a nondimensional function (plastic) of a/b, n, and R_m/t. In Kumar *et al.*[5,6] and Brust *et al.*,[7] h_2 values are given separately for pipes with through-wall circumferential cracks subjected to axial tension and to bending loads. These values were generated using a number of FEM analyses based on a thin-shell formulation[5,6] and full three-dimensional analysis.[7] Kumar *et al.*[5,6] and Brust *et al.*[7] also provide the equation for the elastic component of the center-crack-opening displacement, δ_e, given by

$$\delta_e = 4a\frac{R_m}{I}V_I(a/b, R_m/t)\frac{M}{E} \tag{3}$$

where I is the moment of inertia of the uncracked pipe cross-section and V_I is a nondimensional function (elastic) of a/b and R_m/t. In Kumar *et al.*[5,6] V_I is proposed to be a function of 'effective' crack length, a_e, instead of actual crack length, a. The rationale for using a_e instead of a is that the linear-elastic solutions of the J-integral, COD, and other fracture parameters of interest underestimate the actual values when M/M_0 exceeds 0·5 and the plastic components of the above parameters are too small, e.g. for large n values. This is the apparent reason why Kumar *et al.*[5,6] used the effective crack length a_e given by

$$a_e = a + \kappa r_y \tag{4}$$

where

$$r_y = \frac{1}{2\pi}\frac{n-1}{n+1}\left(\frac{K_I}{\sigma_0}\right)^2, \tag{5}$$

K_I is the mode-I stress-intensity factor, and

$$\kappa = \frac{1}{1 + (M/M_0)^2} \tag{6}$$

under pure bending. Equation (4) was developed to increase the value of the J-integral or COD when the applied moment becomes closer to reference moment, i.e. when M becomes closer to the M_0. There is no sound technical justification for the choice of the $1/[1 + (M/M_0)^2]$ function in eqn (6) except ensuring the continuity of the partial derivatives of J or COD with respect to the applied moment at $M = M_0$. Past work performed by Scott and Brust[9] indicates that the method in Kumar *et al.*[5,6] and Brust *et al.*[7] tends to overestimate experimental COD values even when the actual (rather than effective) crack length is used in the calculations. Therefore, in the PSQUIRT code, it was decided to evaluate the V_I function using the actual crack length. Both elastic (δ_e) and plastic (δ_p) components of the center COD were obtained by adding the contributions from tension and bending, where the bending part includes the induced bending due to axial tension in the presence of a TWC flaw. Using eqns (2) and (3), together with the tabulated h_2 and V_I values given in Kumar *et al.* and Brust *et al.* one can then find the total center COD, δ, as the sum of δ_e and δ_p. However, this still leaves the problem of determining the COA. Kumar *et al.* do not provide any information on crack-opening profiles, which is needed to calculate the COA. In Brust *et al.*,[7] however, there are some experimental data available for CODs measured at discrete points along the crack face (in addition to the center point) that can be potentially used for determining crack-opening shapes.

When crack growth occurs in a pipe, which is somewhat unlikely during normal operating condition, the same equations given above can be applied to compute COD, provided that the crack geometry is continuously updated following J-controlled crack growth. From EPFM theory and ductile tearing, the J-resistance (J–R) curve from a compact-tension specimen can be used to characterize the crack growth in a pipe. This J–R curve can be conveniently modeled by a power-law eqn (7)

$$J_R(\Delta a) = J_{Ic} + C\left(\frac{\Delta a}{k}\right)^m \tag{7}$$

in which $\Delta a = R_m\ \Delta\theta$ is the crack length extension during crack growth, J_{Ic} is the mode-I fracture toughness at crack initiation, and C and m are model parameters obtained from best fit of experimental data. In Eqn (7), k is a dummy parameter with a value of 1 introduced here only to dimensionalize C. In the PSQUIRT code, the power-law modeling of the J–R curve given by eqn (7) is not necessary, but it is useful for the extrapolation of compact-tension specimen data for a large amount of crack growth in a pipe.

For crack-opening-area analysis in the PSQUIRT code, several other J-estimation models besides the GE/EPRI method, which is described in this paper, can be used for predicting center COD. These additional methods involve: (1) the LBB.ENG1 and LBB.ENG2 methods; (2) the Paris/Tada method; (3) the LBB.NRC method; (4) the LBB.GE method, and others.[4] Further details of these methods can be obtained from Ref. 4. However, most of these methods were developed for a crack in the base metal of the pipe. Recently an extension of the LBB.ENG2 method, known as the LBB.ENG3 method, has been developed by the authors to analyze pipes with weld-metal cracks so that the tensile properties of both base and weld metals can be accounted for in predicting crack-opening displacement.[4] Depending on the user's option, each of these methods can be used for performing COA analysis in PSQUIRT.

Several options exist in modeling crack-opening profiles. Paul *et al.*[1] and Norris *et al.*[2] assumed several crack shape possibilities, such as elliptical, diamond-shaped, and rectangular. Comparisons with the experimental data as well as finite-element results, reported in Paul *et al.*[1], suggest that the ellipse may provide the best representation of the crack-opening shape. Recently, this was also verified by the authors when comparing FEM results with an elliptical crack-opening shape for a TWC pipe prior to ductile tearing.[10] Thus, knowing the center COD, crack length, and crack-opening profile, the corresponding COA can be readily calculated.

3.2 Evaluation of COA model in PSQUIRT

The COA model in PSQUIRT was evaluated by comparing its predictions with the available experimental measurements.[1] The experimental data used for comparisons are in the form of COD measurements made during pipe fracture experiments previously conducted at Battelle during the Degraded Piping Program.[11] Two cases of crack geometry were considered. One was a simple through-wall crack which has the same crack length on the inside and outside pipe diameter in terms of percentage of pipe circumference. The other was a complex crack which consists of a 360 degree internal constant-depth surface crack that penetrates the pipe thickness for a shorter through-wall-crack length. See Fig. 2 for the definitions of these two crack geometries and the associated crack-size parameters.

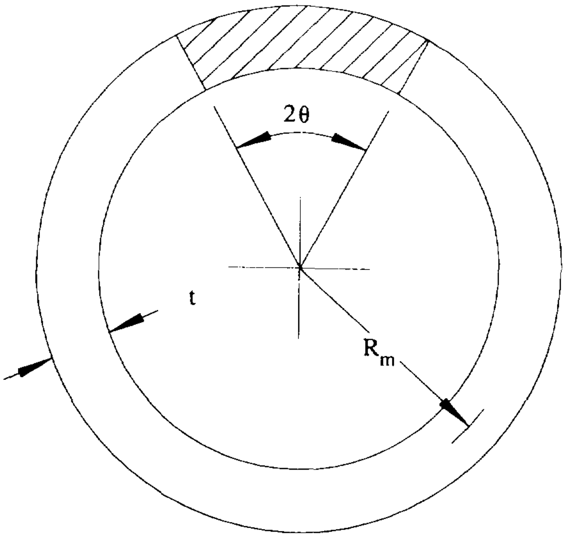
3.2.1 *Through-wall cracked pipes*

Figure 3 shows the results of the COA estimation model in PSQUIRT for Experiment 4111-1[11] analyzed in Paul *et al.*[1] The experiment in this case was performed on a 114 mm (4.5 in.) nominal diameter, SA-333 Grade 6 carbon steel pipe, which was subjected to four-point bending. The solid line in this figure represents the measured center COD as a function of applied load up to the load at crack initiation. The points indicate the predicted center COD based on the Ramberg–Osgood fit of several ranges of tensile stress–strain data. It is seen that in this case, the linear regression fit of the stress–strain data over the whole strain range leads to the best estimate of the COD. The same trend was also found in Experiment 4111-3, the results of which are shown in Fig. 4. However, in this case, the results using the low strain and the total strain curve fits were virtually the same. The material in pipe Experiment 4111-3 was Type 304 stainless steel. Reviewing the results of the estimation method in Figs 3 and 4, it appears that the use of a linear regression curve fit of the stress–strain data between 1% strain and 80% of ultimate strain may serve as the method for prescribing the Ramberg–Osgood constants in PSQUIRT. Similar comparisons with other pipe fracture data are available in Paul *et al.*,[1] including cracks in weld metal.

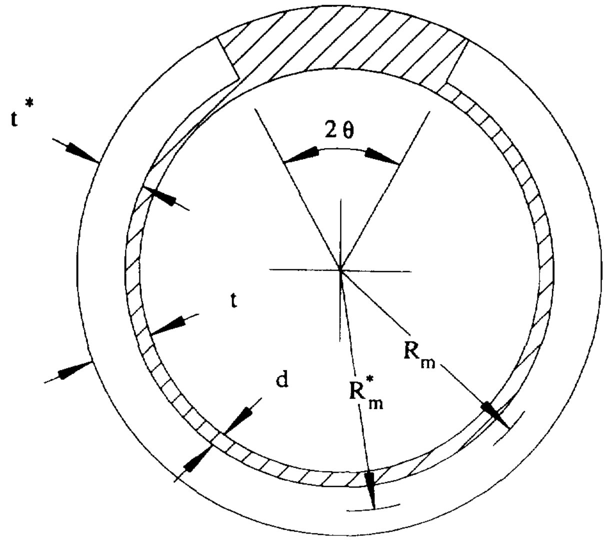
Figure 5 shows the detailed plots of COD at maximum load as a function of crack-tip distance for Experiment 4141-3[11] which were predicted by the estimation method with several assumptions of crack-opening shapes (diamond, ellipse, and rectangle) available in PSQUIRT. Compared with the test data as well as with the finite element results, the elliptical profile is found to best represent the crack-opening shape. Recent COA calculations for a TWC pipe, performed by the authors in Rahman *et al.*[10] and shown here in Fig. 6, also suggest that the equation of an ellipse fits very well the crack-opening shape predicted by the FEM.

3.2.2 *Complex-cracked pipes*

A complex crack is a long circumferential surface crack in a pipe that penetrates the thickness over a short length, see Fig. 2(b). Exact formulas to calculate



(a) Simple through-wall crack



(b) Complex crack

Fig. 2. Various through-wall-crack geometries and definitions of their parameters: (a) simple through-wall crack; (b) complex crack.

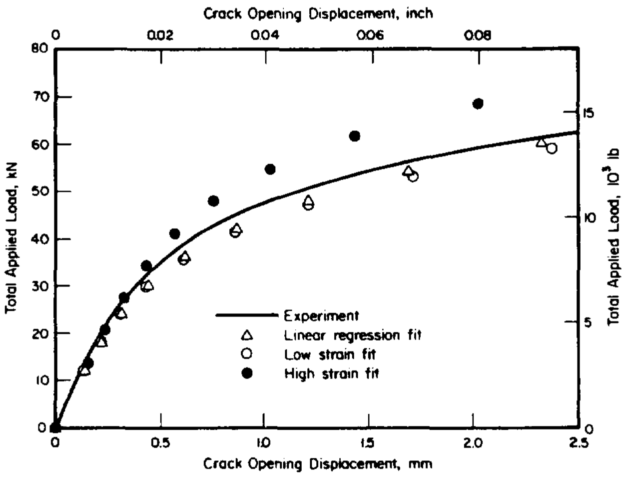


Fig. 3. Center-crack-opening displacement in experiment 4111-1 up to load at crack initiation.

center COD for a complex-cracked pipe have not yet been developed due to the need for more detailed three-dimensional finite element solutions. In the PSQUIRT code, it was assumed that the estimation formulas for simple through-wall-cracked pipes can be applied to analyze complex-cracked pipes. This was done by adjusting the pipe radius (effective radius, $R_m^* = R_m + d/2$, where d is depth of the internal surface crack) and the thickness (effective thickness, $t^* = t - d$) in the cracked plane to account for the presence of the surface crack. A 360 degree surface crack of constant depth was assumed. Any radial crack driving force contribution was ignored. Only growth of the through-wall crack in the circumferential direction was considered. Further details are available in Wilkowski *et al.*[12] and Kramer and Papaspyropoulos.[13]

Figure 7 shows the plots of applied load vs center COD up to a maximum load for a complex-cracked pipe in Experiment 4114-4 from the past degraded piping program [11] under four-point bending. They were obtained from both the predictive estimation formulas and the experimental pipe fracture data.[12] The estimation scheme used was the LBB.ENG2 method.[4] Theoretical results were obtained for two cases of J-resistance curves. One was based on a J-resistance curve of $C(T)$ specimen data ($\hat{C} = 1$) and the other was based on a J-resistance curve of $C(T)$ specimen data multiplied with a relevant reduction

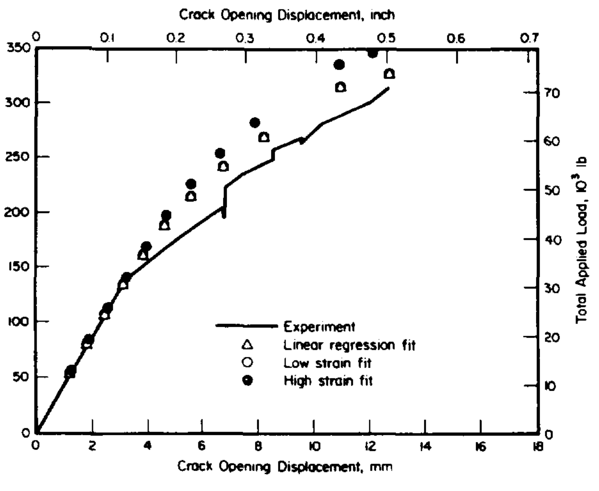


Fig. 4. Center-crack-opening displacement in experiment 4111-3 up to load at crack initiation.

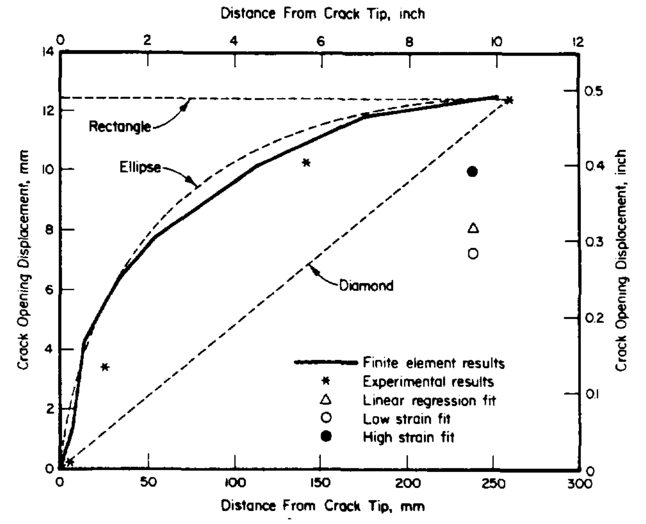


Fig. 5. Crack-opening profile at maximum load in experiment 4141-3.[1,11]

factor, $\hat{C}$, which has a value of less than 1. $\hat{C}$ varies as a function of d/t where d and t represent the depth of surface crack and the thickness of a complex-cracked pipe, respectively. The reduction factor $\hat{C}$ was developed from the comparisons of J-resistance curves from simple through-wall-cracked pipes and complex-cracked pipes.[12,13] The results suggest that the experimental COD is predicted well by the estimation method in the linear–elastic range with either cases of J-resistance curves. If the normal operating stresses are close to linear-elastic, the predicted COD with either case of J-resistance curves is adequate. Hence, for simplicity, the J-resistance curve from $C(T)$ specimen data without any reduction factor can be used for complex-cracked pipes if the fracture behavior is linear-elastic. However, if the normal operating stresses are large enough to induce significant plastic deformation, the simplified estimation method used here for complex-cracked pipe would overpredict experimental COD. Hence, further studies are needed to improve the crack-opening models for complex-cracked pipes.

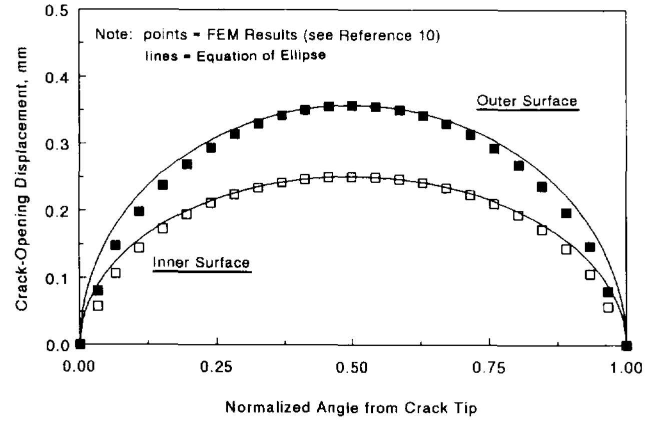


Fig. 6. Comparison of elliptical profile with crack-opening shape from finite element analysis.[10]

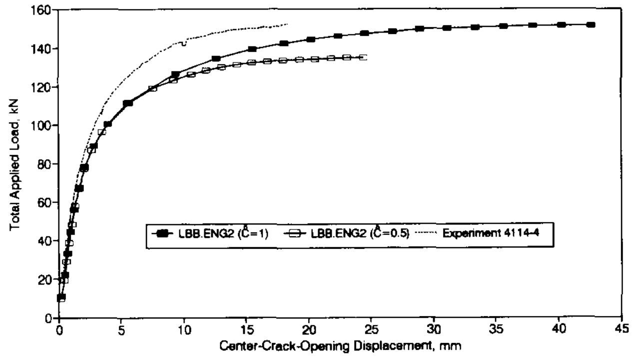


Fig. 7. Center-crack-opening displacement in experiment 4114-4 up to maximum load.

4 THERMAL-HYDRAULIC ANALYSIS BY PSQUIRT

The two-phase critical flow of water through cracks in the piping systems is a highly complex physical phenomenon that has been widely studied during the last 40 years. What makes this problem so difficult is the existence of the two phases in the flow system (i.e. crack path), which can interact in a variety of ways. For instance, a two-phase flow system can exist with either vapor bubbles dispersed in a continuous liquid phase or as liquid droplets dispersed in a continuous vapor phase. The physics of each of these situations is vastly different, yet each represents a two-phase flow. Further complications arise when the two-phase mixture is experiencing critical flow. In this case, the time required for the fluid to reach thermodynamic equilibrium moving into regions of lower pressure is comparable to the time that the fluid is flowing in the crack. This leads to nonequilibrium vapor generation rates for two-phase critical flows. To account for the nonequilibrium effects between the phases, Henry and Fauske proposed a simple model for the nonequilibrium vapor generation rate. In this model, it is assumed that the mixture quality relaxes in an exponential manner toward the equilibrium quality that would be obtained in a long tube. The relaxation coefficient was calculated based on their experiments with the critical flow of a two-phase water mixture in long tubes ($L_a/D_i > 100$ where L_a is the flow-path length and D_i is the inside diameter of the pipe). The Henry–Fauske model is the one that was chosen in PSQUIRT to model the two-phase critical flow of water through cracks in piping systems.

4.1 Henry–Fauske model

The Henry–Fauske model of two-phase flow through long channels was the basis for the thermal-hydraulic analysis conducted in PSQUIRT. Henry's mass flux equation is written in the following format:[14,15]

$$G_c^2 - \frac{1}{\left[\frac{X_c v_{gc}}{\gamma_0 p_c} - (v_{gc} - v_{Lc}) N_1 \frac{dX_E}{dp}\right]_c} = 0 \quad (8)$$

subject to the constraint

$$p_c + p_e + p_a + p_f + p_k + p_{aa} - p_0 = 0 \quad (9)$$

where G_c and p_c are mass flux of fluid and absolute pressure at crack exit plane, p_e, p_a, p_f, p_k, and p_{aa} are pressure losses due to entrance, acceleration, friction, crack-path protrusions, and area change acceleration, p is the internal pressure, p_0 is the absolute pressure at entrance of crack plane, γ_0 is the isentropic expansion coefficient, v_{gc} and v_{Lc} are specific volumes of saturated vapor and liquid at exit pressure, respectively, and X_c and X_E are nonequilibrium vapor generation rate and equilibrium fluid quality, respectively. The variable N_1 in eqn (8) is $20X_E$ for $X_E < 0{\cdot}05$ and 1 for $X_E \geq 0{\cdot}05$. Paul *et al.*[1] has complete definitions of all the pressure-loss equations. Fundamentals of these equations can also be obtained from Paul *et al.*[1] and various other references cited therein. Equations (8) and (9) represent two nonlinear equations with two unknowns, namely G_c^2 and p_c. A Newton–Raphson iteration method[16] was used to solve these simultaneous nonlinear equations.

4.2 Improved model for crack morphology variables

The key crack morphology variables, which are considered in PSQUIRT leak-rate analyses, are surface roughness, number of turns in the leakage path, and entrance loss coefficients. However, examination of service cracks shows that cracks frequently do not grow radially through the pipe thickness. Hence, a fourth parameter 'actual crack path/thickness' representing deviation from straightness was also considered.[17] This parameter was ignored in the past. Figure 8 shows the schematics of the crack-morphology variables considered in PSQUIRT. Only, a brief description of the above crack morphology variables is given below. See Rahman *et al.*[17] for further details.

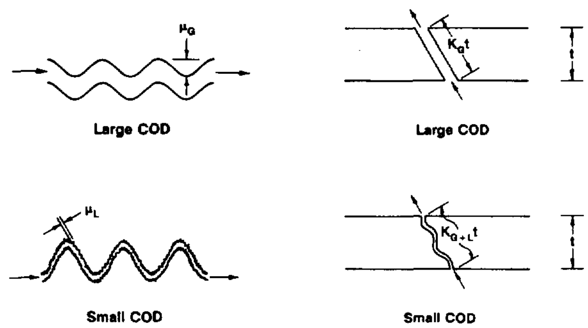


Fig. 8. Schematic of local and global crack-morphology variables for leak-rate analysis.

4.2.1 *Surface roughness*

This input parameter defines the roughness of the crack-face surface to be used in the calculation of the friction factor for the fluid flow through the crack. In the past, the surface roughness was assumed to be invariant with respect to the center-crack-opening displacement. For example, the constant numerical values such as 0·08 and 0·04 mm are frequently used to quantify surface roughness of intergranular stress-corrosion cracks and fatigue-growth cracks, respectively.[1] However, a careful examination of Fig. 8 suggests that the appropriate surface roughness should be large (global) or small (local) depending on whether the COD is large or small, respectively. For this study, the dependence of surface roughness, μ, was achieved by assuming a linear variation between local surface roughness, μ_L and the global surface roughness, μ_G, when the normalized center COD (δ/μ_G) varies between 0·1 and 10.

4.2.2 *Number of turns*

This input parameter defines the number of turns that the fluid must make when following through the crack. In fatigue and stress-corrosion cracks, the number and severity of the bends can in some circumstances account for upwards of one-half the total pressure loss of the fluid when flowing through the crack. Typically, a 45 or 90 degree angle change in flow direction results in about 0·4 or 1·0 velocity head loss, respectively.[1] Norris *et al.*[2] have shown this parameter to be of importance for stress-corrosion cracks. In the past, this parameter was thought to be of lesser importance for fatigue cracks because fatigue cracks generally break through in a fairly flat plane. However, the experimental results shown in Paul *et al.*[1] indicate that the number of bends in the flow path can be significant even for fatigue cracks. This occurs when the variations in the contours of the relatively flat plane of a fatigue crack are large compared to the COD. Therefore, even though the faces of a fatigue crack appear to be fairly flat to the naked eye, they contain many flow paths bends when the crack is tight. Following similar considerations given above for the surface roughness, the appropriate number of turns, n_t, was also assumed to have a linear variation between local number of turns, n_{tL} and global number of turns equal to $0{\cdot}1 n_{tL}$, when δ/μ_G varies between 0·1 and 10.

4.2.3 *Discharge coefficient*

At the entrance to the crack, the discharge coefficient is the ratio of the flow areas associated with the *vena contracta* to the flow area at the crack entrance. For sharp-edged crack entrances, a typical discharge coefficient would be a value of 0·60. For round or smooth-edged crack entrances, a typical discharge coefficient would be close to 0·95.[1]

4.2.4 *Actual crack path/thickness*

This important parameter represents the deviation of flow path from straightness. Depending on the center COD (see Fig. 8), it was assumed to be linearly varying between K_{G+L} and K_G, where K_G is the correction factor for global path deviations for straightness (e.g. a crack following a weld fusion line) and K_{G+L} is the correction factor for local plus global path deviations for straightness (e.g. a crack following the grain boundaries for IGSCC has not only more turns but also a longer flow path).

Note that the piecewise linear variation of the above crack morphology variables is a first attempt to simulate their dependency on COD. The numerical constants in the equations representing the linear variations were based on the review of cracks found in service and expert opinion at the University of Iowa and Battelle. Further studies are needed to verify this linear model.

4.3 Evaluation of the thermal-hydraulic model in PSQUIRT

Figures 9 and 10 compare the thermal-hydraulic predictions by the Henry–Fauske model of PSQUIRT with the data of Collier *et al.*[18] developed at Battelle for flow through a crack in a pipe with the center COD of 108 and 50 μm, respectively.[1] This model was also the basis of the PICEP code[2] developed by Battelle for EPRI, and later the SQUIRT code[1] was developed at Battelle in the IPIRG-1 program.[3] Figures 9 and 10 provide the plots between the calculation error and the measured flow rate, where the calculation error is the predicted minus measured flow rate divided by measured flow rate times 100. Good agreement was obtained between the mean results of the model and the experiments. The scatter in the above figures is found to be more pronounced when the crack opening is smaller. Collier *et al.*[18] attributed this larger uncertainty to the possibility that the crack could have become partially plugged by particles in the water particularly for the tight cracks. Hence, further studies are needed to improve the leak-rate model for very tight cracks.

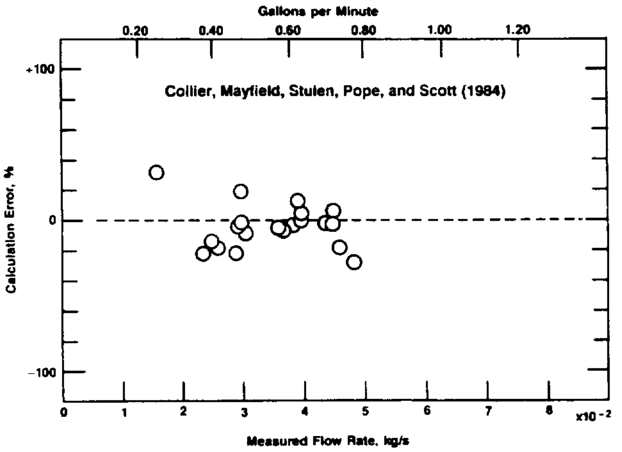


Fig. 9. Comparisons of PSQUIRT leak-rate predictions with experiment for COD = 108 μm.[1]

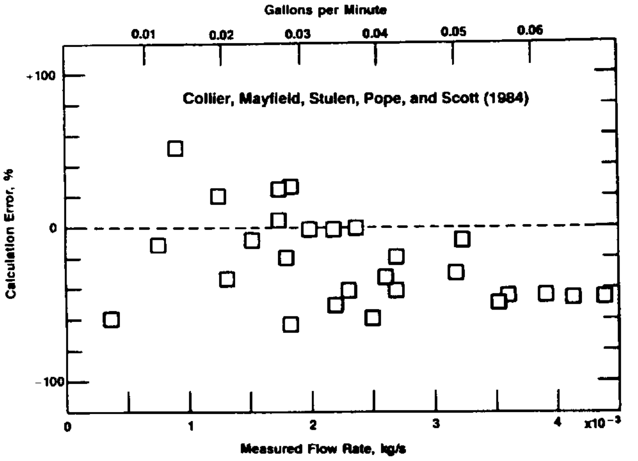


Fig. 10. Comparisons of PSQUIRT leak-rate predictions with experiment for COD = 50 μm.[1]

5 STATISTICAL CHARACTERIZATION OF CRACK-MORPHOLOGY VARIABLES

The random crack-morphology variables, which are currently considered in the PSQUIRT leak-rate model, are: (1) surface roughness (local and global); (2) number of turns in the leakage path (local and global); and (3) actual crack path-to-thickness ratio (global and global plus local).

In a recent study, measurements of surface roughness values for actual cracks in pipes removed from service have been reported.[17,19] Examination of photomicrographs available in the current literature were compiled and the corresponding number of 90-degree turns per unit pipe thickness were estimated. Several pipe materials, such as stainless steel and carbon steel, and several types of cracking mechanisms, such as corrosion-fatigue, thermal-fatigue, and intergranular stress corrosion cracking, were used to determine the associated crack-morphology parameters. Values of path deviations from straightness through the thickness, that can occur in real cracks in pipe, were also determined. Further details regarding the evaluation of crack-morphology parameters and extensive documentation of the sources for the raw data are given in Rahman *et al.*[17]

Using these data, a statistical analysis was conducted by Rahman *et al.*[17,19] to determine the probabilistic characteristics of crack-morphology parameters. The means and standard deviations of these variables were estimated for both austenitic and ferritic pipes with corrosion-fatigue and intergranular stress corrosion cracking types of mechanisms, respectively. Table 1 shows the above statistics of the crack-morphology variables for these cracking mechanisms which are used by the PSQUIRT code. Note that these statistical values provide only partial characterization of the random crack-morphology variables. The database was not large enough to determine their probability distributions accurately. Hence, several distributions should be tried to evaluate their effects. In the PSQUIRT code, there are several types of probability distribution one can use to model the crack morphology variables and perform a probabilistic leak-rate analysis.

6 ORGANIZATION OF THE PSQUIRT CODE

PSQUIRT is a probabilistic simulation code in which all random inputs are generated from their own probability distributions. Using these inputs, repeated deterministic analyses are performed to generate samples of output variables of interest. PSQUIRT is combination of several independent programs, such as SCRAMP, SQUIRT5, SQUIRT6, and FDACS, for conducting pre-processing of input, thermal-hydraulic and fracture-mechanics analyses, and post-processing of output, respectively. They are explained below.

6.1 The SCRAMP module

The SCRAMP code stands for simulation of crack morphology parameters. It generates independent samples of various crack-morphology parameters according to their probability distributions which are needed to conduct thermal-hydraulic analysis. The random crack-morphology variables are: (1) local surface roughness, μ_L; (2) global surface roughness, μ_G; (3) local number of turns per unit thickness, n_{tL}; (4) global path deviation factor, K_G; and (5) global plus local path deviation factor, K_{G+L}. The statistical properties of these variables are given in Table 1. The current version of SCRAMP can generate samples from both normal and lognormal probability distribution functions. These distributions functions can be truncated and/or shifted according to the prescribed input.

Table 1. Statistics of crack morphology parameters for various cracking mechanisms

Random variable	IGSCC[a]		Corrosion fatigue	
	Mean	Standard deviation	Mean	Standard deviation
μ_L, μ mm	4699	3937	8814	2972
μ_G, μ mm	80,010	39,014	40,513	17,653
n_{tL}, mm^{-1}	28	19	7	8
K_G	1·07	0·10	1·017	0·0163
K_{G+L}	1·33	0·17	1·06	0·03

[a] IGSCC = intergranular stress-corrosion cracking.

6.2 The SQUIRT5 and SQUIRT6 modules

The SQUIRT5 and SQUIRT6 codes stand for seepage quantification of upsets in reactor tubes. The SQUIRT5 and SQUIRT6 programs are the modified and extended versions of the deterministic code SQUIRT (Version 2.2). The Version 2.2 of SQUIRT was released to the NRC during the short cracks in piping and piping welds program and is documented in Rahman *et al.*[20]

6.2.1 *The SQUIRT5 module*

The SQUIRT5 code was developed to compute the crack length and center-crack-opening displacement in a pipe when the pipe loads and leakage rate are specified. This usually involves numerical iteration between thermal-hydraulic and fracture-mechanics parts of the code to solve for unknown crack size. SQUIRT5 was automated to:

(1) read the generated samples of crack-morphology parameters from SCRAMP analysis;

(2) conduct deterministic thermal-hydraulic and fracture-mechanics analyses for each input set (sample set) of crack-morphology parameters and perform repeated analyses for a specified number of times; and

(3) generate output samples of leakage flaw size and center COD.

6.2.2 *The SQUIRT6 module*

The SQUIRT6 code was developed to predict the leakage rate for a pipe with known crack geometry and applied loads. The combination of SCRAMP, SQUIRT6, and FDACS codes can be used to conduct probabilistic evaluation of the leak rate from a crack with the specified dimensions and random crack-morphology variables. SQUIRT6 was automated to:

(1) read generated samples of crack-morphology parameters from SCRAMP analysis;

(2) conduct deterministic thermal-hydraulic analyses for each input set (sample set) of crack-morphology parameters and repeat such analyses for specified number of times; and

(3) generate output samples of leakage flow rate.

In conducting both SQUIRT5 and SQUIRT6 analyses, several interface routines were developed to update the crack-morphology variables as a function of center-crack-opening displacement. This updating procedure was automated and can be continued as many times as needed during the iteration from which the leak-rate or the leakage flaw size can be determined.

6.3 The FDACS module

The FDACS code stands for frequency distribution analysis of crack size. It is a program to conduct standard statistical analyses, such as computing the mean, standard deviation, and histogram, of the simulated or actual samples of a variable of interest. Following SCRAMP and SQUIRT5 or SQUIRT6 analyses, leakage flaw size or leak rate can be calculated. FDACS can be used to determine their statistical properties and develop frequency distributions or histograms. It can compute both relative and cumulative frequency distribution functions.

During the development of the PSQUIRT code, two executable modules were created. The first program is defined as the PSQUIRT1 code, which is a combination of SCRAMP, SQUIRT5, and FDACS modules. The second program is defined as the PSQUIRT2 code, which is a combination of SCRAMP, SQUIRT6, and FDACS modules. The PSQUIRT1 code performs probabilistic assessments of crack length and center-crack-opening displacement for a pipe with known material properties, specified leak-rate, and applied loads. The PSQUIRT2 code conducts probabilistic evaluations of leakage flow rate for a pipe with given crack geometry and loads. Figures 11 and 12 show the flowcharts for PSQUIRT1 and PSQUIRT2 codes developed in this study.

7 NUMERICAL EXAMPLES

Consider two pipes, one a TP304 stainless steel pipe and the other an A516 Grade 70 carbon steel pipe, with outer diameter, $D_0 = 711{\cdot}2$ mm (28 in.) wall thickness, $t = 35{\cdot}8$ mm (1·41 in.) which are operating at an internal pressure of 7·24 MPa (1·05 ksi) and a temperature of 288°C (550°F). It is anticipated that the cracking mechanisms are IGSCC and corrosion-fatigue for the stainless steel and carbon steel pipes, respectively. Table 1 provides the statistics of crack-morphology variables for both types of cracks. In this example, it was assumed that these variables would follow a lognormal probability distribution with the mean and standard deviation given in Table 1. The normal operating stresses for these pipes were assumed to be 50% of the Service Level A limit from ASME Section III design code.[21] These correspond to the total stresses (axial plus bending stresses) of 87·66 MPa (12·72 ksi) for the stainless pipe and 109·1 MPa (15·83 ksi) for the carbon steel pipe.

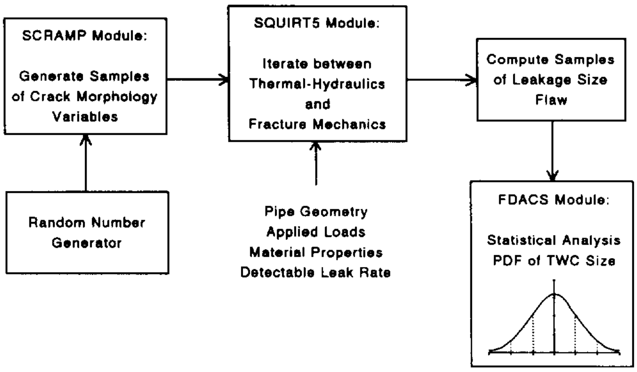


Fig. 11. Flow chart describing various stages of PSQUIRT1 analysis.

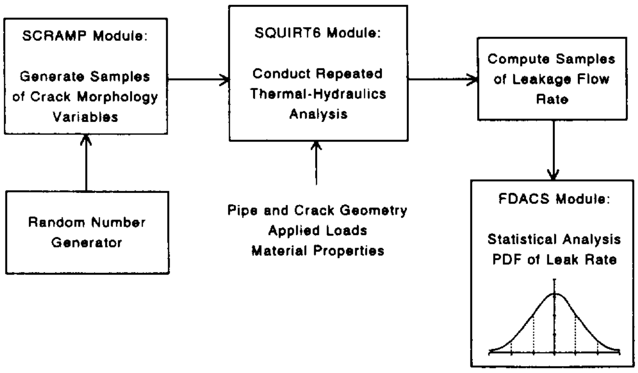


Fig. 12. Flow chart describing various stages of PSQUIRT2 analysis.

It was assumed that the material's stress–strain response can be represented by the Ramberg–Osgood model given by eqn (1). Also, the J–R curve from the compact tension specimen was deemed to be adequately characterized by a power-law equation given by eqn (7). Table 2 provides the material properties of both pipes considered in these examples. The material properties were deterministic.

7.1 PSQUIRT1 analysis: calculation of flaw size

Assuming a simple through-wall crack in a pipe for the applied loads under normal operating conditions,

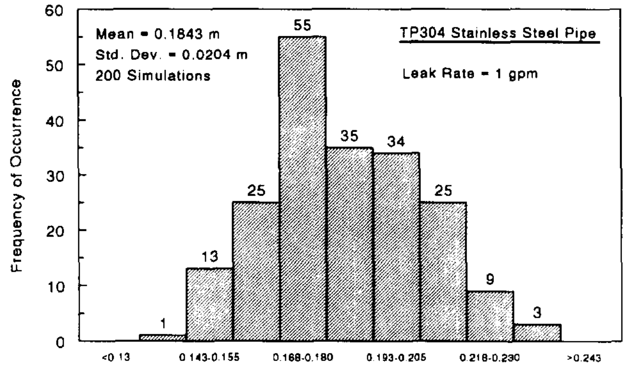
Table 2. Stress–strain and J–R curve parameters for stainless and carbon steel pipes at 288°C[a]

Variable	Stainless steel pipe	Carbon steel pipe
E, MPa	182,700	193,100
σ_0, MPa	161·64	245·13
α	7·22	1·78
n	4·04	5·81
J_{Ic}, kJ m^{-2}	1050	200
C, kJ m^{-2}	297·56	412·73
m	0·47	0·50

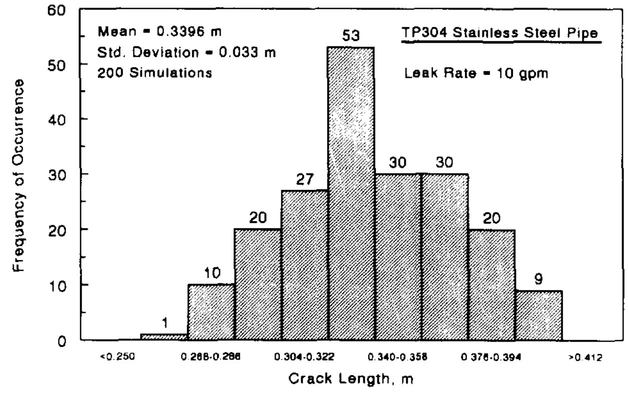
[a] Stress–strain curve is represented by: $\epsilon/\epsilon_0 = \sigma/\sigma_0 + \alpha(\sigma/\sigma_0)^n$, $\epsilon_0 = \sigma_0/E$; and J–R curve is represented by: $J = J_{Ic} + C(\Delta a/k)^m$, where $k = 1$ mm, Δa is in mm.

the PSQUIRT1 code was used to determine the probabilistic characteristics of flaw size for a specified leak rate. Figure 13 shows the histograms of two different crack lengths for the stainless steel pipe corresponding to leak rates of 1 and 10 gpm. As expected, when the leak-rate increases, the flaw size also increases and is clearly shown from the rightward shift of probability mass in Fig. 13. Similar histograms were also developed by PSQUIRT1 for the carbon steel pipe and are shown in Fig. 14. The sample sizes were 200 and 99 for the stainless steel and carbon steel pipes, respectively.

Using Figs 13 and 14, the effects of crack-morphology variability on the leakage flaw size can be evaluated. They can be used for subsequent fracture evaluations either by a probabilistic or a deterministic method. For example, considering probabilistic variations of flaw size given in Figs 13 and 14, one can compute conditional failure probability based on the fracture stability of a leaking crack.[17,19] Alternatively, the mean minus the standard deviation times a specified value of this flaw size can also be used for more conventional deterministic analysis. See Rahman *et al.*[17] for further details on the flaw size distributions and their impact on the reliability assessments of pipes.

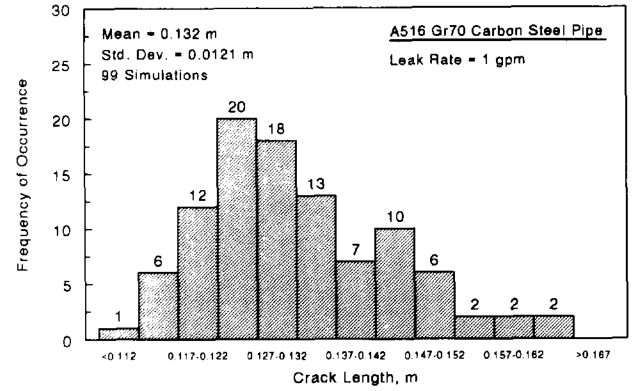


(a) Detectable leak rate = 1 gpm

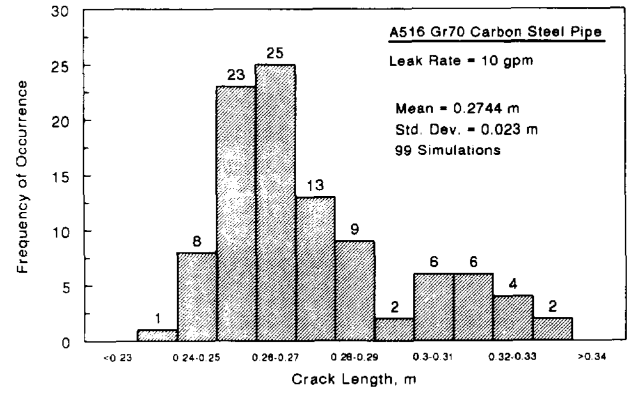


(b) Detectable leak rate = 10 gpm

Fig. 13. Histogram of leakage flaw size for TP304 stainless steel pipe (PSQUIRT1 analysis): (a) detectable leak rate = 1 gpm (b) detectable leak rate = 10 gpm.



(a) Detectable leak rate = 1 gpm



(b) Detectable leak rate = 10 gpm

Fig. 14. Histogram of leakage flaw size for A516 Gr 70 carbon steel pipe (PSQUIRT1 analysis): (a) detectable leak rate = 1 gpm (b) detectable leak rate = 10 gpm.

7.2 PSQUIRT2 analysis: calculation of leakage rate

Using the PSQUIRT2 code, the crack-morphology variables were randomly generated according to the lognormal probability distributions and the corresponding leak rates were calculated for each of the two pipes subject to the applied stresses defined above. In both pipes, the initial through-wall crack was postulated based on a flaw large enough to produce about 1 gpm leak-rate (using mean values of random crack-morphology). Figures 15 and 16 show the histograms of leak rate for the stainless steel and carbon steel pipes, respectively, obtained from 1000 simulated samples in each case. The mean and standard deviation were estimated to be 1·21 and 0·41 gpm, respectively, for the stainless steel pipe, and 1·45 and 0·57 gpm, respectively, for the carbon steel pipe. Correspondingly, the coefficient of variations

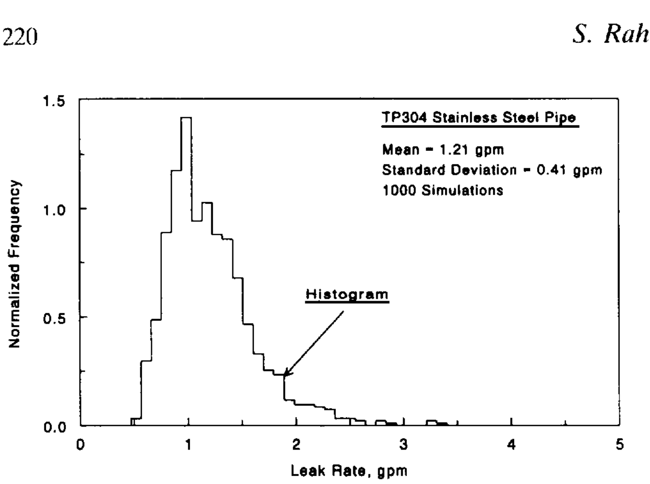


Fig. 15. Histogram of leak rate for TP304 stainless steel pipe (PSQUIRT2 analysis).

(defined as the ratio of standard deviation and mean) were 33·9 and 39·3% for the stainless steel and carbon steel pipes, respectively. It appears that the uncertainties in crack-morphology can produce significant variability in the leak-rate calculations.

8 SUMMARY AND CONCLUSIONS

This paper describes the development of a computer code titled PSQUIRT for probabilistic evaluations of leak rate in nuclear piping for LBB applications. It is based on (1) the Henry–Fauske model of two-phase flow for thermal-hydraulic analysis and (2) a GE/EPRI estimation model for elastic–plastic fracture-mechanics analysis. In both analyses, uncertainties arise due to the incomplete knowledge of the crack-morphology variables and the pipe material properties. The relevant parameters required to conduct these analyses were modeled as random variables. Standard statistical analyses were performed to determine the probabilistic characteristics of these variables. Henceforth, the above thermal-hydraulic and fracture-mechanics models were put in a probabilistic format to allow statistical variability of the input and determination of their effects in pipe fracture and LBB evaluations. A standard Monte Carlo simulation was used to perform the probabilistic analysis.

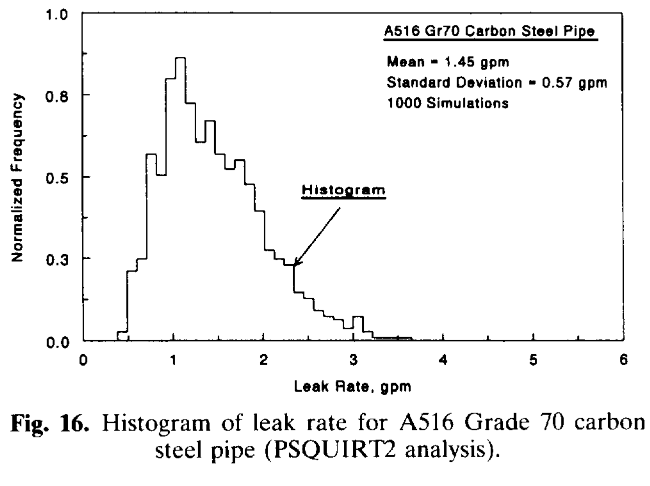


Fig. 16. Histogram of leak rate for A516 Grade 70 carbon steel pipe (PSQUIRT2 analysis).

Numerical examples are presented to illustrate the capabilities of the PSQUIRT code. Past results from the deterministic analyses showed that both fracture-mechanics and thermal-hydraulic models of PSQUIRT can provide reasonably accurate predictions of leak rate when compared with the test data. Probabilistic analyses were performed using PSQUIRT for a stainless steel and a carbon steel pipe. Histograms were developed for the leakage rate and flaw size in these pipes for LBB applications. The results showed that the variability of leak rate can be significant due to statistical scatter of the crack-morphology parameters. Hence, the random nature of the crack-morphology should be accounted for in performing LBB calculations.

From the histograms generated by PSQUIRT, the subsequent fracture stability of a leaking crack can be evaluated by either a probabilistic or a deterministic approach. In the probabilistic approach, the statistical variation of flaw size can be accounted for by using the histogram or its analytical approximation by classical distribution functions. As an alternative, the mean value minus the standard deviation times a specified value of this flaw size can also be used for a more traditional deterministic analysis. Regardless, these histograms provide important information for LBB evaluations.

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