3D Simple Point, Topology Preservation, and Skeletonization

Punam Kumar Saha
Professor
Departments of ECE and Radiology
University of Iowa
pksaha@engineering.uiowa.edu

References

[1] P. K. Saha, B. Chanda, and D. D. Majumder, "Principles and algorithms for 2D and 3D shrinking," NCKBCS Library, Indian Statistical Institute Calcutta, India, Technical Report TR/KBCS/2/91, 1991.

[2] P. K. Saha, B. B. Chaudhuri, B. Chanda, and D. D. Majumder, "Topology preservation in 3D digital space," *Pattern Recognition,* vol. 27, pp. 295-300, 1994.

[3] P. K. Saha and B. B. Chaudhuri, "Detection of 3-D simple points for topology preserving transformations with application to thinning," *IEEE Transactions on Pattern Analysis and Machine Intelligence,* vol. 16, pp. 1028-1032, 1994.

[4] P. K. Saha, B. B. Chaudhuri, and D. D. Majumder, "A new shape preserving parallel thinning algorithm for 3D digital images," *Pattern Recognition,* vol. 30, pp. 1939-1955, 1997.

[5] D. Jin and P. K. Saha, "A new fuzzy skeletonization algorithm and its applications to medical imaging," *Proc of the Image Analysis and Processing–ICIAP*, pp. 662-671, 2013.

[6] P. K. Saha, G. Borgefors, and G. Sanniti di Baja, "A survey on skeletonization algorithms and their applications," *Pattern Recognition Letters,* 2015.

[7] P. K. Saha and B. B. Chaudhuri, "3D digital topology under binary transformation with applications," *Computer vision and image understanding,* vol. 63, pp. 418-429, 1996.

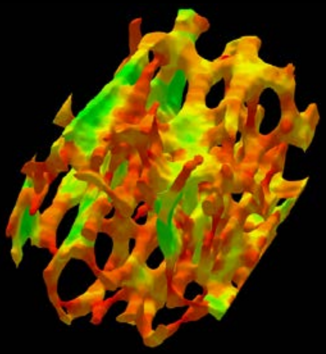
[8] D. Jin, C. Chen, and P. K. Saha, "Filtering Non-Significant Quench Points Using Collision Impact in Grassfire Propagation," in *Image Analysis and Processing—ICIAP 2015,* Springer International Publishing, pp. 432-443, 2015.

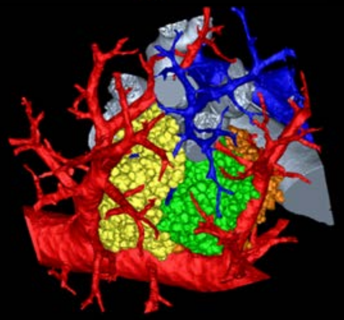
[9] D. Jin, K. S. Iyer, C. Chen, E. A. Hoffman, and P. K. Saha, "A robust and efficient curve skeletonization algorithm for tree-like objects using minimum cost paths," *Pattern Recognition Letters,* 2015.

[10] C. Chen, D. Jin, and P. K. Saha, "Fuzzy Skeletonization Improves the Performance of Characterizing Trabecular Bone Micro-architecture," in *Advances in Visual Computing,* Springer International Publishing, pp. 14-24, 2015.

Outline

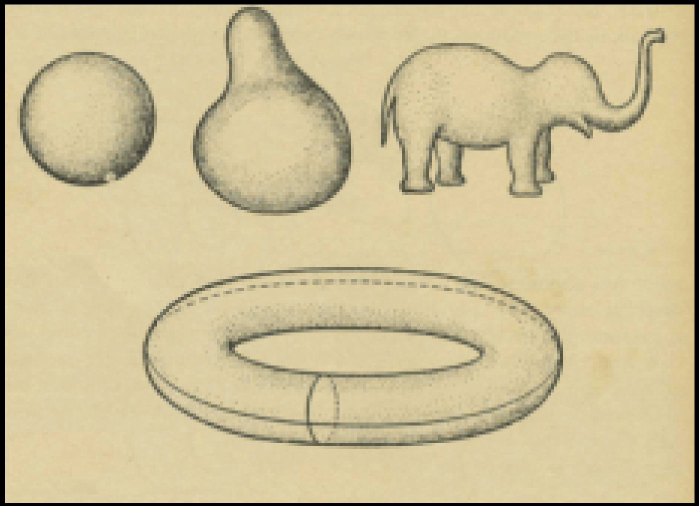
- Introduction to topological transformation and topological equivalence
- Basic notions of digital topology
- Euler characteristic
- Simple point
- A simple point characterization in 3-D
- Number of tunnels in $3 \times 3 \times 3$ neighborhood
- Local topological numbers
- Efficient algorithms
- Fuzzy Skeletonization





Continuous Deformation

Topology. The study of those properties of geometric figures or solid bodies that remain invariant under certain transformations.

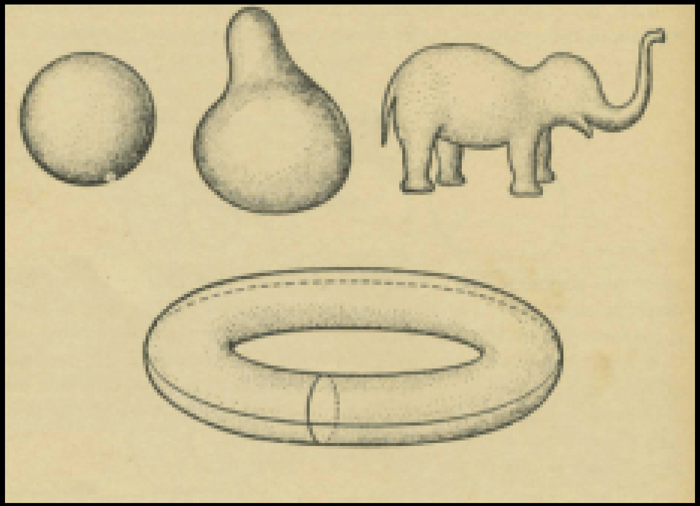


Continuous deformation. A transformation which shrinks, stretches, bents, twists, etc. in any way without tearing

- Envision a figure drawn on a rubber sheet
- A deformation of the sheet by stretching, twisting, bending, etc. which doesn't tear the sheet will change the figure into some other shape

Topological Transformation and Equivalence

Topology. The study of those properties of geometric figures or solid bodies that remain invariant under certain transformations.



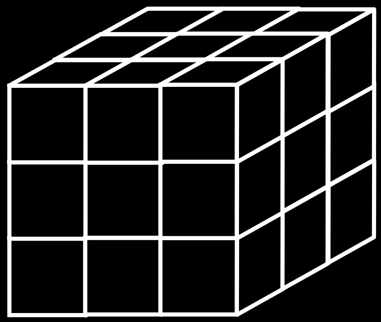
Topological transformation. A transformation that carries one geometric figure into another figure is a topological transformation if the following conditions are met:

1) the transformation is one-to-one
2) the transformation is bicontinuous (i.e. continuous in both directions)

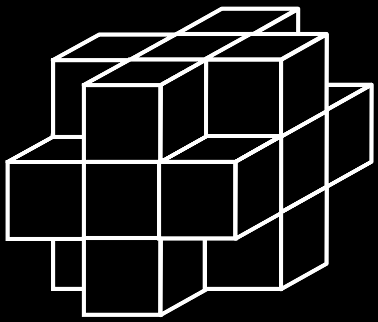
Topologically equivalent. Two different shapes are topologically equivalent if one can be changed to the other by a topological transformation

Basic Definitions in 3-D

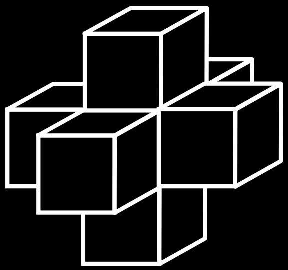
- A cubic grid constitutes the set Z^3
- An element of Z^3 is referred to as a point represented by its x-, y-, z-coordinates
- Each cube centered at an element in Z^3 is referred to as a voxel



26-adjacency



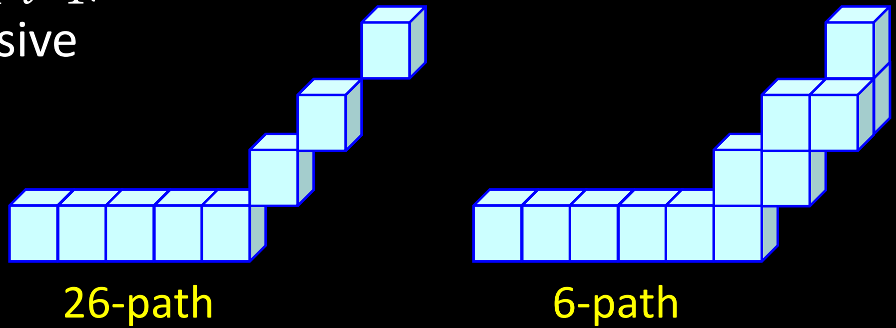
18-adjacency



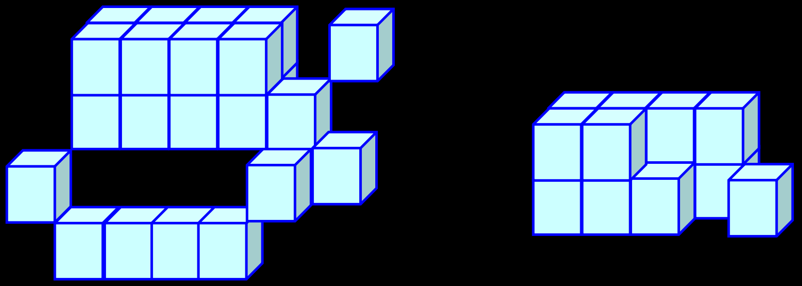
6-adjacency

Basic Definitions in 3-D

- An α-path π, where $\alpha \in \{6, 18, 26\}$, is a nonempty sequence $\langle p_0, \cdots, p_{l-1} \rangle$ of voxels where every two successive voxels are α-adjacent

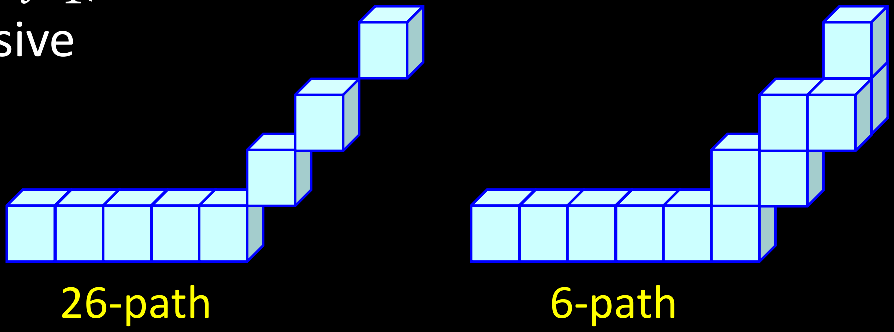


- An α-component of a set of voxels S is a maximal subset of S where every two voxels are α-connected in S

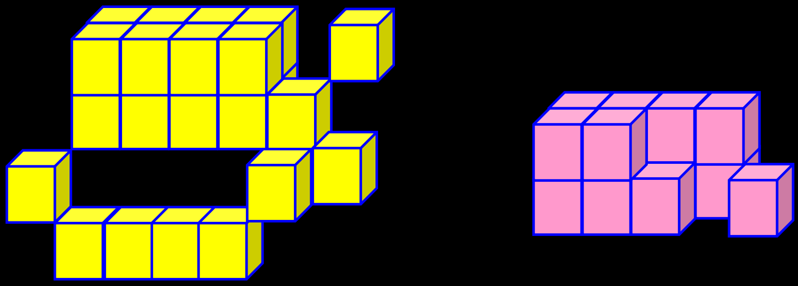


Basic Definitions in 3-D

- An α-path π, where $\alpha \in \{6, 18, 26\}$, is a nonempty sequence $\langle p_0, \cdots, p_{l-1} \rangle$ of voxels where every two successive voxels are α-adjacent

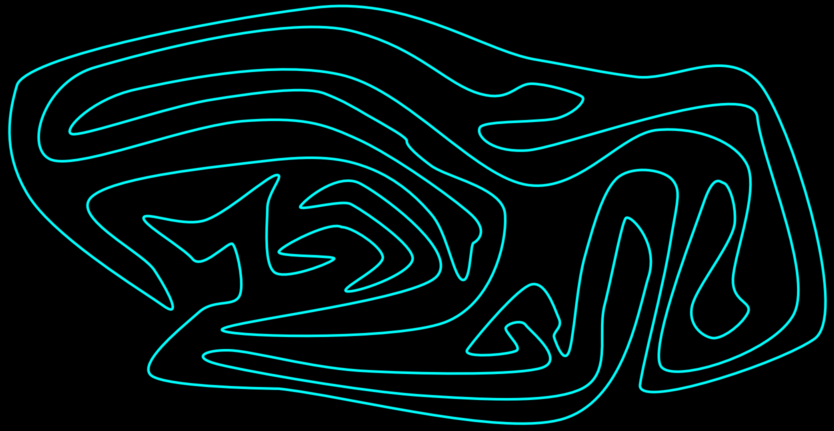


- An α-component of a set of voxels S is a maximal subset of S where every two voxels are α-connected in S



Adjacency Pairs in Digital Topology

- Digital topology loosely refers to the use of mathematical topological properties and features such as connectedness, topology preservation, boundary etc., for images defined in digital grids



Jordan curve

Adjacency Pairs in Digital Topology

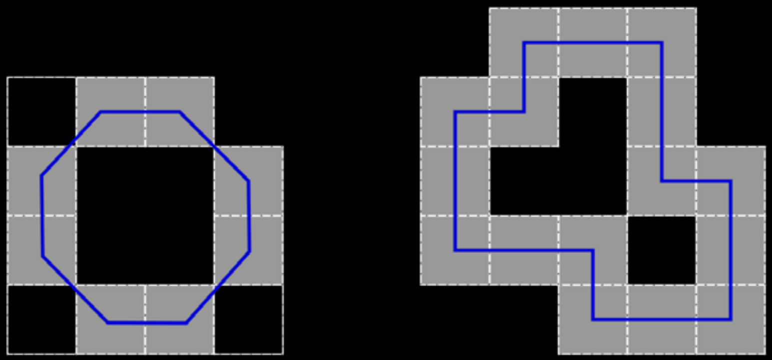
- Digital topology loosely refers to the use of mathematical topological properties and features such as connectedness, topology preservation, boundary etc., for images defined in digital grids

- Adjacency pairs. Rosenfeld's approach to digital topology is to use a pair of adjacency relations (κ_1, κ_0) where κ_1 is used for object points while κ_0 is used for background points

Theorem. Jordan curve partitions of a plane into inside and outside

Why the Adjacency Pair?

Rosenfeld convincingly demonstrated that use of a proper adjacency pair leads to workable framework of digital topology, which holds several important mathematical topological properties, including the Jordan curve theorem

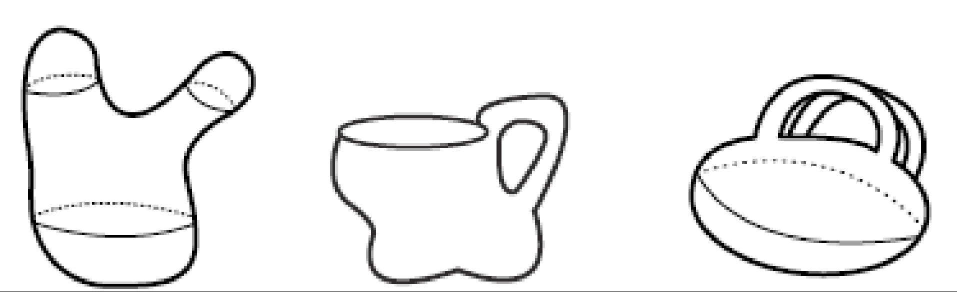


- One proper adjacency pair is (26,6)
- (26,6) is the most popular adjacency pairs in 3-D

The modern trend is to use the cubicial complex representation of digital images to define topological transformation

Cavities and Tunnels in 3-D

- Cavity. A background or white component surrounded by an object component

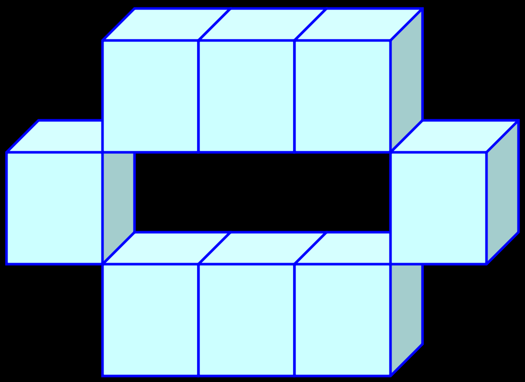


- Tunnel. Difficult to define a tunnel. However, the number of tunnels in an object is well-defined - the rank of the first homology group of the object.
  - Intuitively, a tunnel would be the opening in the handle of a coffee mug, formed by bending a cylinder to connect the two ends to each other or to another connected object
  - A hollow torus has two tunnels: the first arises from the cavity inside the ring and the second from the ring itself

Euler Characteristic

The Euler characteristic of a polyhedral set X, denoted by $\chi(X)$, is defined as follows

1) $\chi(\phi) = 0$
2) $\chi(X) = 1$, if X is non-empty and convex
3) for any two polyhedral X, Y, $\quad \chi(X \cup Y) = \chi(X) + \chi(Y) - \chi(X \cap Y)$



Euler Characteristic: Alternative Definitions

The Euler characteristic of a polyhedron with each element being convex

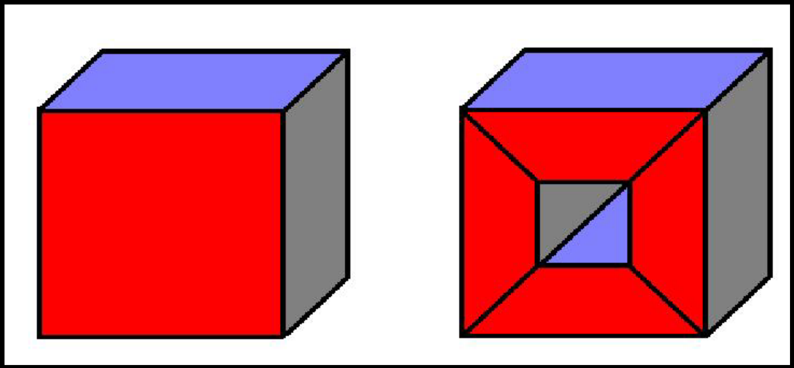
$$\chi(X) = \#\text{points} - \#\text{edges} + \#\text{faces} - \#\text{volumes},$$

and

$$\chi(X) = \#\text{components} - \#\text{tunnels} + \#\text{cavities}$$

8 – 12 + 6 – 1 = 1

1 – 0 + 0 = 1

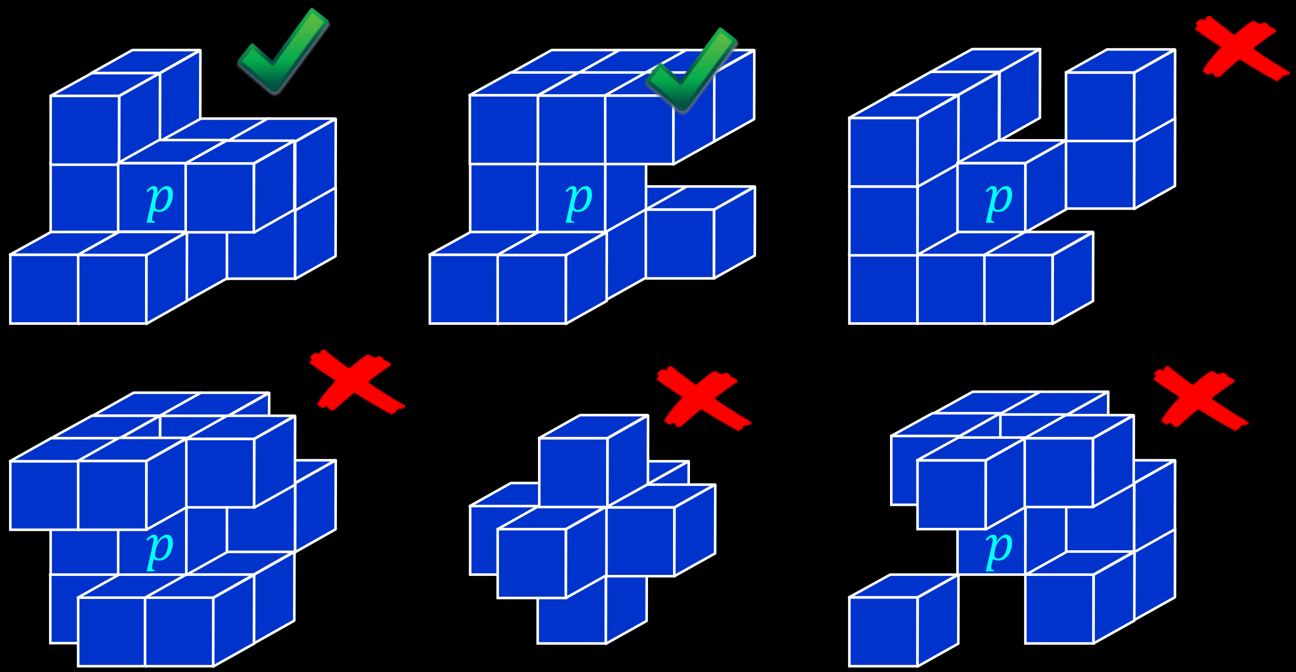


16 – 32 + 20 – 4 = 0

1 – 1 + 0 = 0

3-D Simple Point

Simple Point. A point whose deletion or addition preserves the topology in the local neighborhood in terms of components, tunnels, and cavities



The major challenge. Presence of tunnels in 3-D that is not there in 2-D

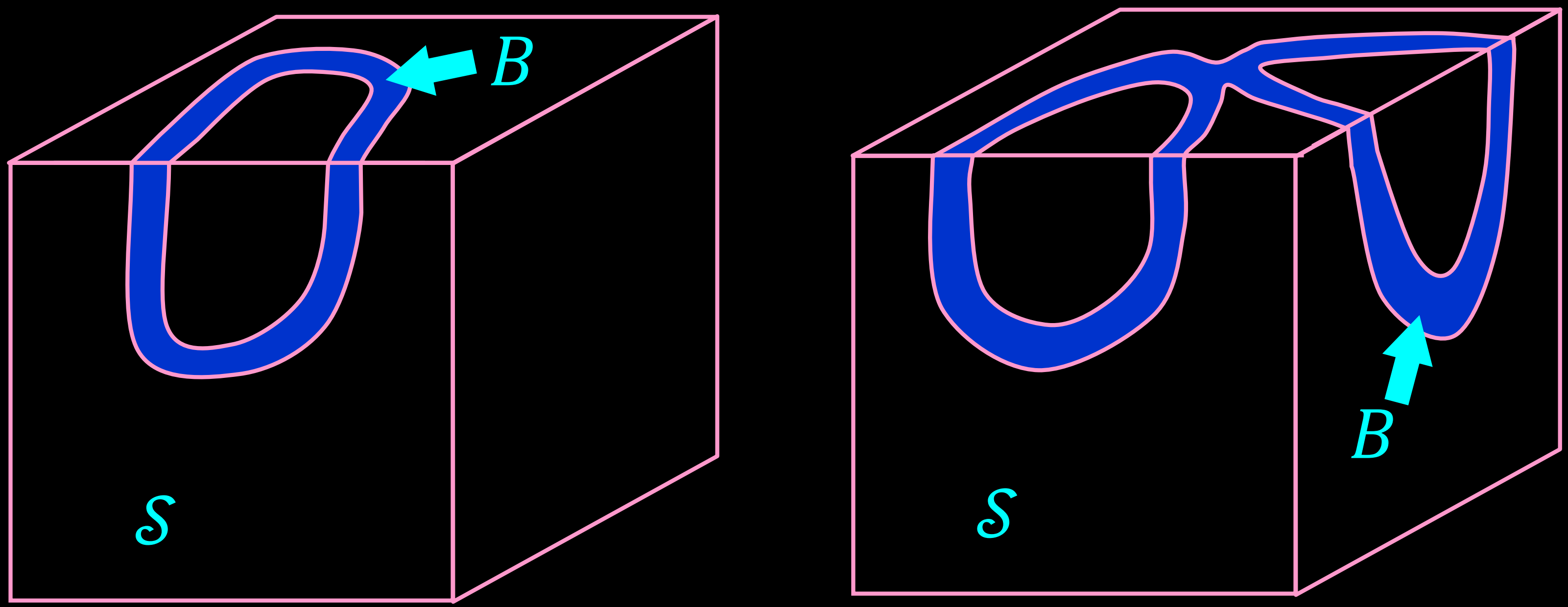
3-D Simple Point Characterization by Morgenthaler (1981)

A point $p \in Z^3$ is a (26,6) simple point in a 3-D binary image $(Z^3, 26,6, B)$ if and only if the following conditions are satisfied

- In $N_{26}^*(p)$, the point p is 26-adjacent to exactly one black (object) component
- In $N_{26}^*(p)$, the point p is 6-adjacent to exactly one white (background) component
- $\chi\left(\left(Z^3, 26,6, \left(B \cap N(p)\right) \cup \{p\}\right)\right) = \chi\left(\left(Z^3, 26,6, \left(B \cap N(p)\right) - \{p\}\right)\right)$

$$\chi(X) = \#\text{components} - \#\text{tunnels} + \#\text{cavities}$$

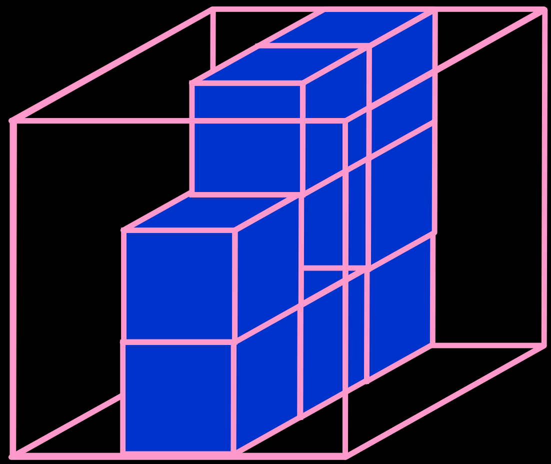
Tunnels on the Surface of a Topological Sphere



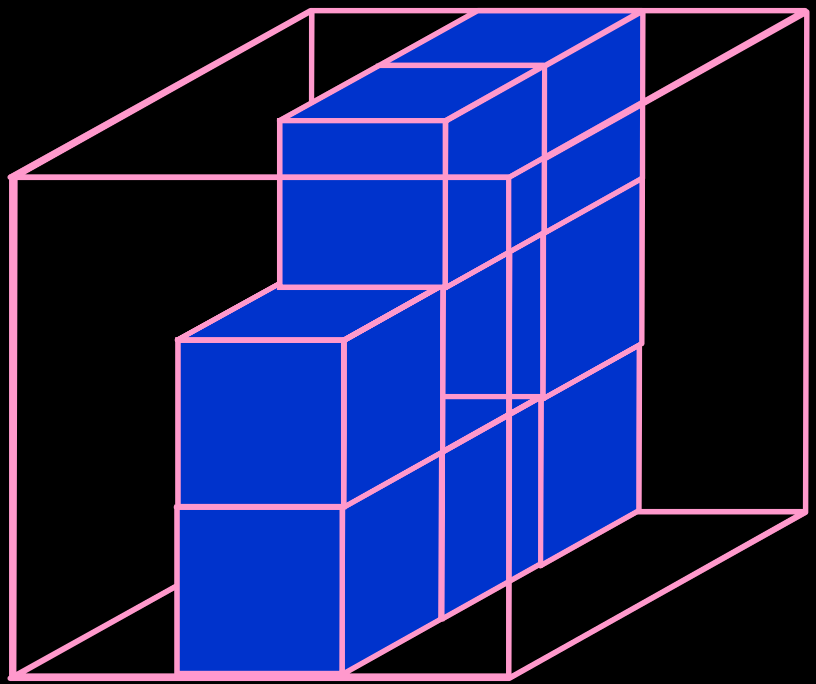
- Saha, Chanda, Dutta Majumder, "Principles and algorithms for 2D and 3D shrinking," Indian Statistical Institute, Calcutta, India, TR/KBCS/2/91, 1991.
- Saha, Chaudhuri, Chanda, Dutta Majumder, "Topology preservation in 3D digital space," Pat Recog, **27**:295-300, 1994.
- Saha and Chaudhuri, "Detection of 3-D simple points … with application to thinning," IEEE Trans Pat Anal Mach Intel, **16**:1028-1032, 1994.
- Saha and Chaudhuri, "3D digital topology under binary transformation with applications," *Comp Vis Imag Und,* **63**:418-429, 1996

Tunnels on the Surface of 3x3x3 Neighborhood (Digital Case)

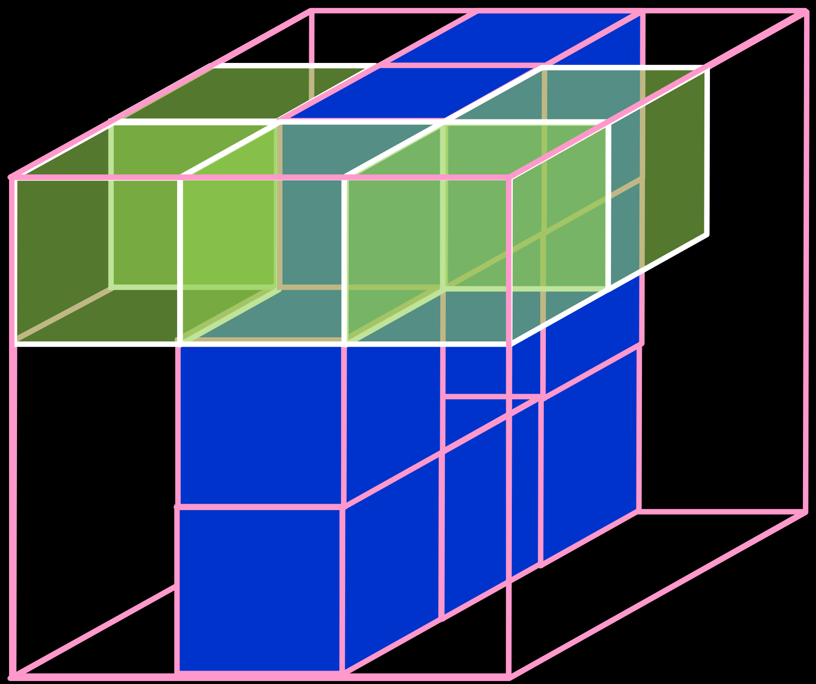
- In a $3 \times 3 \times 3$ neighborhood, if the central voxel is white, all black voxels lie on its outer surface
- For computation of tunnels, a white component must be 6-adjacent to the central voxel
- Ooops still there is some problem!!



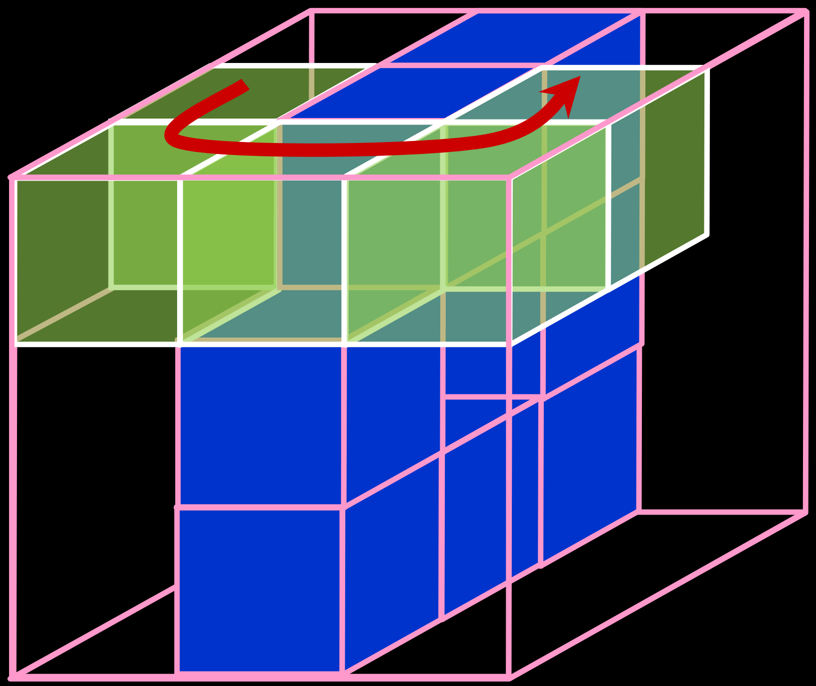
Tunnels on the Surface of 3x3x3 Neighborhood (Digital Case)



Tunnels on the Surface of 3x3x3 Neighborhood (Digital Case)

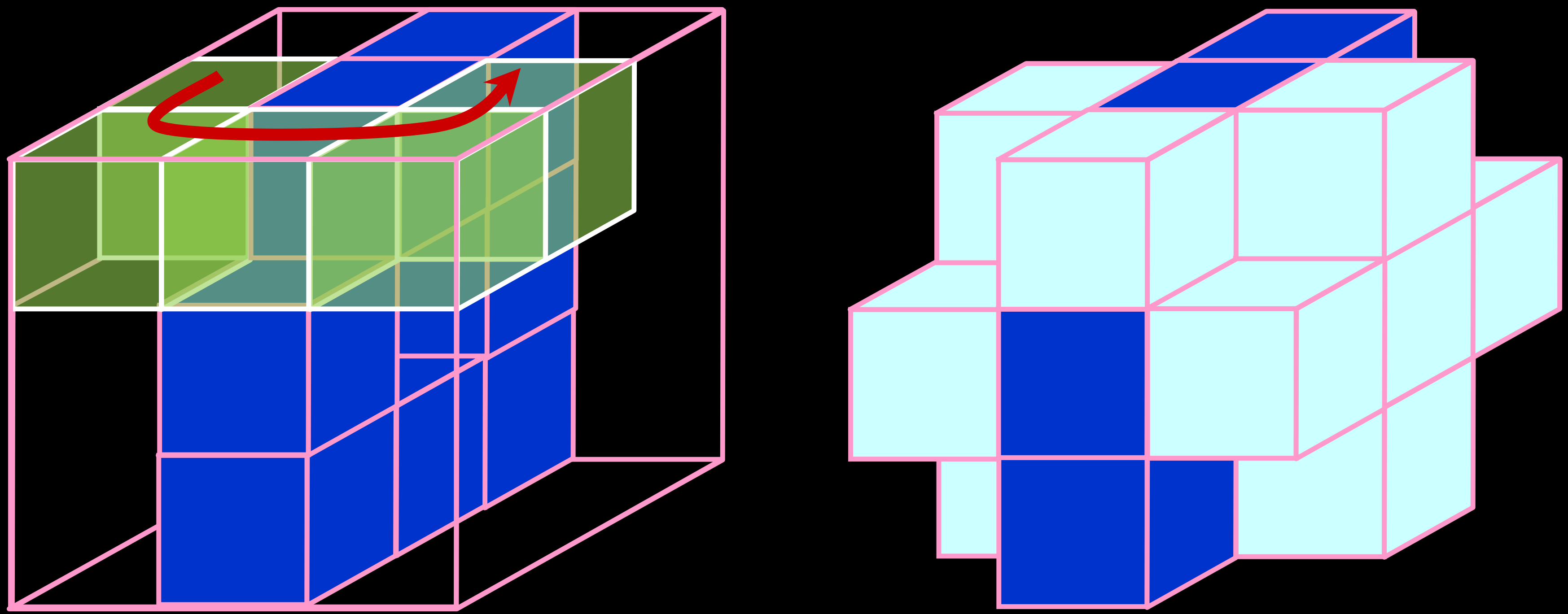


Tunnels on the Surface of 3x3x3 Neighborhood (Digital Case)



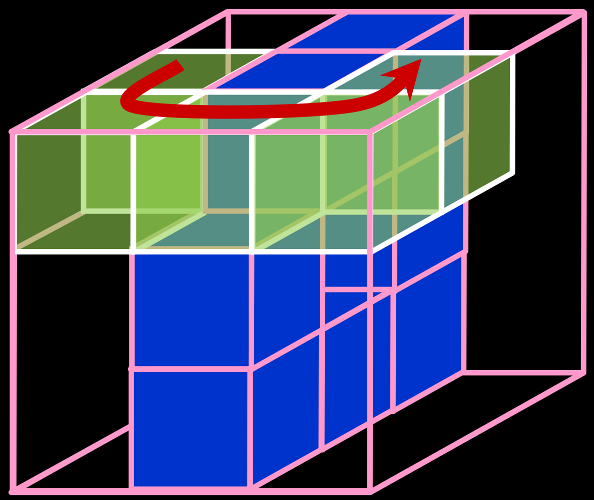
- Saha, Chanda, Dutta Majumder, "Principles and algorithms for 2D and 3D shrinking," Indian Statistical Institute, Calcutta, India, TR/KBCS/2/91, 1991.
- Saha, Chaudhuri, Chanda, Dutta Majumder, "Topology preservation in 3D digital space," Pat Recog, **27**:295-300, 1994.
- Saha and Chaudhuri, "Detection of 3-D simple points ... with application to thinning," IEEE Trans Pat Anal Mach Intel, **16**:1028-1032, 1994.
- Saha and Chaudhuri, "3D digital topology under binary transformation with applications," *Comp Vis Imag Und,* **63**:418-429, 1996

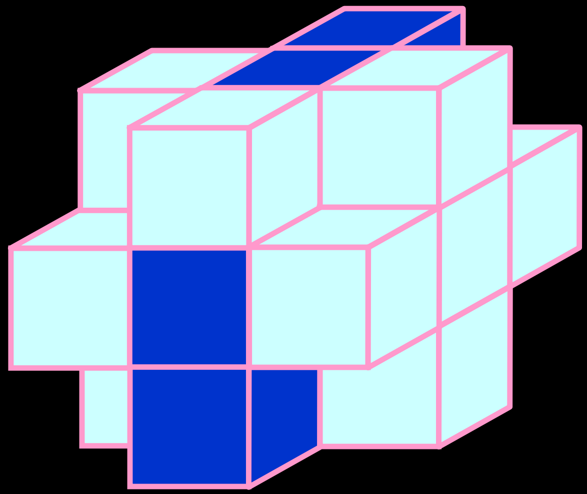
Tunnels on the Surface of 3x3x3 Neighborhood (Digital Case)



- Saha, Chanda, Dutta Majumder, "Principles and algorithms for 2D and 3D shrinking," Indian Statistical Institute, Calcutta, India, TR/KBCS/2/91, 1991.
- Saha, Chaudhuri, Chanda, Dutta Majumder, "Topology preservation in 3D digital space," Pat Recog, **27**:295-300, 1994.
- Saha and Chaudhuri, "Detection of 3-D simple points ... with application to thinning," IEEE Trans Pat Anal Mach Intel, **16**:1028-1032, 1994.
- Saha and Chaudhuri, "3D digital topology under binary transformation with applications," *Comp Vis Imag Und,* **63**:418-429, 1996

Tunnels on the Surface of 3x3x3 Neighborhood (Digital Case)





Theorem. If a voxel or point $p \in Z^3$ has at a white 6-neighbor, the number of tunnels $\eta(p)$ in $N_{26}^*(p)$ is one less than the number of 6-components of white points in $N_{18}^*(p)$ that intersect with $N_6^*(p)$, or, zero otherwise.

- Saha, Chanda, Dutta Majumder, "Principles and algorithms for 2D and 3D shrinking," Indian Statistical Institute, Calcutta, India, TR/KBCS/2/91, 1991.
- Saha, Chaudhuri, Chanda, Dutta Majumder, "Topology preservation in 3D digital space," Pat Recog, **27**:295-300, 1994.
- Saha and Chaudhuri, "Detection of 3-D simple points … with application to thinning," IEEE Trans Pat Anal Mach Intel, **16**:1028-1032, 1994.
- Saha and Chaudhuri, "3D digital topology under binary transformation with applications," *Comp Vis Imag Und,* **63**:418-429, 1996

3-D Simple Point Characterization by Saha *et al.* (1991, 1994)

Theorem. If a voxel or point $p \in Z^3$ has at a white 6-neighbor, the number of tunnels $\eta(p)$ in $N_{26}^*(p)$ is one less than the number of 6-components of white points in $N_{18}^*(p)$ that intersect with $N_6^*(p)$, or, zero otherwise.

A point $p \in Z^3$ is a (26,6) simple point in a 3-D binary image $(Z^3, 26, 6, B)$ if and only if the following conditions are satisfied

- p has a white (background) 6-neighbor, i.e., $N_6^*(p) - B \neq \phi$
- p has a black (object) 26-neighbor, i.e., $N_{26}^*(p) \cap B \neq \phi$
- The set of black 26-neighbors of p is 26-connected, i.e., $N_{26}^*(p) \cap B$ is 26-connected
- The set of white 6-neighbors of p is 6-connected in the set of white 18-neighbors, i.e., $N_6^*(p) - B$ is 6-connected in $N_{18}^*(p) - B$

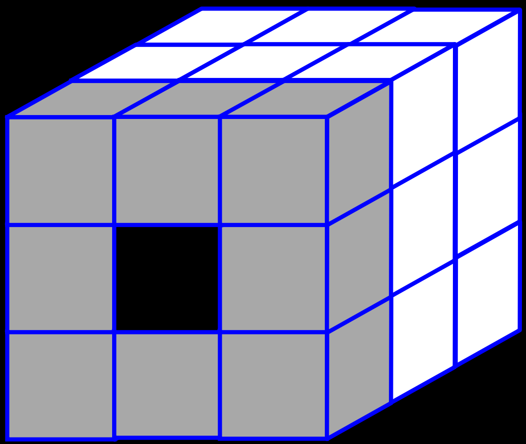
- Saha, Chanda, Dutta Majumder, "Principles and algorithms for 2D and 3D shrinking," Indian Statistical Institute, Calcutta, India, TR/KBCS/2/91, 1991.
- Saha, Chaudhuri, Chanda, Dutta Majumder, "Topology preservation in 3D digital space," Pat Recog, **27**:295-300, 1994.
- Saha and Chaudhuri, "Detection of 3-D simple points … with application to thinning," IEEE Trans Pat Anal Mach Intel, **16**:1028-1032, 1994.
- Saha and Chaudhuri, "3D digital topology under binary transformation with applications," *Comp Vis Imag Und,* **63**:418-429, 1996

Local Topological Numbers

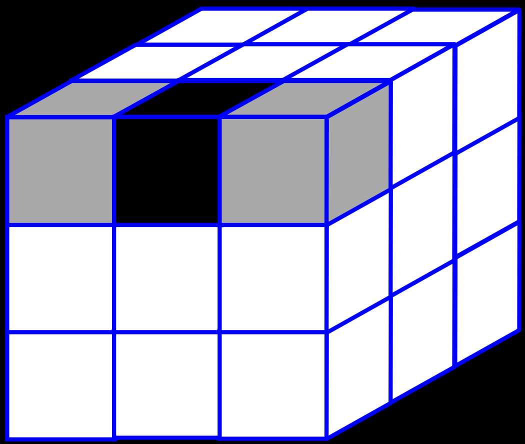
- $\xi(p)$: the number of objects components in the $3 \times 3 \times 3$ neighborhood after deletion of p
- $\eta(p)$: the number of tunnels in the $3 \times 3 \times 3$ neighborhood after deletion of p
- $\delta(p)$: the number of cavities in the $3 \times 3 \times 3$ neighborhood after deletion of p

- Saha, Chanda, Dutta Majumder, "Principles and algorithms for 2D and 3D shrinking," Indian Statistical Institute, Calcutta, India, TR/KBCS/2/91, 1991.
- Saha and Chaudhuri, "3D digital topology under binary transformation with applications," *Comp Vis Imag Und,* **63**:418-429, 1996

Efficient Computation of 3-D Simple Point and Local Topological Numbers



Dead surface

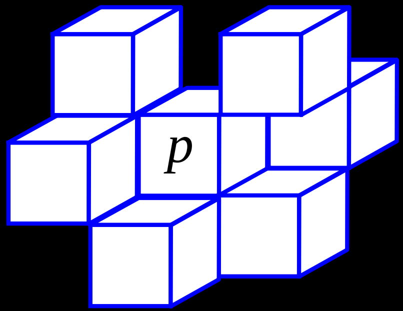


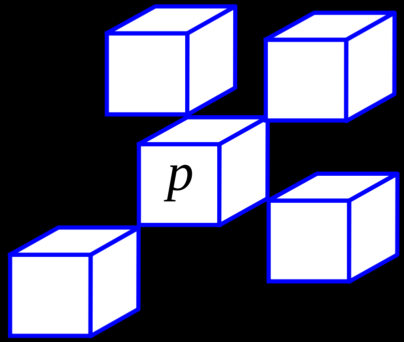
Dead edge

Theorem. 3-D simplicity and local topological numbers of a point is independent of its dead points.

- Saha, Chanda, Dutta Majumder, "Principles and algorithms for 2D and 3D shrinking," Indian Statistical Institute, Calcutta, India, TR/KBCS/2/91, 1991.
- Saha, Chaudhuri, Chanda, Dutta Majumder, "Topology preservation in 3D digital space," Pat Recog, **27**:295-300, 1994.
- Saha and Chaudhuri, "Detection of 3-D simple points … with application to thinning," IEEE Trans Pat Anal Mach Intel, **16**:1028-1032, 1994.
- Saha and Chaudhuri, "3D digital topology under binary transformation with applications," *Comp Vis Imag Und,* **63**:418-429, 1996

Effective Neighbors





- e (edge)-neighbor: 18-adjacent but not 6-adjacent, i.e., share an edge with p
- Effective e-neighbor: An e-neighbor not belonging to a dead surface

- v (vertex)-neighbor: 26-adjacent but not 18-adjacent, i.e., share a vertex with p
- Effective v-neighbor: A v-neighbor not belonging to a dead surface or a dead edge

Theorem. Object/background configuration 6-neighbors, effective e- and v-neighbors is the necessary and sufficient information to decide on 3-D simplicity and local topological numbers of a point.

Efficient Algorithm

- Determine the object/background configuration of 6-neighbors
- Determine the object/background configuration of effective e-neighbors
- Determine the object/background configuration of effective v-neighbors
- Use look up table to determine 3-D simplicity and the local topological numbers $\xi(p)$, $\eta(p)$, and $\delta(p)$

- Saha, Chanda, Dutta Majumder, "Principles and algorithms for 2D and 3D shrinking," Indian Statistical Institute, Calcutta, India, TR/KBCS/2/91, 1991.
- Saha, Chaudhuri, Chanda, Dutta Majumder, "Topology preservation in 3D digital space," Pat Recog, **27**:295-300, 1994.
- Saha and Chaudhuri, "Detection of 3-D simple points … with application to thinning," IEEE Trans Pat Anal Mach Intel, **16**:1028-1032, 1994.
- Saha and Chaudhuri, "3D digital topology under binary transformation with applications," *Comp Vis Imag Und,* **63**:418-429, 1996

Topology Preservation in Parallel Skeletonization

The principal challenge in topology preservation for parallel skeletonization

- a characterization of simple point guarantees topology preservation when one simple point is deleted at a time
- however, these characterizations fail to ensure topology preservation when a set of simple points are deleted in parallel

Our Approach

- Sub-iterative scheme. Divide an iteration into subiterations based on $2 \times 2 \times 2$ subfield partitioning of the image grid

- Saha, Chaudhuri, and Dutta Majumder, "A new shape preserving parallel thinning algorithm for 3D digital images," Patt Recog, **30**:1939-1955, 1997

Fuzzy Skeletonization, and its Applications

Outline

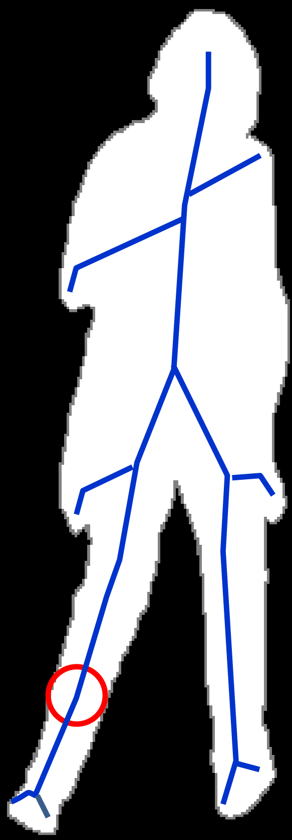
- Fuzzy Skeletonization
- Applications of Digital Topology and Geometry in Object Characterization

Outline

- Fuzzy Skeletonization
- Applications of Digital Topology and Geometry in Object Characterization

Principle of Skeletonization

- Object: A closed and bounded subset of R^3
- Maximal Included Ball: A ball included in the object that cannot be cannot be fully included by another ball inside the object
- Skeleton: Loci of the centers of maximal included balls
- Blum's Grassfire Transform: A process that yields the skeleton of a binary objects



Blum’s Grassfire Propagation

- Blum’s grassfire transform is defined by fire propagation on a grass field, where the field resembles a binary object.
  - grassfire is simultaneously initiated at all boundary points
  - grassfire propagates inwardly at a uniform speed
  - the skeleton is defined as the set of quench points where two or more opposite fire fronts meet

Fuzzy Grassfire Propagation

- Fuzzy Object: A membership value is assigned at each voxel
- The membership value is interpreted as the fraction of object occupancy in a given voxel or local material density
- Fuzzy Grassfire Propagation
  - grassfire is simultaneously initiated at the boundary of the support of a fuzzy object
  - the speed of fire-front at at given voxel is inversely proportion to its material density, i.e., membership value
  - grassfire stops at quench voxels when its natural speed of propagation is interrupted by colliding impulse from opposing fire-fronts

Outline of the Algorithm

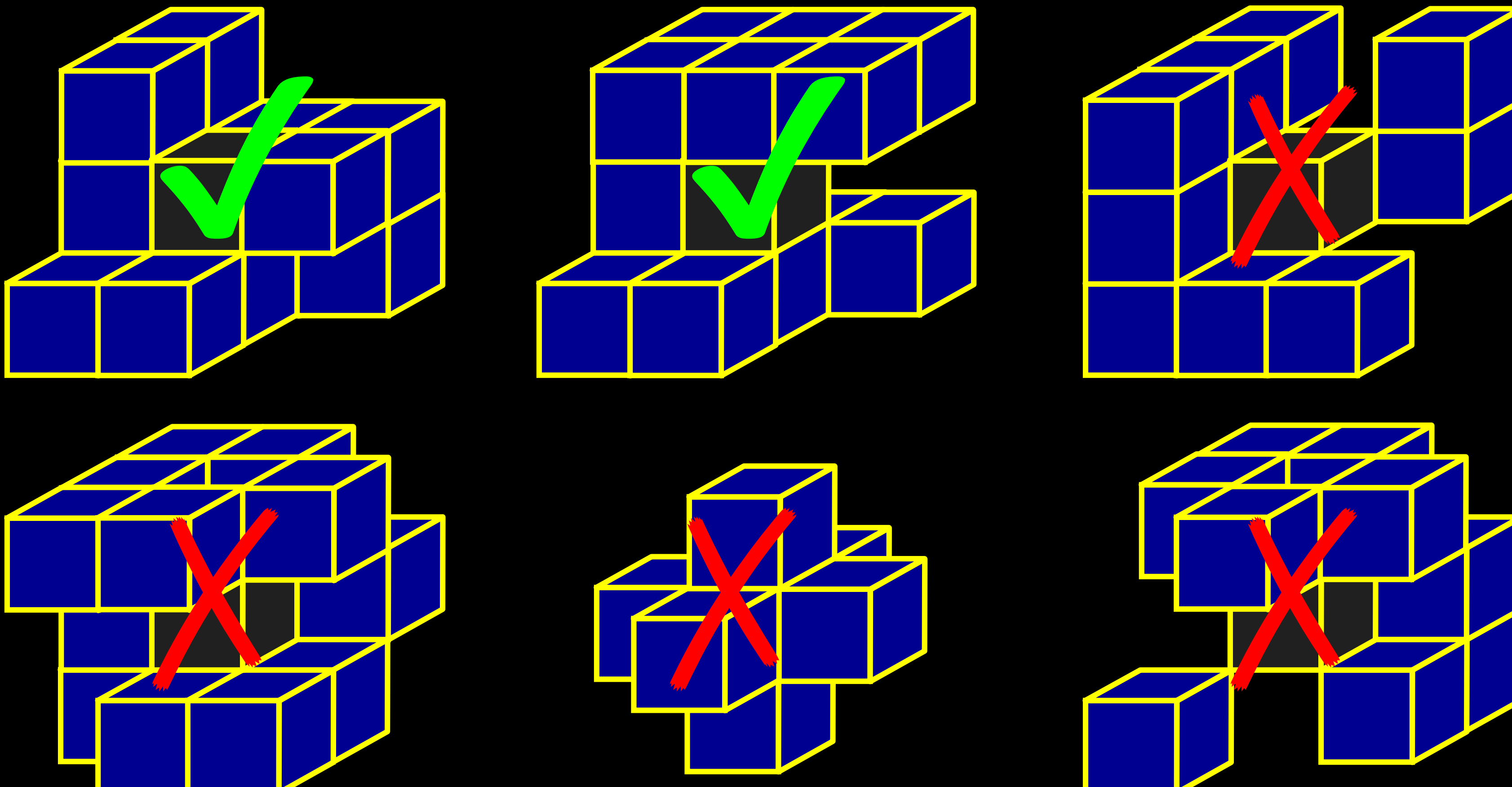
- Primary skeletonization
  - Locate fuzzy quench voxels in the decreasing order of FDT values and filter those using local shape factor
  - Sequentially remove simple points that are not necessary for topology preservation in the increasing order of FDT values
- Final skeletonization
  - Convert two-voxel thick structures into single-voxel structures
  - Remove voxels with conflicting topological and geometric properties
- Skeleton pruning
  - Compute global shape factor to detect spurious branches
  - Delete spurious branches

Simple Points: Topology Preservation

Theorem: A point *p* is a 3-D simple point if and only if it satisfies the following four conditions:

- *p* has a black 26-neighbor
- *p* has a white 6-neighbore
- The set of black 26-neighbors of *p* is26-connected
- The set of white 6-neighbors of *p* is 6-connected in the set of white 18-neighbors of *p*

Simple Points: Examples

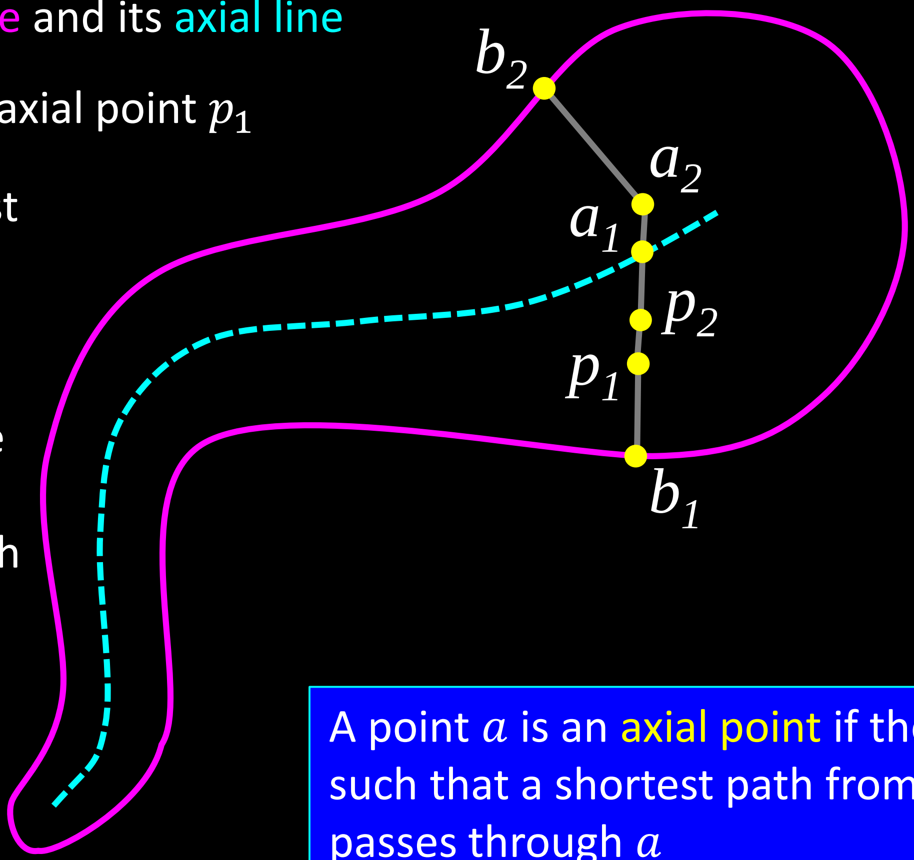


Fuzzy Quench or Axial Points

- During fuzzy grassfire propagation, the speed of a fire-front at a given voxel equates to the inverse of local material density
- Fuzzy distance transform defines the time when the fire-front reaches at a given voxel
- This process is violated only at quench or axial points where the propagation is interrupted by colliding impulse from opposite fire-fronts

How to Locate an Axial Point

1. Consider a shape and its axial line
2. Consider a non-axial point p_1
3. Find the shortest path $p_1 b_1$ to boundary
4. If we extend the path $p_1 b_1$ to p_2 the shortest path $p_2 b_1$ passes through p_1



5. Now, consider an axial point a_1
6. If extend the shortest path $a_1 b_1$ to a_2, the shortest path from a_2 to the boundary does not passes through a_1

A point a is an axial point if there is no point a' such that a shortest path from a' to the boundary passes through a

Fuzzy Quench or Axial Points

- A point p is a quench or axial point if there is no point p' such that a shortest path from p' to the boundary passes through p.
- Specifically, a point p is a quench or axial point if there is no point q in the neighborhood of p such that

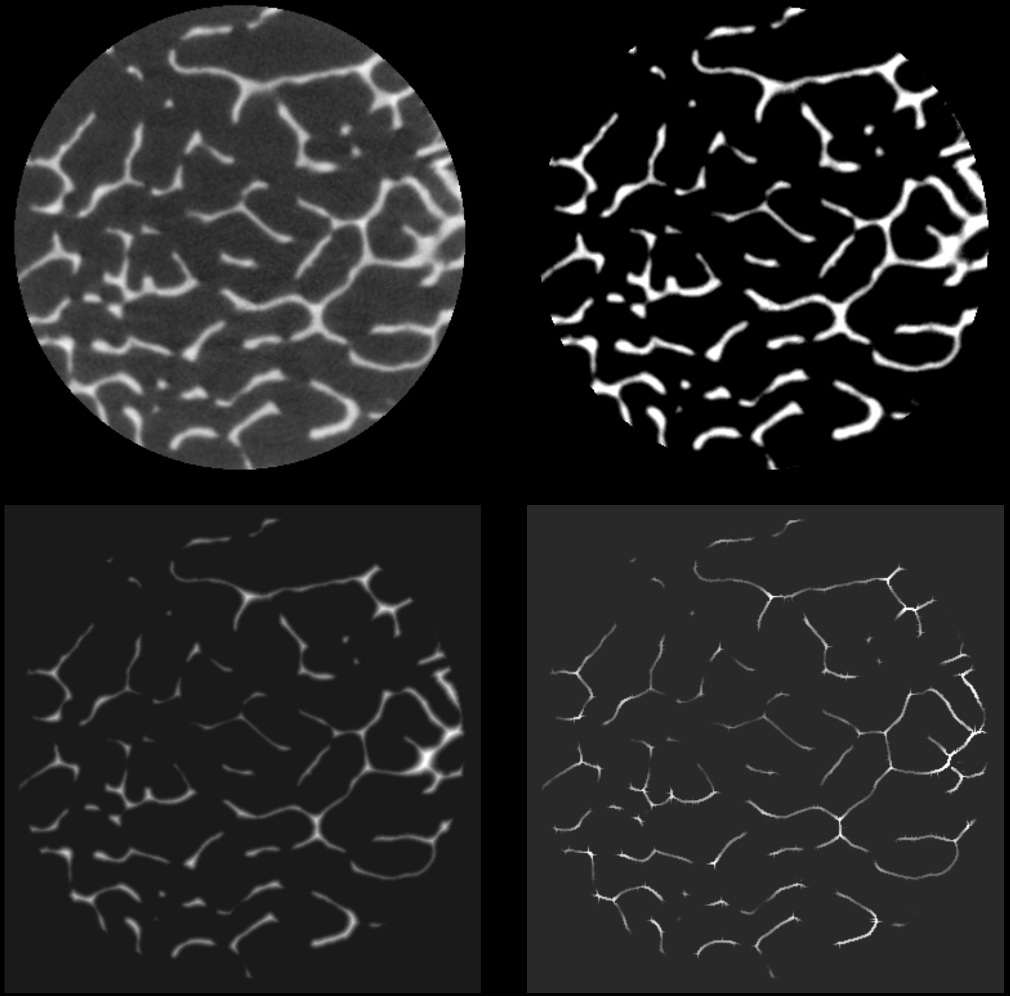
$$\Omega_{\mathcal{O}}(q) = \Omega_{\mathcal{O}}(p) + \mu_{d_{\mathcal{O}}}(p, q)$$

where $\Omega_{\mathcal{O}}$ is the FDT function and $\mu_{d_{\mathcal{O}}}$ is the length of a link

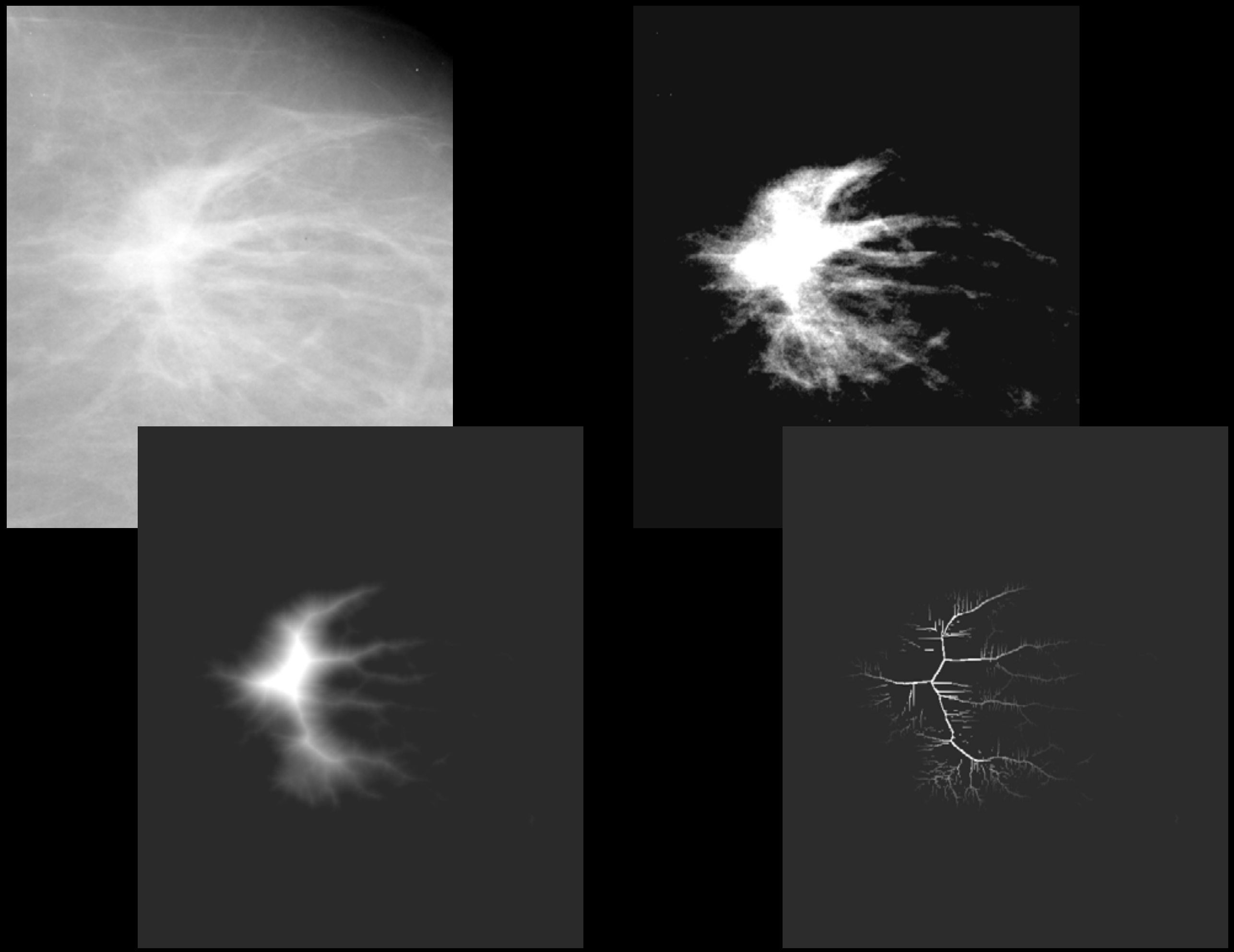
- Arcelli and Sanniti di Baja introduced a criterion to detect the centers of maximal balls (CMBs) in a binary digital image using 3×3 weighted distance transform
- Borgefors extended it to 5×5 weighted distances
- This concept was generalized to fuzzy sets by Saha and Wehrli, Svensson, and Jin and Saha

- Arcelli, Sanniti di Baja, "Finding local maxima in a pseudo-Euclidean distance transform", Comput Vis Graph Imag Proc, 43: 361-367, 1988
- Borgefors, "Centres of maximal discs in the 5-7-11 distance transform", Proc of the Scandinavian Conf on Imag Anal ,**1**: 105-105), 1993
- Saha, Wehrli, "Fuzzy distance transform in general digital grids and its applications", Proc of 7th Joint Conf Info Sc, Research Triangular Park, NC, 201-213, 2003
- Svensson, "Aspects on the reverse fuzzy distance transform", Patt Recog Lett **29**: 888-896, 2008
- Jin, Saha. "A new fuzzy skeletonization algorithm and its applications to medical imaging." Proc. of 17th Int Conf on Imag Anal Proc (ICIAP), 662-671, 2013.

Examples

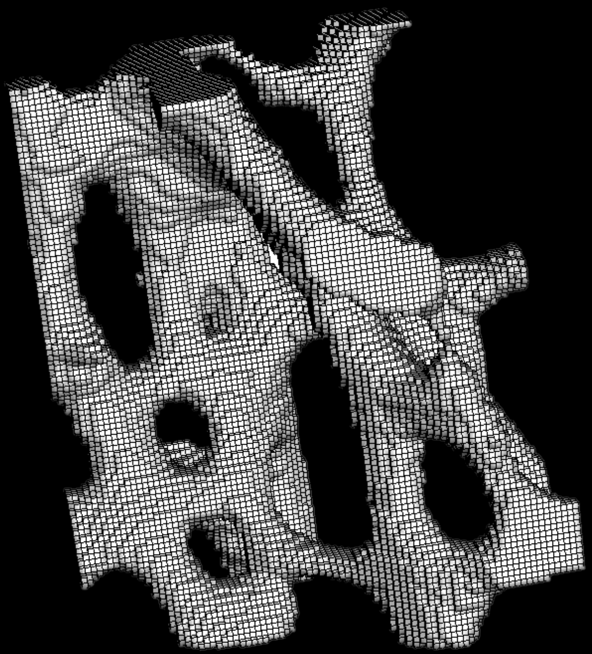


Examples

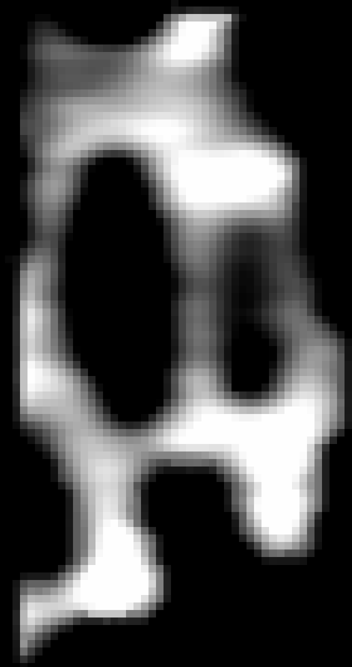


Filtering Quench or Axial Voxels

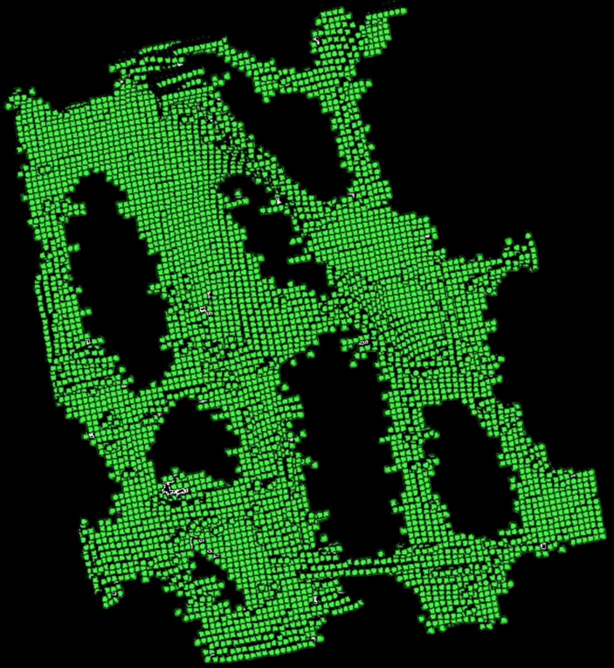
- Too many spurious quench voxels



Support of the fuzzy object



An image slice of the fuzzy object



All quench voxels

Local Shape Factor for Quench Voxels

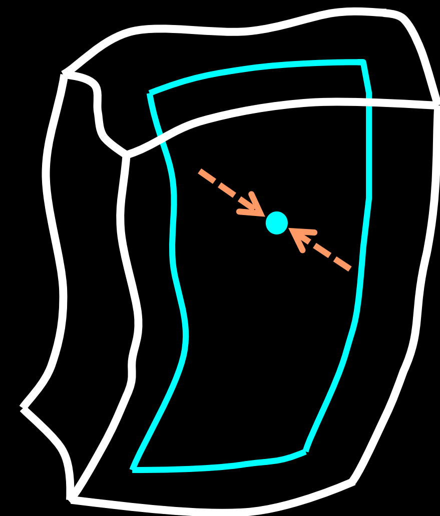
- At quench voxels, natural speed of fire-front propagation is interrupted by colliding impulse from opposite fire-fronts
- Local Shape Factor is defined as the measure of this "degree of colliding impulse"

$$LSF(p) = 1 - f_{+}\left(\max_{q \in N^{*}(p)} \frac{\Omega_{\mathcal{O}}(q) - \Omega_{\mathcal{O}}(p)}{\mu_{d_{\mathcal{O}}}(p, q)}\right)$$

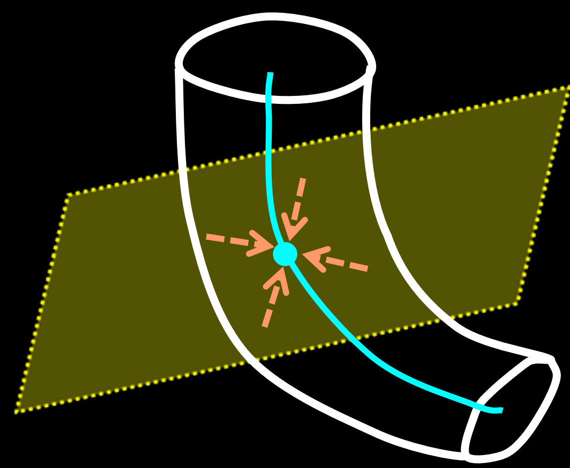
- Local shape factor determines the significance of individual quench voxels

Surface and Curve Quench Voxels

- Surface Quench Voxels
  - two opposite fire fronts meet



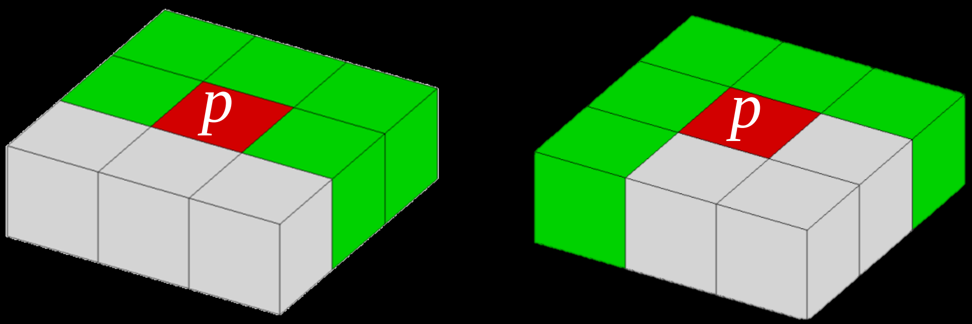
- Curve Quench Voxels
  - fire fronts meet from all directions on a plane



Filtering Quench Voxels

- Define a suitable support mask that fits the geometric type of the quench voxel
- Determine the significance in terms of LSF over the support mask

Support mask for a surface quench voxel



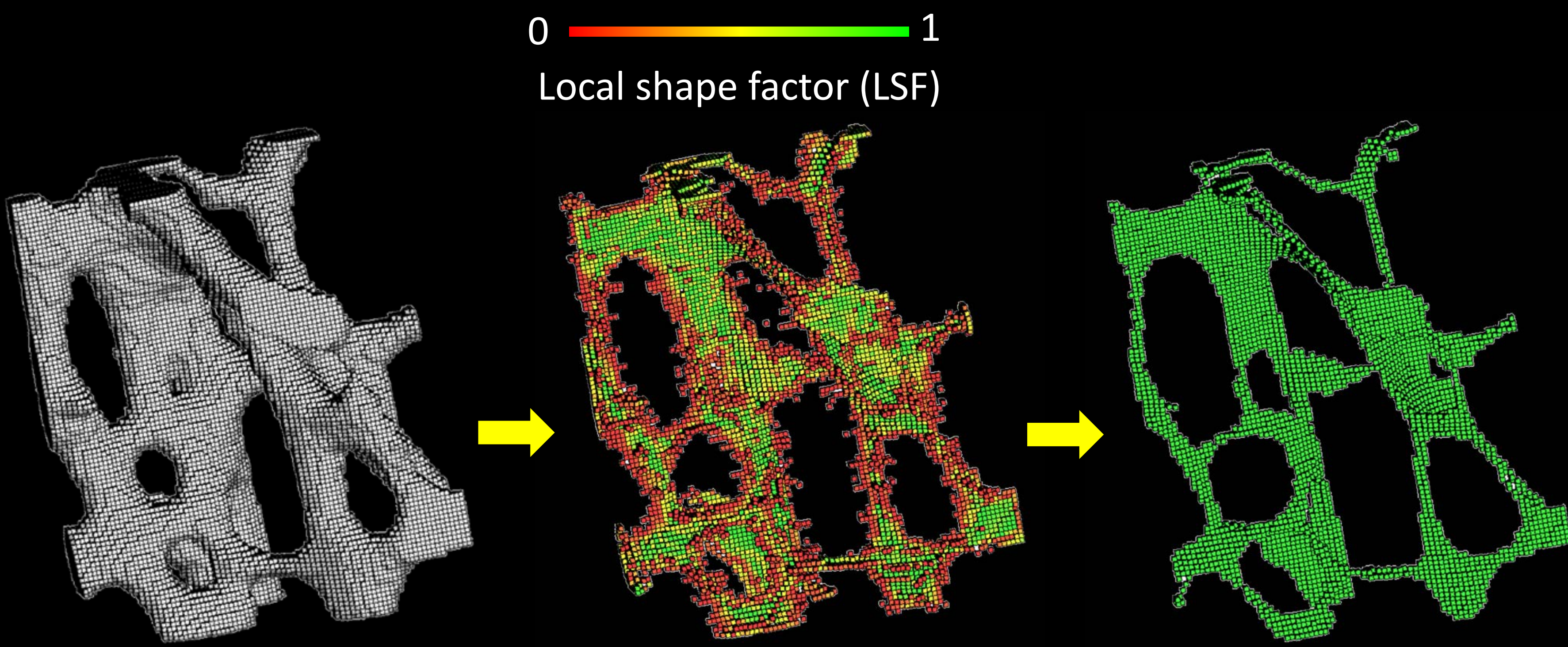
Overall significance

- compute minimum LSF over the support mask

or

- compute the average LSF over the support mask

Filtered Axial Voxels



Support of the fuzzy object

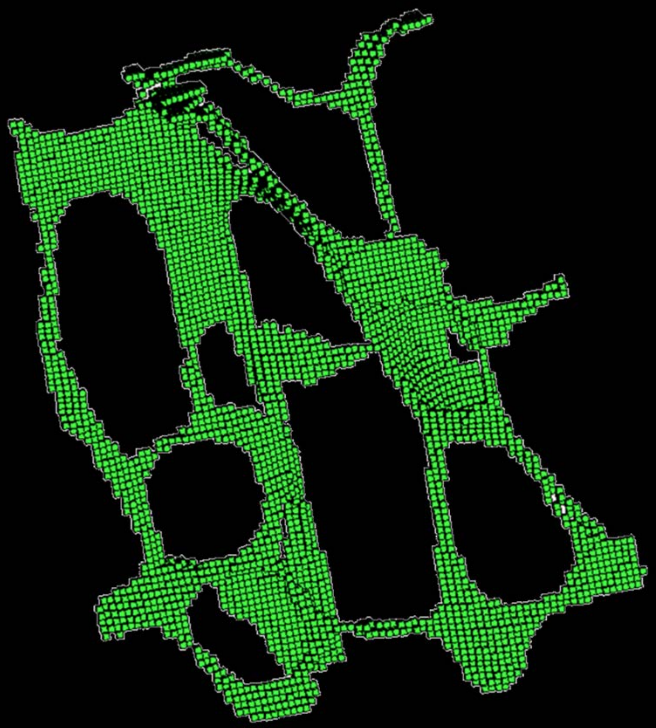
Initial quench voxels; red voxels have lower LSF values representing noisy quench voxels

Filtered quench voxels

Skeletal Pruning

- Compute global shape factor of each branch by adding LSF values of individual voxels and prune spurious branches

before



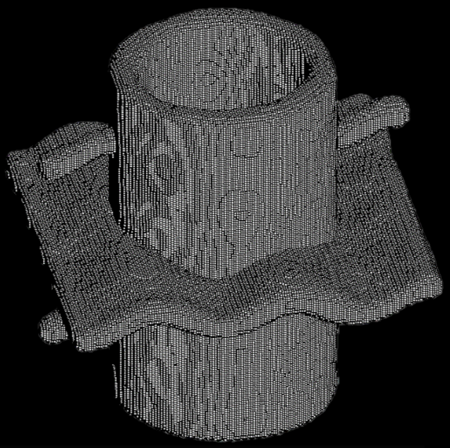
after

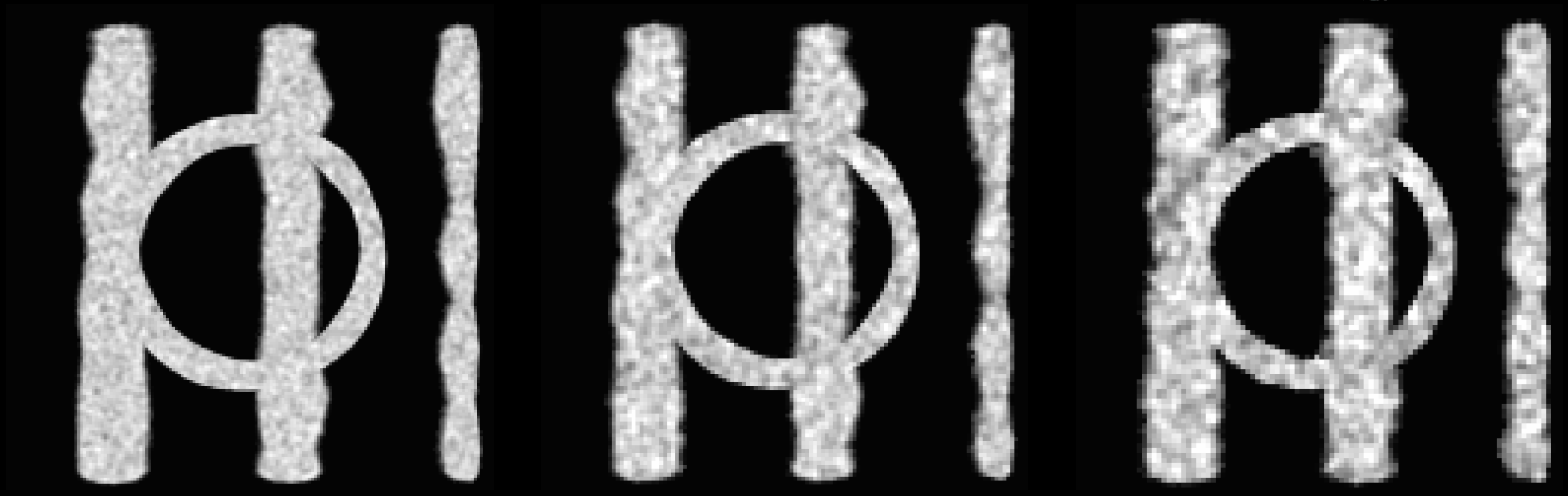
A Few Examples

A Few Examples

Evaluation

- Ground truth: High resolution 3-D binary objects with known skeletons
- Test phantoms: Down-sampling binary objects and addition of white Gaussian noise to generate fuzzy objects

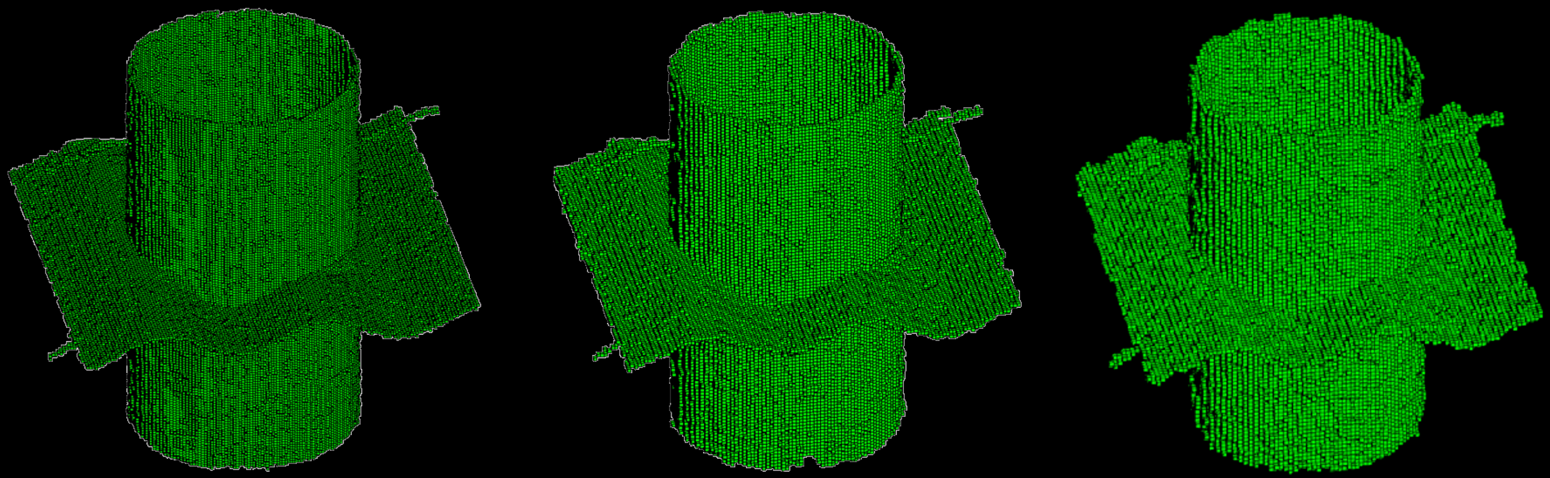




low noise/blur medium noise/blur high noise/blur

Results

- Skeletons at low, medium, and high noise/blur



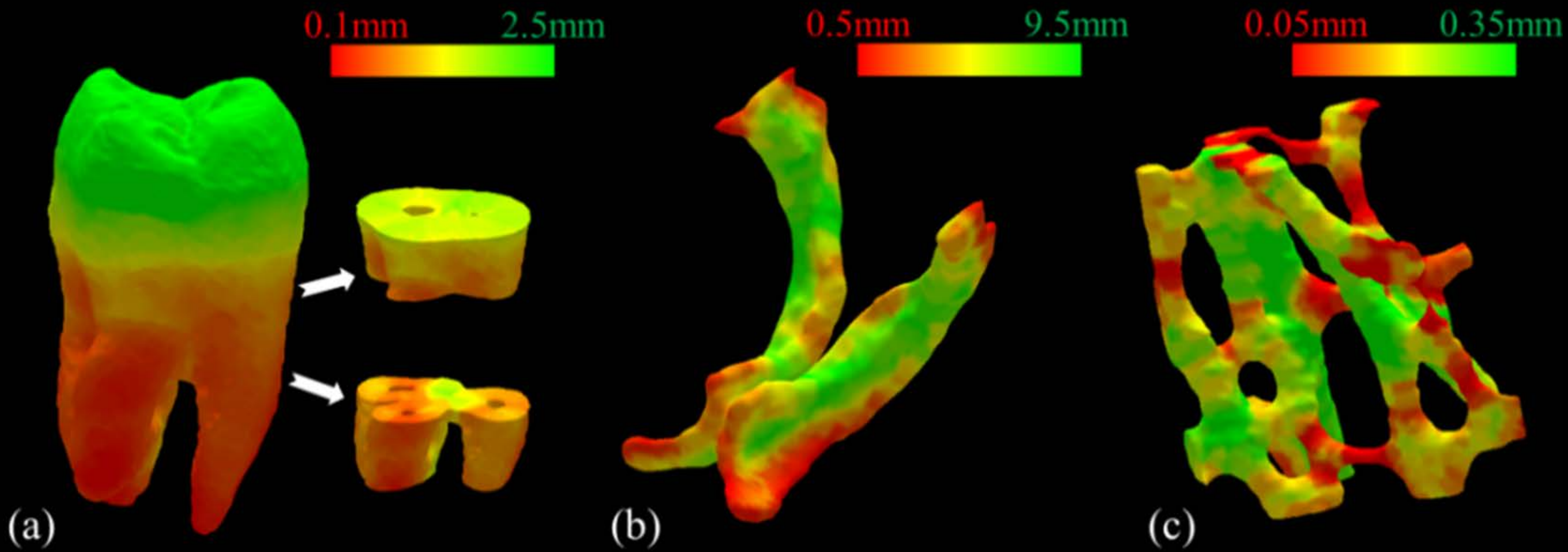
- Fuzzy skeletonization errors in voxel unit

Downsampling	No noise	SNR 24	SNR 12	SNR 6
3×3×3	0.49	0.52	0.54	0.58
4×4×4	0.52	0.53	0.54	0.58
5×5×5	0.57	0.58	0.59	0.60

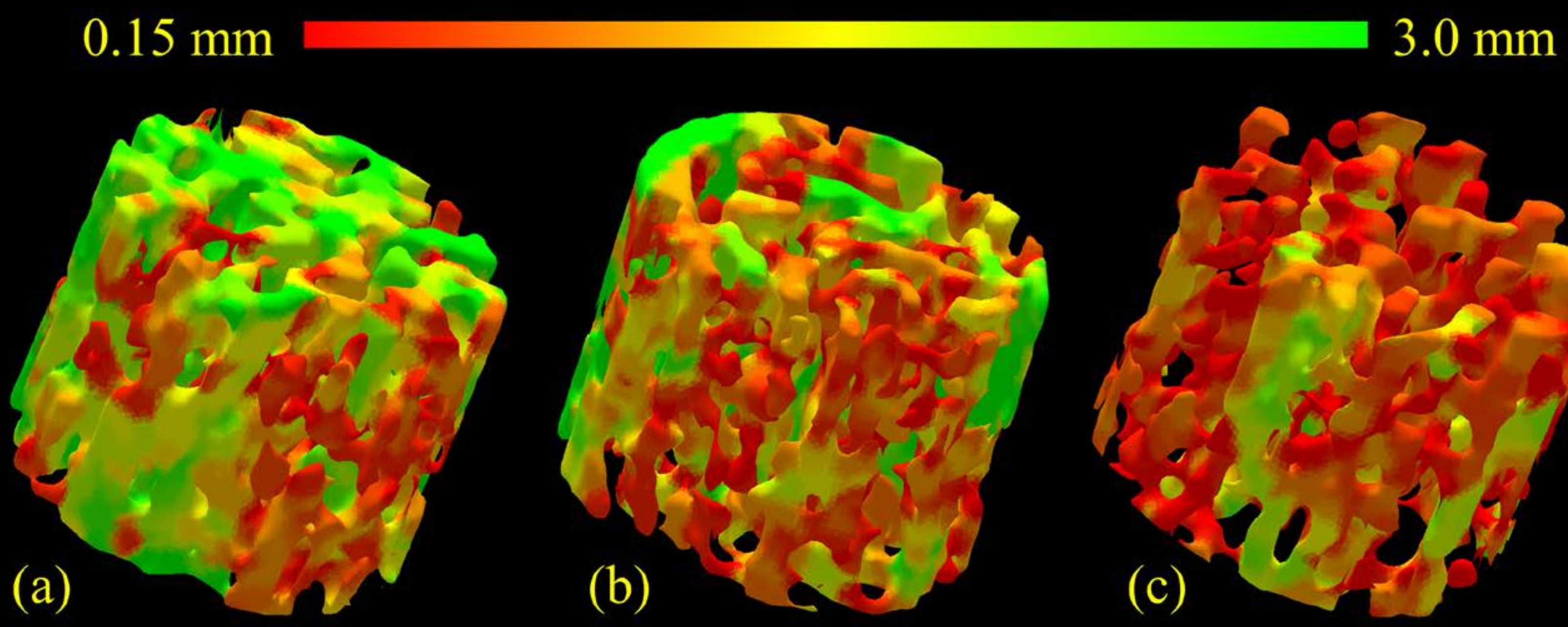
Results of Application on Online 3D Figures

Applications

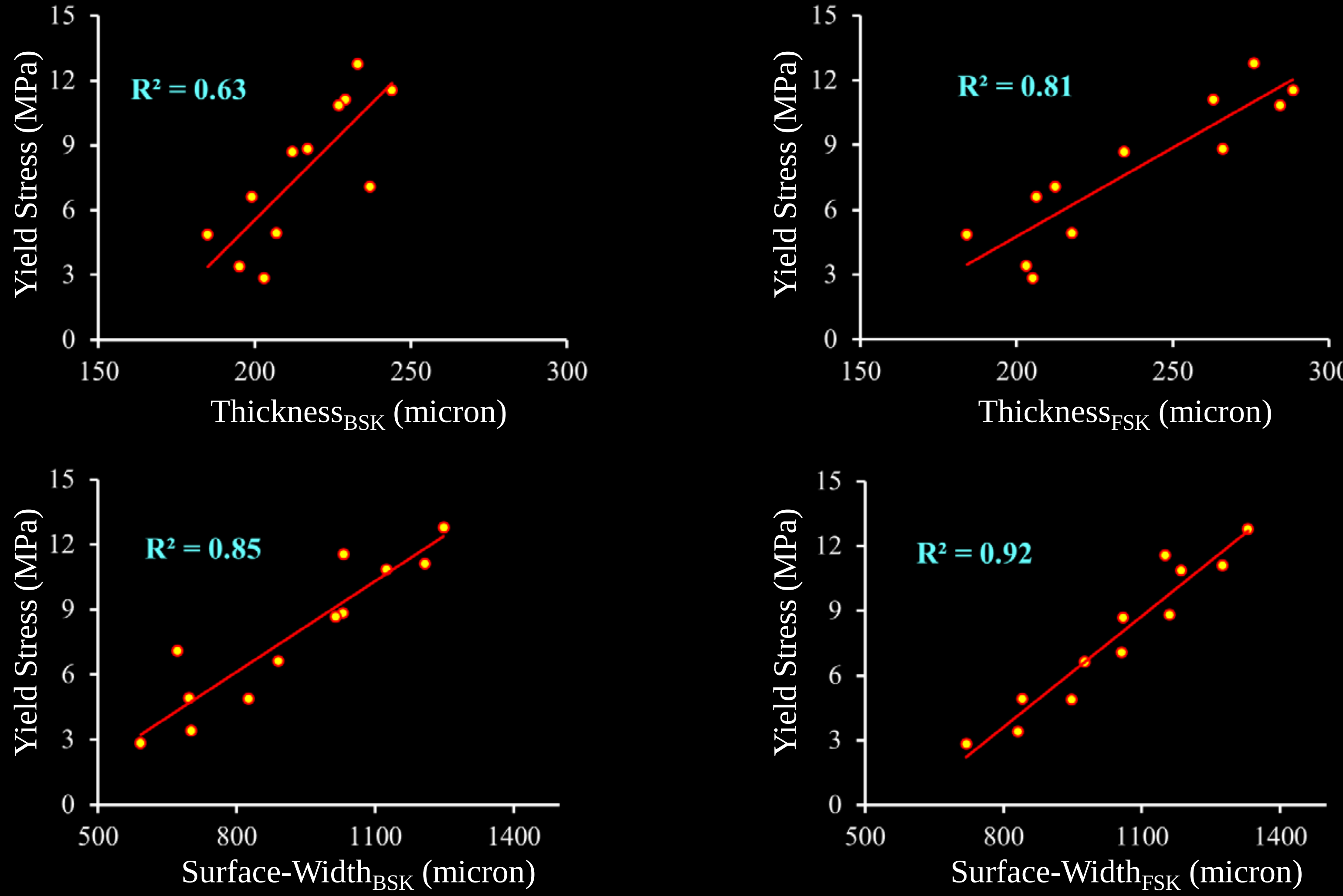
Local Thickness Computation for Fuzzy Objects



Local Width Computation for Fuzzy Objects



Fuzzy Skeletonization Improves the Sensitivity of Derived Measures



Summary

- The issues of sequential topological transformation in 3-D cubic grid (3-D simple point) are solved
- Local topological properties introduced by Saha et al. are useful to characterize 1-D and 2-D digital manifolds and their junctions embedded in a 3-D digital space
- Topology preservation in parallel skeletonization is effectively solved using a subfield approach
- Digital topology and geometry play important roles in medical image processing
  - solves several classical problems of medical imaging
  - expands the scope of target information
  - provides a strong theoretical foundation to a process enhancing its stability, fidelity, and efficiency
- A comprehensive framework for fuzzy skeletonization is developed along the spirit of fuzzy grassfire propagation
- Experimental results show that the fuzzy skeletons are computed with sub-voxel accuracies under various levels of SNR and downsampling rates
- Fuzzy skeletonization improves the performance of individual trabecular thickness and width computation at *in vivo* CT imaging