Theory of Object Class Uncertainty and its Application

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References


Outline

• Theory Of Object Class Uncertainty
• Applications To Optimum Thresholding
• Applications To Snake
Object Class Uncertainty

Postulate. In an image with fuzzy boundaries, at optimum partitioning of object classes, voxels with high class uncertainty appear in the vicinity of object boundaries.

Computation of Object Class Uncertainty

A priori probability an object pixel having intensity $g$

$$p_O(g) = P(f(c) = g \mid c \in F_O),$$

where $P$ represents “probability,” and $F_O$ represents the true object class.

A priori probability a background pixel having intensity $g$

$$p_B(g) = P(f(c) = g \mid c \in F_B),$$

where $F_B$ represents the true object class.

$\theta$: A priori probability of any pixel belonging to object

$p(g)$: A priori probability of any pixel having intensity $g$

$$p(g) = \theta p_O(g) + (1 - \theta)p_B(g).$$
Computation of Object Class Uncertainty

A *posteriori* probability:

\[
P(c \in F_O | f(c) = g) = \frac{\theta p_O(g)}{p(g)}
\]

\[
P(c \in F_B | f(c) = g) = \frac{(1 - \theta)p_B(g)}{p(g)}
\]

\[h(g) : \text{“object class uncertainty” at intensity } g\]

\[h(g) = -\frac{\theta p_O(g)}{p(g)} \log \frac{\theta p_O(g)}{p(g)} - \frac{(1 - \theta)p_B(g)}{p(g)} \log \frac{(1 - \theta)p_B(g)}{p(g)}\]
**Optimum Thresholding**

**Postulate.** In an image with fuzzy boundaries, at optimum partitioning of object classes, voxels with high class uncertainty appear in the vicinity of object boundaries.

\[
E(t) = \sum_{c \in C} H_t(f(c))(1 - \Delta_{\text{rank}}(c)) + \left(1 - H_t(f(c))\right)\Delta_{\text{rank}}(c),
\]

where \(H_t\) is the uncertainty map at a threshold \(t\) and \(\Delta_{\text{rank}}\) is a rank-normalized gradient operator.

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**A theory to combine information theoretic measures with image gradient features**

Rank-Normalized Gradient

- DoG measures are sensitive to the standard deviation parameter of the normalizing Gaussian function
- Rank-based normalization of the gradient parameter
  - A parameter-free approach of normalization

\[
\Delta_{\text{rank}} = \frac{LC(\Delta(c))}{LC(\Delta_{\text{max}})},
\]

where
- \( \Delta \) is the intensity gradient operator
- \( LC(x) = \sum_{y \leq x} L(y) \), and \( L(y) \) is the histogram count for the intensity gradient value \( y \)

Optimum Thresholding Algorithm

**Principle.** Minimization of Uncertainty Homogeneity Energy (MHUE) $E(t)$

$$t_{opt} = \arg \min \limits_t E(t)$$

An efficient computation of the Energy function $E(t)$

$$E(t) = \sum_{c \in C} H_t(f(c))(1 - \Delta_{rank}(c)) + (1 - H_t(f(c)))\Delta_{rank}(c)$$

∀ intensity value $i$,

$$X(i) = \sum_{c \in C \mid f(c) = i} 1 - \Delta_{rank},$$

$$Y(i) = \sum_{c \in C \mid f(c) = i} \Delta_{rank},$$

Efficient formulation of $E(t)$

$$E(t) = \sum_i H_t(i)X(i) + (1 - H_t(i))Y(i)$$

Note: Number of possible intensity values in an image is far less than the number of pixels/voxels in the image

Results

Digitized Mammogram

MSII\textsuperscript{†}

MHUE

optimum uncertainty (OU) map

\textsuperscript{†}Leung, Lam, “Maximum segmented image information thresholding,” Graph Mod Imag Proc, 60: 57-76, 1998
Results

Inverted CT

MSII

MHUE

OU Map

Threshold determined by the MSII method

Threshold determined by the MHUE method
Multiple Object Segmentation

CT Slice  MSII  MHUE  OU Map

$SIH$  $EOJ$
Application on MR Slice Data

- Flair
- MHUE segmentation
- OU Map

Threshold determined by the MSh method
Thresholds determined by the MHUE method
Phantom Experiment

Phantoms

MHUE

OU Map
Phantom Experiment

Phantoms

MSII

Threshold determined by the MSII method
Application of Object Class Uncertainty to Snake

Outline

• Brief Overview of Snake
• Basic Challenges
• Object Feature Force
• Object Class Uncertainty
• Smart Force
• Smart Snake – Methods and Design
• Experimental Results
Curves in Motion

- **Initialization**
  - *Squeezing Snake*: Object contained entirely inside the region enclosed by the initial contour
  - *Expanding Snake*: Object entirely includes the region enclosed by the initial contour
  - **Automatic**
    - Expand from a seed point using balloon force
    - Converge from the boundary of image frame
Internal Energy

The spline properties

• An elastic rubber band possessing elastic potential energy
  – Responsible for shrinking of the contour
• Behaves like a thin metal strip giving rise to bending energy
  – Bending energy is minimum for a circle.
• Total internal energy of the snake $\nu$ can be defined as

$$E_{\text{int}} = \alpha(s) \left\| \frac{\partial \nu(s)}{\partial s} \right\| + \beta(s) \left\| \frac{\partial^2 \nu(s)}{\partial s^2} \right\|^2$$
Snake: Basic Formulation

- **Snake**: a deformable spline $\nu$

- **Basic Snake Equation**

  $$E_{\text{snake}} = E_{\text{int}} + E_{\text{image}} + E_{\text{con}}$$

- **Internal Energy**
  - String (elastic) Force
  - Rigidity Force

  $$E_{\text{int}} = \alpha(s) \left\| \frac{\partial \nu(s)}{\partial s} \right\| + \beta(s) \left\| \frac{\partial^2 \nu(s)}{\partial s^2} \right\|^2$$

- **Image Energy**
  - Gradient
  - Intensity

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An Overlooked Territory

• Theory and algorithms to optimally fit in *a priori* object/background feature information

Attempts to overcome this limitation

• A blind balloon force† to move the snake in homogeneous regions
• Failure to arrest uncontrolled snake propagation once leaked through a weak boundary zone
• Sub optimal performance near boundaries with narrow concavities

Main Contribution

• Introduction of object/background feature based SMART FORCE into snake

Nature of smart force

• Expanding within the object
• Compressing inside the background
• Weakens at the vicinity of the object-background interface
Design of the Smart Force

• Probably, we need...
  • Optimum object-background classification
  • Confidence level of the classification

• We have used...
  • Object Class Uncertainty† Based Smart Force

Smart force

\[ F_{\text{smart}, \nu}(p) = \begin{cases} 
1 - h(f(p))\mathbf{t}_\nu(p), & \text{if } p \in O, \\
-1 + h(f(p))\mathbf{t}_\nu(p), & \text{otherwise}. 
\end{cases} \]

\[ \mathbf{t}_\nu(p) : \text{unit vector radially outward at the location } p \text{ on the contour } \nu \]

- Expanding (inside object)
- Contracting (inside background)
- Weak force (at interface)

Properties of Smart Force

- Direction adaptive
  - Expands inside the object
  - Compresses within background
  - Resists uncontrolled post-leaking propagation

- Optimal response to the chaos in acquired signal

- Complementary with Image Gradient force
  - stronger inside homogeneous regions
  - weak near boundaries
Estimation of Uncertainty Force

- Prior Information about object and background intensity distribution acquired

\[ m_O : \text{Object mean} \quad \sigma_O : \text{Object standard deviation} \]
\[ m_B : \text{Background mean} \quad \sigma_B : \text{Background standard deviation} \]
Image Force Field and Snake

- Smart force (SS)
- Balloon snake (BS)
Comparative Results

Phantom with high object-background contrast at different levels of noise and blurring
Comparative Results
Comparative Results

phantom
Smart force
SS
BS
Object Class Uncertainty Induced Smart Snake
Comparison with Balloon Snake

Segmentation result (red) using balloon snake

Segmentation result (red) smart snake
Comparison with Balloon Snake

Result (red) using balloon snake

Results (red) using smart snake
Carotid Data Segmentation using Smart Snake
Summary

• Introduced object class uncertainty theory
  – Combines information theoretic measure with image features

• A fundamental postulate is stated
  – In most real life imaging applications, under optimum classification,
    image elements with the maximum class uncertainty appear in the
    vicinity of object boundaries.
  – Supported by results of application on several real images and 250
    computer generated realistic phantoms
  – Potential application in multiple image and data classification tasks
Summary (Contd.)

• Application to optimum thresholding
  – Potential application in local threshold selection
  – Results of application using both real and phantom data

• Introduced object class uncertainty based smart force into snake model
  – Direction adaptive
  – Strength adaptive to fit with the inherent chaos in signal
  – Acts in complementary fashion with image gradient information

• Preliminary results of application of class uncertainty based smart snake on several natural and medical data