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ME:7268 Turbulent Flows – Student Project Instructions

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1. Introduction

Additional analyses of the Four-dimensional particle tracking velocimetry (4DPTV) static drift β = 10 deg results for the 5415 sonar dome vortices have been done to realize its full potential for the assessment of the turbulence structure and vortex breakdown and interactions and for providing data for scale resolved computational fluid dynamics (CFD) validation.

Figure 1: DTMB 5415 geometry.

The focus is on the strongest primary sonar dome vortex (SDVP) at x/L = 0.12 (just downstream of the sonar dome) and its interaction with the second strongest (SDVS) of the multiple sonar dome vortices. Both vortices are counterclockwise and due to cross flow separations with SDVP and SDVS onset from the windward (port) and leeward (starboard) sides of the sonar dome.

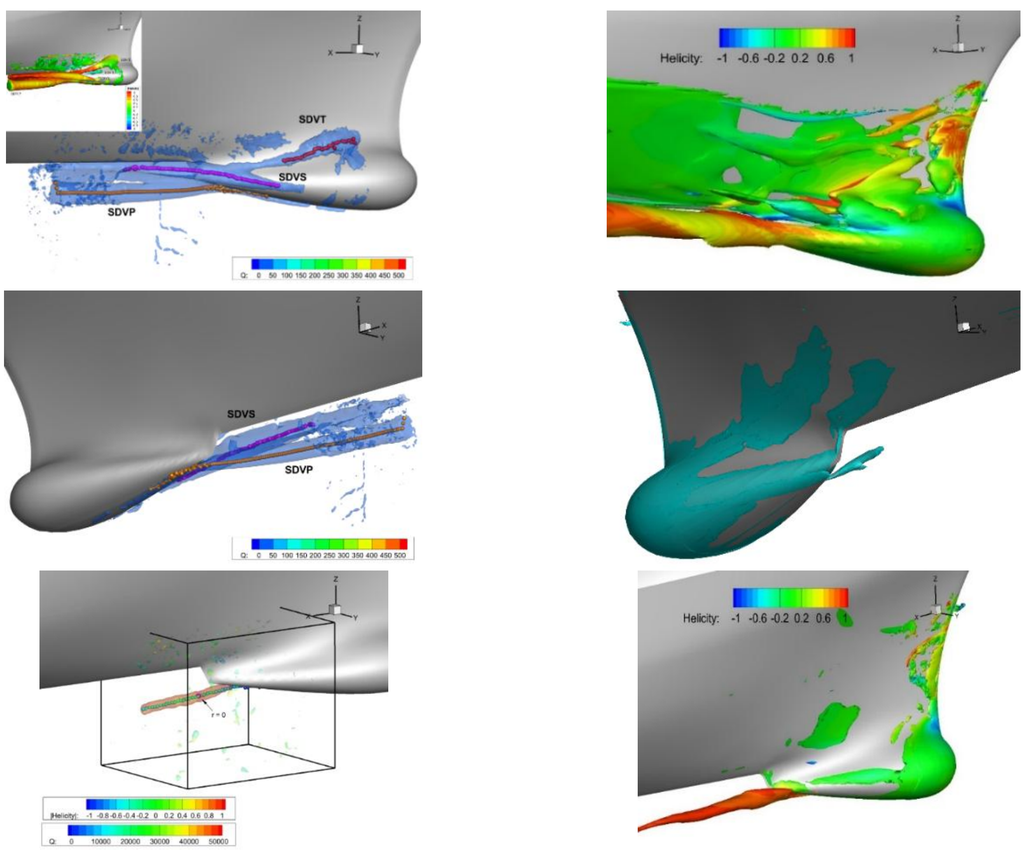

Figure 2: SDVP, SDVS, SDVT, and SDVT2 vortices (top and middle) and turbulence analysis location for SDVP core (bottom).

Figure 2 (top and middle) shows the overall structure and onset and progression of the 5415 sonar dome vortices, including the primary vortices SDVP, SDVS, SDVT, and SDVT2, whereas Figure 1 (bottom) shows the location for the turbulence structure and vortex breakdown and interaction analysis for the SDVP vortex core at x/L = 0.12. The analysis methods are largely based on Bernard (2019) and Pope (2000) with detailed derivations provided by Stern et al. (2023).

The students will be required to partially follow the analysis conducted in Stern et al. (2024) to obtain the turbulence characteristics starting from the raw timeseries data of the flow at different points in the 4DPTV volume, as shown in Figure 3.

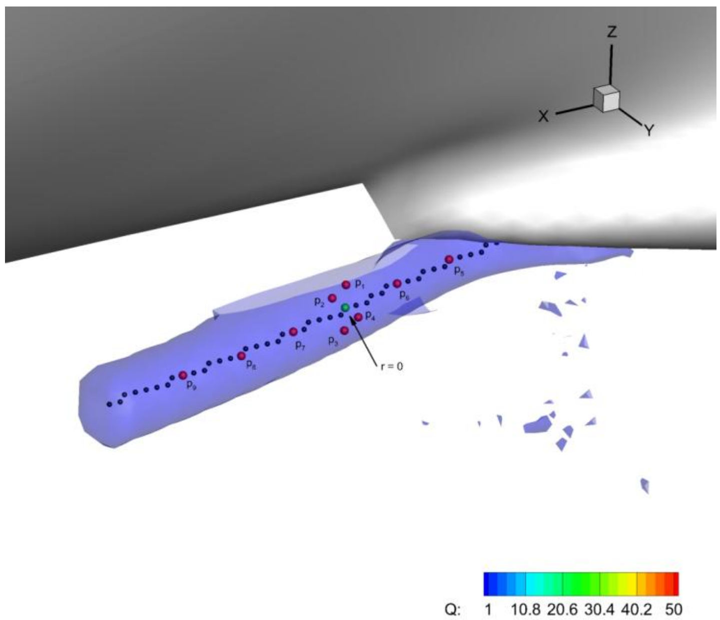

Figure 3: 4DPTV sampling points for turbulence analysis.

2. Velocity time series

Velocity ($U(t)$, $V(t)$, and $w(t)$) time histories of SDVP vortex (upstream, downstream, and around core) are extracted using the Butterworth filtered instantaneous four-dimensional particle-tracking-velocimetry (4DPTV) dataset. **This step is already completed, and the SDVP vortex core timeseries data (at desired locations) is provided to students. The velocity field is non-dimensionalized using the carrier velocity ($\boldsymbol{U_C = 1.531\ m/s}$).**

3. Microscale, Macroscale, and temporal autocorrelation

Starting from the provided velocity time histories, students are required to perform the following:

a. Reynolds's decomposition of the velocity

   i. Find time-averaged velocity components, $\overline{U}$, $\overline{V}$, and $\overline{W}$

   ii. Get fluctuating velocity components

$$u(t) = U(t) - \overline{U}$$

$$v(t) = V(t) - \bar{V}$$

$$w(t) = W(t) - \bar{W}$$

iii. Determine macroscale estimates

$$k = \frac{1}{2}(\langle u^2 \rangle + \langle v^2 \rangle + \langle w^2 \rangle)$$

$$u_0 = \sqrt{k}$$

$$u' = \sqrt{\langle u^2 \rangle}$$

$$l_0 = 0.025\, m = 25\, mm$$

$$\varepsilon = u_0^3 / l_0$$

$$Re_{l_0} = u_0 l_0 / \upsilon$$

b. Perform temporal autocorrelation on the fluctuating velocity component time history

$$R_E(\tau) = \frac{\langle u(t)u(t+\tau)\rangle}{\langle u^2 \rangle}$$

c. Student should calculate temporal microscale and macroscale (τ_E and T, respectively) with the following equations

$$\tau_E = \left[\frac{-2}{R_E''(0)}\right]^{\frac{1}{2}}$$

a. Pope pg. 198 details method of using a osculating parabola at $\tau = 0$ to determine second derivative or $R_E''(0)$ which also shows that τ_E is where the parabola intersects the horizontal axis
    i. Student can curve fit a parabola (built-in functions in python/MATLAB to the autocorrelation function

$$T = \int_0^\infty R_E(\tau)\, d\tau$$

d. Use Taylor's frozen turbulence hypothesis to convert temporal scales into Taylor microscale and macroscale length scales. Additionally, estimates can be calculated for dissipation and the Kolmogorov length scale

$$\lambda_f = \bar{U}\tau_E$$

$$\Lambda_f = \bar{U}T$$

$$\varepsilon = {30\nu u'^2}\Big/{\lambda_f{}^2}$$

$$\eta = \left({\nu^3}/{\varepsilon}\right)^{\frac{1}{4}}$$

$$R_\lambda = \frac{\sqrt{2}\, u' \lambda_f}{\nu}$$

Note: $u' = (\frac{2}{3}k)^{\frac{1}{2}}$ is used here instead of $\langle u^2 \rangle$ or u_0.

Summary:

Macroscale parameters	
k	$\frac{1}{2}(\langle u^2 \rangle + \langle v^2 \rangle + \langle w^2 \rangle)$
u_0	$\sqrt{k}$
u'	$\sqrt{\langle u^2 \rangle}$
l_0	$0.025\, m = 25\, mm$
ε	u_0^3/l_0
Re_{l_0}	$u_0 l_0/\upsilon$
τ_E	$\left[\frac{-2}{R_E''(0)}\right]^{\frac{1}{2}}$
T	$\int_0^\infty R_E(\tau)\, d\tau$
λ_f	$\bar{U}\tau_E$
Λ_f	$\bar{U}T$
R_λ	$\frac{\sqrt{2}\, u' \lambda_f}{\nu}$
η	$\left(\nu^3/\varepsilon\right)^{\frac{1}{4}}$

Microscale parameters	
τ_E	$\left[\frac{-2}{R_E''(0)}\right]^{\frac{1}{2}}$
T	$\int_0^\infty R_E(\tau)\, d\tau$
λ_f	$\bar{U}\tau_E$
Λ_f	$\bar{U}T$
R_λ	$\frac{\sqrt{2}\, u' \lambda_f}{\nu}$
ε	$30\nu u'^2 / {\lambda_f}^2$
η	$\left(\nu^3/\varepsilon\right)^{\frac{1}{4}}$

4. Fourier transform of temporal autocorrelation and 1D energy spectrum

a. Take Fourier transform of $R_E(\tau)$

$$\hat{R}_E(2\pi\omega) = 2\int_0^\infty R_E(\tau)\, cos(2\pi\omega\tau)\, d\tau$$

$$\hat{R}_E(2\pi\omega) = \frac{1}{\overline{u^2}} \lim_{T\to\infty} \frac{1}{2T} \underbrace{\int_{-T}^{T} u(t)e^{-i\omega' t}dt}_{\boxed{\hat{u}(\omega)}} \underbrace{\int_{-\infty}^{\infty} u(t)e^{i\omega' t}dt}_{\boxed{\hat{u}^*(\omega)}}$$

Fourier transform code/function will be provided to students as it should be important to keep the windowing and scaling. A MATLAB/Python code will be available to download on the course website. Students wanting to use another program should talk to the TA and double check results.

b. Verify that the FFT of the autocorrelation and PSD of $u(t)$ are equivalent.

c.

d. Transform $\hat{R}_E(2\pi\omega)$ to 1D energy spectrum in Time $\hat{E}_{11}(\omega)$ and space $E_{11}(k_1)$ (using Taylor's frozen hypothesis again)

$$\hat{E}_{11}(\omega) = 2\langle u^2\rangle \hat{R}_E(2\pi\omega) = \lim_{T\to\infty}\frac{1}{T}|\hat{u}(\omega)|^2$$

$$E_{11}(k_1) = \frac{\overline{U}}{2\pi}\hat{E}_{11}(\omega)$$

e. Scale $E_{11}(k_1)$ and plot for Kolmogorov scaling and compensated scaling (using both macroscale and microscale estimates for ε)

   a. $E_{11}(k_1)/(\varepsilon\nu^5)^{\frac{1}{4}}$ vs. $k_1\eta$

   b. $\varepsilon^{-\frac{2}{3}} k_1^{\frac{5}{3}} E_{11}(k_1)$ vs. $k_1\eta$

      i. Log-linear

      ii. Linear-log

5. Anisotropy analysis

Starting from the given Reynolds stress tensor, determine/verify the following:

a. Decompose the Reynolds stress tensor into its isotropic and anisotropic components:

$$R_{ij} = a_{ij} + \frac{2}{3}k\delta_{ij}$$

b. Determine the anisotropic Reynolds stress tensor:

$$b_{ij} = \frac{a_{ij}}{2k}$$

And confirm that it respects the realizability constraints:

$$-\frac{1}{3} \leq b_{11}, b_{22}, b_{33} \leq \frac{2}{3}, -\frac{1}{2} \leq b_{ij} \leq \frac{1}{2}\ i \neq j$$

c. Plot the Reynolds stress tensor and anisotropic stress tensor components in the original frame of reference. Comment the plot.

d. Determine the eigenvalues and eigenvectors of R_{ij} and b_{ij}. Verify that the eigenvalues respect this relation:

$$\lambda_{b_i} = -\frac{1}{3} + \frac{\lambda_{u_i}}{\lambda_{u_1} + \lambda_{u_2} + \lambda_{u_3}}$$

Verify that the eigenvectors are the same for R_{ij} and b_{ij}. Why?

e. Plot the eigenvalues of the Reynolds stress tensor and anisotropic stress tensor. Comment the plot.

f. Plot the Lumley triangle and the anisotropic invariant map of the Reynolds stress. Comment the plot.

g. Plot the Reynolds stress ellipsoid, its principal axes, and the directions of the mean velocity and vorticity of the flow. What does the ellipsoid represent?

   i. Determine the angles generated by the Reynolds stress principal axes and the mean velocity vector.
   ii. Determine the angles generated by the Reynolds stress principal axes and the mean vorticity vector.