Chapter 3 Turbulent flow equations

Instantaneous equations

- Continuity and Navier -Stokes equations
- Mechanical energy equation
- Energy equation
- Vorticity equation
- Enstrophy equation
- Pressure equation

Reynolds averaged equations

- Continuity and RANS
- KE mean flow equation
- TKE equation
- Dissipation ε equation
- Reynolds stress equation
- Mean vorticity equation
- Fluctuating vorticity equation
- Vorticity transport equation
- Enstrophy equation

Instantaneous / Deterministic / DNS : Fundamental physics
evaluate statistical methods

NS : $\nabla \cdot \underline{v} = 0$

$$\rho \frac{D\underline{v}}{Dt} = -\nabla \hat{p} + \mu \nabla^2 \underline{v}$$

ME : $$\rho \frac{Dk}{Dt} = \underbrace{\rho \underline{g} \cdot \underline{v} + \frac{\partial}{\partial x_j}(v_i \sigma_{ij})}_{\text{rate of work: body force } + \sigma_{ij}} - \varphi$$

$\varphi = 2\mu \varepsilon_{ij}\varepsilon_{ij}$
= rate of viscous dissipation
= loss ME

Energy: $$\rho \frac{D\hat{u}}{Dt} = \nabla \cdot (k \nabla T) + \varphi$$

Same term opposite sign

φ $\hat{u}$ increase due ME loss viscous dissipation

+ Vorticity, Enstrophy, & Pressure

RANS / Statistical : idealized flow physics — homogeneous/isotropic + shear, free shear, BL, channel/pipe
canonical flow physics
industrial physics
turbulence modeling

$\tilde{u}_i = v_i + u$ (mean $\langle v_i \rangle$)

RANS : $$\frac{Dv_i}{Dt} = -g\delta_{i3} + \rho^{-1}\frac{\partial}{\partial x_j}\overline{\sigma}_{ij}$$

$\overline{\sigma}_{ij} = -\overline{p}\delta_{ij} + 2\mu E_{ij} - \rho\overline{u_i u_j}$

$E_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i})$

MKE : $$\frac{DK}{Dt} = \frac{\partial T}{\partial x_j} - 2\nu E_{ij}E_{ij} + \overline{u_i u_j}\, v_{i,j} - g v_3/\rho$$

$K = \frac{1}{2}v_i v_i$

transport, viscous dissipation, loss due turbulence, loss due PE

Same term opposite sign

TKE : $$\frac{Dk}{Dt} = -\nabla T + P - \varepsilon$$

$\varepsilon = 2\nu \varepsilon_{ij}\varepsilon_{ij}$ $\quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$

transport + shear production − dissipation
gain loss

Mean flow (RANS) : $\overline{u_i u_j}$, MKE

MKE : P loss

TKE : P gain & ε loss

Chapter 4 : Turbulence at Small Scales

Richardson Cascade

& Kolmogorov hypotheses & Scaling

$$\frac{D\varepsilon}{Dt} = P_\varepsilon^1 + P_\varepsilon^2 + P_\varepsilon^3 + P_\varepsilon^4 \underbrace{+ \Pi_\varepsilon + T_\varepsilon + D_\varepsilon}_{\varepsilon \text{ redistribution or loss at boundary}} - Y_\varepsilon$$

P_ε^i = production (mostly > 0)

Y_ε = viscous loss ε

& RS, $\overline{\Omega}_i$, ω_i, vorticity transport, & enstrophy

Chapter 5 Energy Decay Isotropic Turbulence

$$\frac{dk}{dt} = -\varepsilon \qquad \frac{d\varepsilon}{dt} = P_\varepsilon^4 - Y_\varepsilon$$

$R_{ij}(\underline{x}, \underline{y}, t)$ equation

Karman-Howarth equation

Energy spectrum equation

$R_{ij}(\underline{x}, \underline{y}, t) = \langle u_i(\underline{x}, t) u_j(\underline{y}, t) \rangle \neq f(t)$ if $\langle \rangle$ = time average

Chapter 6 Turbulent Transport & Its Modeling

Gradient transport

Homogeneous Shear flow : $\frac{dk}{dt} = P - \varepsilon \quad \frac{d\varepsilon}{dt} = P_\varepsilon^1 + P_\varepsilon^2 + P_\varepsilon^4 - Y_\varepsilon$

Vorticity transport