A baseball pitcher throws a curveball with an initial velocity of 65 mi/h and a spin of 6500 r/min about a vertical axis. A baseball weighs 0.32 lbf and has a diameter of 2.9 in. Using the data of Fig. P7.108 for turbulent flow, estimate how far such a curveball will have deviated from its straight-line path when it reaches home plate 60.5 ft away.

Let us assume the gravity is not involved and baseball moves at horizontal plane (xy plane). The baseball pitcher has to throw the ball with counterclockwise spin around z axis so the dynamic lift force can counteract with the outward centrifugal force to keep the ball on the path.

\[ \omega = \frac{6500(2\pi/60)}{681} \text{ rad/s} \]

\[ \frac{\omega R}{V_0} = 0.86 \Rightarrow C_D = 0.44 \text{ and } C_L = 0.17 \text{ (Fig. 7-108)} \]

**a. approximate solution**

In this approach:
1- Drag and lift coefficients are assumed constant.
2- It is assumed X and Y acceleration are constant i.e. X and Y velocities are linear functions of time t.
3- It is assumed that the ball has uniform circular motion i.e. traversing a circular path at constant speed (U).
4- U velocity can be assumed the average X velocity at start \( V_0 \) and at home plate.

First, we need to find the drag and lift coefficients:

\[ V_0 = 65 \text{ mi/h} = 95 \text{ ft/s} \]

\[ \omega = 6500(2\pi/60) = 681 \text{ rad/s} \]

Therefore: \( \omega R/V_0 = 0.86 \Rightarrow C_D = 0.44 \text{ and } C_L = 0.17 \text{ (Fig. 7-108)} \)

The initial accelerations in the x- and y-directions are

\[ a_{x,0} = -\frac{\text{drag}}{m} = -\frac{0.44(0.00238/2)(95)^2(\pi/4)(2.9/12)}{0.32/32.2 \text{ slug}} \approx -22.0 \text{ ft/s}^2 \]

\[ a_{y,0} = -\frac{\text{lift}}{m} = -\frac{0.17(0.00238/2)(95)^2(\pi/4)(2.9/12)}{0.32/32.2} \approx -8.5 \text{ ft/s}^2 \]

Assuming \( a_x \) is constant, the X velocity at home plate would be:
\[ a_x = -22 \text{ ft/s}^2 \text{ and } x = 60.5 \text{ ft} \Rightarrow x = 1/2a_x t^2 + v_{0x} t \Rightarrow t = 0.69 \text{ sec} \Rightarrow v = a_x t + v_{0x} = 79.8 \text{ ft/s} \]

The radius of circular motion can be found if the circular velocity \( U \) and centripetal acceleration are known. \( U \) can be estimated from averaged X velocity. In addition, the centripetal acceleration for circular motion would be same as \( a_{y0} \). Therefore, the radius of the path can be estimated:

\[
R = \frac{U^2}{a_{y0}} = \frac{V_{ave}^2}{a_{y0}} = \frac{(95 + 79.8)^2}{2} = 898.9 \text{ ft}
\]

Based on above figure:

\[
\theta = \sin^{-1}\left(\frac{\Delta x}{R}\right) = \sin^{-1}\left(\frac{60.5}{898.9}\right) = 3.86^\circ
\]

\[
\Delta y = R(1 - \cos \theta) = 898.9(1 - \cos 3.86) = 2.03 \text{ ft}
\]

**b. exact solution**

To find the exact solution, we should solve the problem in normal and tangential coordinate. In this approach, we assume:

1. Drag and lift coefficients are constant.
2. The tangential and normal acceleration are constant.

For tangential direction, the drag force should be balanced with inertial force:

\[
F_t = ma_t \Rightarrow -C_D \frac{1}{2} \rho AV^2(t) = m \frac{dV(t)}{dt} \Rightarrow \frac{dV}{V^2} = -\frac{\rho AC_D}{2m} dt \Rightarrow \frac{1}{V} = -\frac{\rho AC_D}{2m} t + \text{Const}
\]

\[
A = \frac{\pi D^2}{4} = 0.046 \text{ ft}^2
\]

\[
\rho = 0.00238 \text{ slug/ft}^3
\]

\[
m = \frac{W}{g} = \frac{0.32}{32.2} = 0.0099 \Rightarrow \frac{1}{V} = 0.00243t + 0.0105 \text{ or: } V = \frac{1}{0.00243t + 0.0105}
\]

\[
C_D = 0.44
\]

\[
V(t = 0) = 65 \frac{mi}{h} = 95 \frac{ft}{s}
\]

The calculated velocity along the path shows it reduces by passing time due to the drag force (resistance). In addition, the velocity and consequently the acceleration along the path is nonlinear function of time.
For normal direction, the lift force should be balanced with centrifugal force:

\[ F_n = ma_n \Rightarrow C_L \frac{1}{2} \rho AV^2(t) = m \frac{V^2(t)}{r} \Rightarrow r = \frac{2m}{\rho AC_L} \]

The radius of curvature can be defined as:

\[ r = \frac{[1 + \left(\frac{dy}{dx}\right)^2]^{3/2}}{d^2y/dx^2} \]

Therefore:

\[ \frac{[1 + \left(\frac{dy}{dx}\right)^2]^{3/2}}{d^2y/dx^2} = \frac{2m}{\rho AC_L} \]

By solving above differential equation, the function of path can be found:

\[
\frac{dy}{dx} = \alpha \Rightarrow \frac{d\alpha}{\rho AC_L} \left[1 + \alpha^2\right]^{3/2} = \frac{2m}{\rho AC_L} \Rightarrow \frac{d\alpha}{\sqrt{1+\alpha^2}} = \frac{\rho AC_L}{2m} dx \Rightarrow \alpha = \frac{\rho AC_L}{2m} x + \text{Const}
\]

\[ \Rightarrow \alpha(x = 0) = \frac{dy}{dx}(x = 0) = 0 \Rightarrow \frac{\alpha}{\sqrt{1+\alpha^2}} = \frac{\rho AC_L}{2m} x \Rightarrow \frac{\left(dy/dx\right)^2}{1+\left(dy/dx\right)^2} = \left(\frac{\rho AC_L}{2m}\right)^2 x^2 \]

\[ \Rightarrow \left(1 - \left(\frac{\rho AC_L}{2m}\right)^2 x^2\right) \left(dy/dx\right)^2 = \left(\frac{\rho AC_L}{2m}\right)^2 x^2 \Rightarrow dy = \frac{\left(\frac{\rho AC_L}{2m}\right)^2 x}{\sqrt{1 - \left(\frac{\rho AC_L}{2m}\right)^2 x^2}} dx \]

\[ \Rightarrow y = -\left(\frac{2m}{\rho AC_L}\right) \sqrt{1 - \left(\frac{\rho AC_L}{2m}\right)^2 x^2} + \text{Const} \]

\[ y(x = 0) = 0 \Rightarrow y = -\left(\frac{2m}{\rho AC_L}\right) \sqrt{1 - \left(\frac{\rho AC_L}{2m}\right)^2 x^2} + \left(\frac{2m}{\rho AC_L}\right) \]

Finally at \( x=60.5 \): \( y=1.722 \text{ ft} \)

Comparing the exact solution with the one from simple approach shows that \( y \) is over-predicted by 18% using simple approach.