Problem 1:

Consider the flow of air into the floor attachment nozzle of a typical household vacuum cleaner. The width of the nozzle inlet slot is \( w = 2.0 \) mm, and its length is \( L = 35.0 \) cm. The slot is held a distance \( b = 2.0 \) cm above the floor, as shown in Fig a. The total volume flow rate through the vacuum hose is \( Q = 0.110 \) m\(^3\)/s. Consider the flow field in the center plane of the attachment (the xy-plane in Fig a) as a 2D sink line flow near a wall, as shown in Fig b. The strength of the sink \( (m) \) can be estimated by \( m = (Q/L)/(2\pi) \). Calculate (a) the velocity and (b) gage pressure distribution along the \( x \) axis. (c) What is the maximum magnitude of speed along the floor, and where does it occur? (The vacuum cleaner should be most effective where speed is maximum or pressure is minimum)
Solution:

We can consider sinks as sources with negative source strength. Then:

\[ V_x = 2V_{source} \cos \theta = 2 \frac{-m}{r_1} \frac{x}{r_1^2} = -2m \frac{x}{x^2 + b^2} = -2 \left( \frac{Q}{2\pi L} \right) \frac{x}{x^2 + b^2} = - \frac{Q}{\pi L} \left( \frac{x}{x^2 + b^2} \right) \]

\[ P_{\infty} + \rho \frac{V_x^2}{2} = P + \rho \frac{V_x^2}{2} \Rightarrow P_{\infty} = P - \rho \frac{V_x^2}{2} = \rho \left( \frac{V_x^2}{2} - V_x^2 \right) = \rho \left( V_x^2 - \frac{Q^2}{\pi^2 L^2} \left( \frac{x}{x^2 + b^2} \right)^2 \right) \]

Max \( V_x \):

\[ \frac{dV_x}{dx} = 0 \Rightarrow \frac{d}{dx} \left( \frac{x}{x^2 + b^2} \right) = 0 \Rightarrow (1)(x^2 + b^2) - (x)(2x) = 0 \Rightarrow x^2 + b^2 - 2x^2 = 0 \Rightarrow x = \pm b = \pm 0.02m \]

\[ V_x(x = \pm b) = -\frac{Q}{\pi L} \left( \frac{\pm b}{b^2 + b^2} \right) = \mp \frac{Q}{2\pi L b} = \mp \frac{0.110}{2\pi (0.35)(0.02)} = \mp 2.5 \text{ m/s} \]
Problem 2:

8.31 A Rankine half-body is formed as shown in Fig. P8.31. For the conditions shown, compute (a) the source strength $m$ in m$^2$/s; (b) the distance $a$; (c) the distance $h$; and (d) the total velocity at point A.

**Solution:** The vertical distance above the origin is a known multiple of $m$ and $a$:

$$y_{x=0} = 3\ m = \frac{\pi m}{2U} = \frac{\pi m}{2(7)} = \frac{\pi a}{2},$$

or $m \approx 13.4\ \text{m}^2/\text{s}$ and $a \approx 1.91\ \text{m}$  \textit{Ans.} (a, b)

The distance $h$ is found from the equation for the body streamline:

At $x = 4\ m$, $r_{\text{body}} = \frac{m(\pi - \theta)}{U}\sin\theta = \frac{13.4(\pi - \theta)}{7}\sin\theta = \frac{4.0}{\cos\theta}$, solve for $\theta \approx 47.8^\circ$

Then $r_A = \frac{4.0}{\cos(47.8^\circ)} = 5.95\ \text{m}$ and $h - r\sin\theta \approx 4.41\ \text{m}$ \textit{Ans.} (c)

The resultant velocity at point A is then computed from Eq. (8.18):

$$V_A = U\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta_A\right)^{1/2} = 7\left[1 + \left(\frac{1.91}{5.95}\right)^2 + 2\left(\frac{1.91}{5.95}\right)\cos47.8^\circ\right]^{1/2} \approx 8.7\ \text{m/s} \ \textit{Ans.} (d)$$