For the piping system of Fig. P6.118, all pipes are concrete with a roughness of 0.04 inch. Neglecting minor losses, compute the overall pressure drop \( p_1 - p_2 \) in lbf/in\(^2\). The flow rate is 20 ft\(^3\)/s of water at 20°C.

**Solution:** For water at 20°C, take \( \rho = 1.94 \) slug/ft\(^3\) and \( \mu = 2.09\times10^{-5} \) slug/ft-s. Since the pipes are all different make a little table of their respective \( L/d \) and \( \varepsilon/d \):

<table>
<thead>
<tr>
<th>(a)</th>
<th>( L = 1000 \text{ ft} )</th>
<th>( d = 12 \text{ in} )</th>
<th>( L/d = 1000 )</th>
<th>( \varepsilon/d = 0.00333 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>1500 ft</td>
<td>8 in</td>
<td>2250</td>
<td>0.00500</td>
</tr>
<tr>
<td>(c)</td>
<td>800 ft</td>
<td>12 in</td>
<td>800</td>
<td>0.00333</td>
</tr>
<tr>
<td>(d)</td>
<td>1200 ft</td>
<td>15 in</td>
<td>960</td>
<td>0.00267</td>
</tr>
</tbody>
</table>

With the flow rate known, we can find everything in pipe (a):

\[
V_a = \frac{Q_a}{A_a} = \frac{20}{(\pi/4)(1 \text{ ft})^2} = 25.5 \text{ ft/s, \quad Re}_a = \frac{1.94(25.5)(1)}{2.09\times10^{-5}} = 2.36\times10^6, \quad f_a \approx 0.0270
\]

Then pipes (b,c,d) are in parallel, each having the same head loss and with flow rates which must add up to the total of 20 ft\(^3\)/s:

\[
h_{fb} = \frac{8f_bL_bQ_b^2}{\pi^2g d_b^5} = h_{fc} = \frac{8f_cL_cQ_c^2}{\pi^2g d_c^5} = h_{fd} = \frac{8f_dL_dQ_d^2}{\pi^2g d_d^5}, \quad \text{and} \quad Q_b + Q_c + Q_d = 20 \text{ ft}^3/\text{s}
\]

Introduce \( L_b, d_b, \) etc. to find that \( Q_c = 3.77Q_b(f_b/f_c)^{1/2} \) and \( Q_d = 5.38Q_b(f_b/f_d)^{1/2} \)

Then the flow rates are iterated from the relation

\[
\sum Q = 20 \text{ ft}^3/\text{s} = Q_b[1 + 3.77(f_b/f_c)^{1/2} + 5.38(f_b/f_d)^{1/2}]
\]

First guess: \( f_b = f_c = f_d; \quad Q_b \approx 1.97 \text{ ft}^3/\text{s}; \quad Q_c \approx 7.43 \text{ ft}^3/\text{s}; \quad Q_d \approx 10.6 \text{ ft}^3/\text{s} \)

Improve by computing \( Re_b \approx 349000, f_b \approx 0.0306, Re_c \approx 878000, f_c \approx 0.0271, Re_d \approx 1002000, f_d \approx 0.0255 \). Repeat to find \( Q_b \approx 1.835 \text{ ft}^3/\text{s}, Q_c \approx 7.351 \text{ ft}^3/\text{s}, Q_d \approx 10.814 \text{ ft}^3/\text{s} \). Repeat once more and quit: \( Q_b \approx 1.833 \text{ ft}^3/\text{s}, Q_c \approx 7.349 \text{ ft}^3/\text{s}, Q_d \approx 10.819 \text{ ft}^3/\text{s} \), from which \( V_b \approx 5.25 \text{ ft/s}, V_c \approx 9.36 \text{ ft/s}, V_d \approx 8.82 \text{ ft/s} \). The pressure drop is

\[
p_1 - p_2 = \Delta p_a + \Delta p_b = f_a \frac{L_a \rho V_a^2}{d_a^2} + f_b \frac{L_b \rho V_b^2}{d_b^2} = 17000 + 1800 \approx 18800 \text{ psf} \approx 131 \frac{\text{lbf}}{\text{in}^2} \quad \text{Ans.}
\]
6.130 In Fig. P6.130 lengths $AB$ and $BD$ are 2000 and 1500 ft, respectively. The friction factor is 0.022 everywhere, and $p_A = 90$ lb/ft$^2$ gage. All pipes have a diameter of 6 in. For water at $20^\circ C$, determine the flow rate in all pipes and the pressures at points $B$, $C$, and $D$.

**Solution:** For water at $20^\circ C$, take $\rho = 1.94$ slug/ft$^3$ and $\mu = 2.09E-5$ slug/ft·s. Each pipe has a head loss which is known except for the square of the flow rate:

Pipe AC: $h_f = \frac{8fLQ_A^2}{\pi^2gd^5} = \frac{8(0.022)(1500)Q_A^2}{\pi^2(32.2)(6/12)^5} = K_{AC}Q_A^2$, where $K_{AC} \approx 26.58$

Similarly, $K_{AB} = K_{CD} = 35.44$, $K_{BD} = 26.58$, and $K_{BC} = 44.30$.

Loop ABC: $35.44Q_{AB}^2 + 44.3Q_{BC}^2 - 26.58Q_{AC}^2 = 0$
Loop BCD: $44.3Q_{BC}^2 + 35.44Q_{CD}^2 - 26.58Q_{BD}^2 = 0$
Junctions A,B,C: $Q_{AB} + Q_{AC} = 2.0$;
$Q_{AB} = Q_{BC} + Q_{BD} + 1.0$; $Q_{AC} + Q_{BC} = Q_{CD} + 0.5$

After solving the above equations:

$Q_{AB} = 0.949$ ft$^3$/s (toward B); $Q_{AC} = 1.051$ ft$^3$/s (toward C)
$Q_{BC} = 0.239$ (toward B); $Q_{CD} = 0.312$ (toward D); $Q_{BD} = 0.188$ (to D)  \textit{Ans. (a)}

The pressures start at A, from which we subtract the friction losses in each pipe:

$p_B = p_A - \rho g K_{AB} Q_{AB}^2 = 90 \times 144 - 62.4(35.44)(0.949)^2 = 10969$ psf $\div 144 = 76$ psi

Similarly, we obtain $p_C = 11127$ psf $= 77$ psi; $p_D = 10911$ psf $\approx 76$ psi  \textit{Ans. (b)}