4.37 A viscous liquid of constant density and viscosity falls due to gravity between two parallel plates a distance 2h apart, as in the figure. The flow is fully developed, that is, \( w = w(x) \) only. There are no pressure gradients, only gravity. Set up and solve the Navier-Stokes equation for the velocity profile \( w(x) \).

**Solution:** Only the z-component of Navier-Stokes is relevant:

\[
\rho \frac{dw}{dt} = 0 = \rho g + \mu \frac{d^2w}{dx^2}, \quad \text{or} \quad w'' = -\frac{\rho g}{\mu}, \quad w(-h) = w(+h) = 0 \quad \text{(no-slip)}
\]

The solution is very similar to Eqs. (4.142) to (4.143) of the text:

\[
w = \frac{\rho g}{2\mu} (h^2 - x^2) \quad \text{Ans.}
\]