The purpose of this document is to summarize and demonstrate V&V procedures to be used in CFDLab reports.

Nomenclature:

$S_{g1}$: solution from fine grid

$S_{g2}$: solution from medium grid

$S_{g3}$: solution from coarse grid

$R_g$: grid convergence ratio

$r_g$: grid refinement ratio

$P_g$: order of accuracy for grid

$P_{gen}$: theoretical order of accuracy, 2 for 2nd order and 1 for 1st order schemes

$P$: ratio of accuracy of grid and theoretical accuracy

$\delta_{REg1}^*$: grid error from Richardson Extrapolation based on fine mesh solution

$U_g$: grid uncertainty based on FS method
Formulae:

\[ \varepsilon_{g21} = S_{g2} - S_{g1} \]

\[ \varepsilon_{g32} = S_{g3} - S_{g2} \]

\[ R_g = \frac{\varepsilon_{g21}}{\varepsilon_{g32}} \]

If monotonically converged \((0 < R_g < 1)\), then:

\[ P_g = \frac{\ln\left( \frac{\varepsilon_{g32}}{\varepsilon_{g21}} \right)}{\ln(r_g^*)} \]

\[ P = \frac{P_g}{P_{gest}} \]

\[ \delta^*_{REg1} = \frac{\varepsilon_{g21}}{(r_g^P - 1)} \]

\[ U_g = \begin{cases} 
(2.45 - 0.85P)|\delta^*_{REg1}| & \text{if } 0 < P \leq 1 \\
(16.4P - 14.8)|\delta^*_{REg1}| & \text{if } P > 1 
\end{cases} \]
Following examples demonstrate grid studies for friction factor of laminar pipe flows.

Example:

\[ r_g = \sqrt{2} \]

\[ \varepsilon_{g21} = S_{g2} - S_{g1} = -0.0027916 \]

\[ \varepsilon_{g32} = S_{g3} - S_{g2} = -0.0157815 \]

\[ \ln\left(\frac{\varepsilon_{g32}}{\varepsilon_{g21}}\right) \]

\[ P_g = \frac{\varepsilon_{g21}}{\ln(r_g)} = 2.49907 \]

\[ P = \frac{P_g}{2} = \frac{2.49907}{2} = 1.249535 \]

\[ \delta^{*}_{Reg1} = \frac{\varepsilon_{g21}}{(r_g \frac{P_g}{2} - 1)} = -0.0006 \]

\[ P > 1 \]

\[ U_g = (16.4P - 14.8) * |\delta^{*}_{Reg1}| = 0.003415424 \]