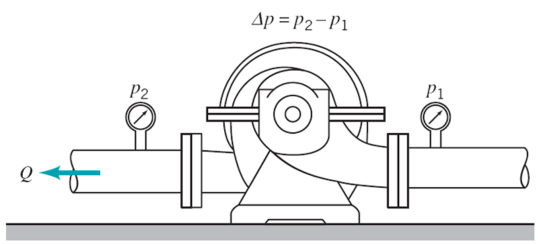
Name: ------------------ **Quiz 8** **Time: 20 minutes**

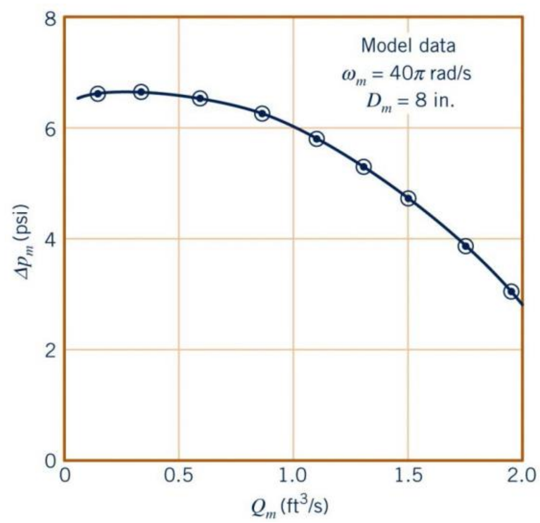
ME:5160 **Fall 2023**

The exam is closed book and closed notes.

The pressure rise, Δp, across a centrifugal pump in Figure can be expressed as $\Delta p = f(D, \omega, \rho, Q)$, where D is the impeller diameter, ω the angular velocity of the impeller (unit for ω is T^{-1}), ρ the fluid density, and Q the volume rate of flow through the pump. (a) By using dimensional analysis find the pi terms. (b) A model pump having a diameter of 8 in. is tested in a laboratory using water (ρ = 998 kg/m^3). When operated at an angular velocity of 40π rad/s the model pressure rise as a function of Q is shown in Figure. Use this curve to predict the pressure rise across a geometrically similar pump (prototype) for a prototype flowrate of 6 ft^3/s. The prototype has a diameter of 12 in. and operates at an angular velocity of 60π rad/s. The prototype fluid is also water.



Centrifugal pump



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Solution: Format (+3)

(a)

Δp	D	ω	ρ	Q
$ML^{-1}T^{-2}$	L	T^{-1}	ML^{-3}	L^3T^{-1}

$$k - r = 5 - 3 = 2 \qquad \text{(+1)}$$

Pi terms

$$\Pi_1 = D^a\omega^b\rho^c\Delta p \doteq (L)^a(T^{-1})^b(ML^{-3})^c(ML^{-1}T^{-2}) \doteq L^0T^0M^0$$

Thus, a = -2, b = -2, and c = -1.

$$\therefore \boldsymbol{\Pi_1 = \frac{\Delta p}{\rho\omega^2D^2}} \qquad \text{(+2)}$$

$$\Pi_2 = D^a\omega^b\rho^cQ \doteq (L)^a(T^{-1})^b(ML^{-3})^c(L^3T^{-1}) \doteq L^0T^0M^0$$

Thus, a = -3, b = -1, and c = 0.

$$\therefore \boldsymbol{\Pi_2 = \frac{Q}{\omega D^3}} \qquad \text{(+2)}$$

(b) Similarity

$$\frac{Q}{\omega D^3} = \frac{Q_m}{\omega_m D_m^3}$$

$$Q_m = \left(\frac{\omega_m}{\omega}\right)\left(\frac{D_m}{D}\right)^3 Q$$

$$Q_m = \left(\frac{40\pi}{60\pi}\right)\left(\frac{8}{12}\right)^3(6) = \mathbf{1.19\ ft^3/s} \qquad \text{(+1)}$$

Prediction equation

$$\frac{\Delta p}{\rho\omega^2D^2} = \frac{\Delta p_m}{\rho_m\omega_m^2D_m^2}$$

$$\Delta p = \left(\frac{\rho}{\rho_m}\right)\left(\frac{\omega}{\omega_m}\right)^2\left(\frac{D}{D_m}\right)^2\Delta p_m$$

From Fig., Δp_m = 5.5 psi at Q_m = 1.19 ft^3/s.

$$\therefore \Delta p = (1)\left(\frac{60\pi}{40\pi}\right)^2\left(\frac{12}{8}\right)^2(5.5) = \mathbf{27.8\ psi} \qquad \text{(+1)}$$