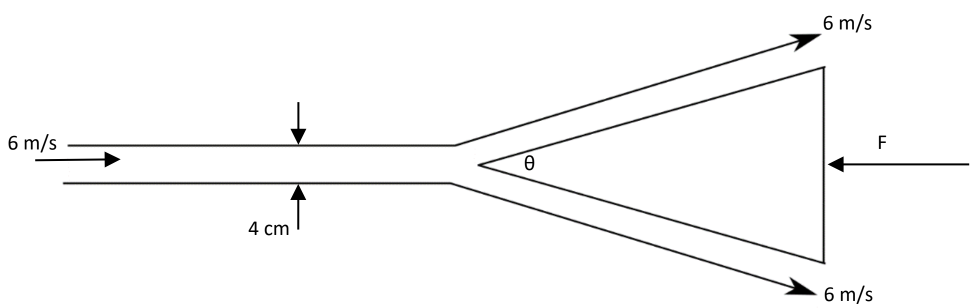
Name: ------------------ **Quiz 3** **Time: 20 minutes**

ME:5160 **Fall 2023**

The exam is closed book and closed notes.

A wedge splits a sheet of water, as shown in the figure. Both wedge and sheet are very long into the paper. The angle of the wedge θ is 48°. (a) What is the thickness of the water sheets at each side of the wedge? (b) What is the mass flow rate per unit depth? (c) What is the force F_x (per meter of depth into the paper) required to hold the wedge stationary? (d) Explain why F_y is equal to zero. (Use water density ρ = 998kg/m^3)



Continuity equation: $-\frac{d}{dt}\int_{CV}\rho d\forall = \int_{CS}\rho \underline{V_R}\cdot\underline{n}dA$

Momentum equation: $\sum F = \frac{d}{dt}\int_{CV}\rho\underline{V}d\forall + \int_{CS}\rho\underline{V}\underline{V_R}\cdot\underline{n}dA$

Hint: 1) the force required to hold the wedge should be in the x-direction.

2) $\cos(24\text{deg}) = 0.9135$

Assumption: non-deforming CV, steady uniform flow with one inlet and two outlets.

Solution:

(a) Apply continuity to find thickness of exit sheets t_{exit} (D is the depth into the paper)

$$\sum V_{in}A_{in} = \sum V_{out}A_{out}$$

$$V_{in}A_{in} = 2V_{exit}A_{exit}$$

$$V_{in} = V_{exit} = V = 6\ \mathrm{m/s}$$ **+3**

$$VDt_{in} = 2VDt_{exit}$$

$$t_{exit} = \frac{t_{in}}{2} = \frac{(4\ \mathrm{cm})}{2} = 2\ \mathrm{cm} = 0.02\ \mathrm{m}$$ **+2**

(b) The mass flow per unit depth is

$$\frac{\dot{m}}{D} = \rho V t_{in} = \left(998\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(6\frac{\mathrm{m}}{\mathrm{s}}\right)(0.04\ \mathrm{m}) = 239.5\frac{\mathrm{kg}}{\mathrm{m\ s}}$$ **+2**

(c) Apply x-momentum integral relation over a control volume surrounding the wedge:

$$\sum F_x = -F = \sum_{out}\dot{m}_j u_j - \sum_{in}\dot{m}_j u_j = 2\frac{\dot{m}}{2}V\cos\frac{\theta}{2} - \dot{m}V = \dot{m}V\left(\cos\frac{\theta}{2} - 1\right)$$ **+2**

Substitute the values:

$$F = -\left(239.5\frac{\mathrm{kg}}{\mathrm{m\ s}}\right)\left(6\frac{\mathrm{m}}{\mathrm{s}}\right)(0.9135 - 1) = 124\ \mathrm{N}$$ **+0.5**

(d) The flow is symmetric with respect to the x-axis since the wedge forms an isosceles triangle and the sheets have the same thickness t_{exit}. **+0.5**