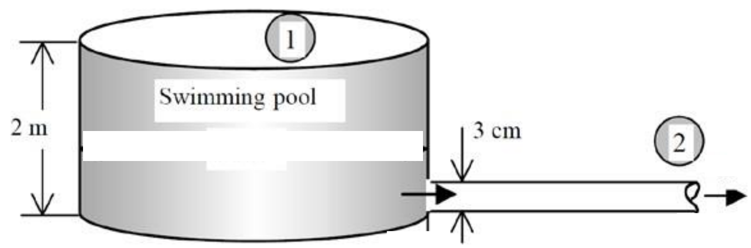
Name:----------------- **Quiz: No. 2** **Time: 15 minutes**

ME:5160, Fall 2023

The exam is closed book and closed notes.

The water in an aboveground swimming pool is to be emptied by unplugging a 3-cm-diameter horizontal pipe attached to the bottom of the pool. Assuming that point 1 at the free surface of the pool and point 2 at the exit of pipe are open to atmosphere with a vertical distance of 2m, determine the maximum flow rate of water through the pipe.

Bernoulli's equation: $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$



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Attendance: (2)

ME:5160, Fall 2023

Solution

Assumptions **1** The orifice has a smooth entrance, and all frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable).

Analysis We take point 1 at the free surface of the pool, and point 2 **(1)** at the exit of pipe. We take the reference level at the pipe exit ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \ \textbf{(3)} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g} \quad \rightarrow \quad V_2 = \sqrt{2gz_1} \ \textbf{(1)}$$

The maximum discharge rate occurs when the water height in the pool is a maximum, which is the case at the beginning and thus $z_1 = h$. Substituting, the maximum flow velocity and discharge rate become

$$V_{2,\max} = \sqrt{2gh} = \sqrt{2(9.81\text{ m/s}^2)(2\text{ m})} = 6.26\text{ m/s} \ \textbf{(1)}$$

$$Q_{max} = A_{pipe} V_{2,\max} \ \textbf{(1)} = \frac{\pi D^2}{4} V_{2,\max} \ \textbf{(0.5)} = \frac{\pi(0.03\text{ m})^2}{4}(6.26\text{ m/s}) = 0.00443\text{ m}^3/\text{s} = \mathbf{4.43\ L/s} \ \textbf{(0.5)}$$