Name: ------------------ **Quiz 11** **Time: 20 minutes**

ME:5160 **Fall 2022**

The exam is closed book and closed notes.

A thin flat plate 20 by 40 cm is immersed in a 1-m/s stream of water at 20°C (ρ = 998 kg/m3, μ =0.001kg/m.s.). Assume that the flow is laminar for Re_x < 5E5 and turbulent for larger Re_x. (a) If the stream is parallel to the short side of the plate, compute the boundary layer thickness at the end of the plate (in stream-wise direction), and the water velocity at a point 1 mm normal to the end of the plate. (b) If the stream is parallel to the long side, compute the wall shear stress at the end of the plate (in stream-wise direction), and the total friction drag considering both front and back sides of the plate.

Equations:

- The Blasius solution for laminar boundary layer flow shows that the dimensionless velocity profile u/U is a function only of the single composite dimensionless variable $\eta = y\left(\frac{U}{\upsilon x}\right)^{1/2}$. Accurate solution for the function is obtained only by numerical integration and some tabulated values of the velocity profile shape are given in the Table below:

$y[U/(\nu x)]^{1/2}$	u/U	$y[U/(\nu x)]^{1/2}$	u/U
0.0	0.0	2.8	0.81152
0.2	0.06641	3.0	0.84605
0.4	0.13277	3.2	0.87609
0.6	0.19894	3.4	0.90177
0.8	0.26471	3.6	0.92333
1.0	0.32979	3.8	0.94112
1.2	0.39378	4.0	0.95552
1.4	0.45627	4.2	0.96696
1.6	0.51676	4.4	0.97587
1.8	0.57477	4.6	0.98269
2.0	0.62977	4.8	0.98779
2.2	0.68132	5.0	0.99155
2.4	0.72899	∞	1.00000
2.6	0.77246		

And the laminar boundary layer thickness is computed as: $\frac{\delta}{x} \sim \frac{5.0}{{Re_x}^{1/2}}$ where $Re_x = \frac{Ux}{\upsilon}$

- Laminar Boundary Layer: $c_f = \frac{2\tau_w}{\rho U^2} = \frac{0.664}{Re_x^{1/2}}$; $C_D = \frac{D}{0.5\rho A U^2} = \frac{1.328}{Re_L^{1/2}}$;
- Turbulent Boundary Layer: $c_f = \frac{2\tau_w}{\rho U^2} \approx \frac{0.027}{Re_x^{1/7}}$; $C_D = \frac{D}{\frac{1}{2}\rho A U^2} = \frac{0.031}{Re_L^{1/7}}$;

  velocity profile: $\frac{u}{U} \approx \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$ where $\frac{\delta}{x} \approx \frac{0.16}{Re_x^{1/7}}$

Solution:

(a)

$$Re_{x=b} = \frac{\rho U b}{\mu} = \frac{(998)(1)(0.2)}{0.001} = 1.996E5 \quad Laminar \qquad \textbf{(3)}$$

$$\delta_{x=b} = \frac{5b}{{Re_b}^{1/2}} = \frac{(5)(0.2)}{(1.996E5)^{1/2}} = 0.00224m \qquad \textbf{(1)}$$

$$\eta = y\left(\frac{\rho U}{\mu x}\right)^{1/2} = 0.001\left(\frac{(998)(1)}{(0.001)(0.2)}\right)^{1/2} = 2.2 \qquad \textbf{(1)}$$

From the table:

$$\frac{u}{U} = 0.68132 \rightarrow u = 0.68132U = 0.68132m/s \qquad \textbf{(1)}$$

(b)

$$Re_{x=L} = \frac{\rho U L}{\mu} = \frac{(998)(1)(0.4)}{0.001} = 3.992E5 \quad Laminar \qquad \textbf{(1)}$$

$$c_{f\,x=L} = \frac{0.664}{Re_x^{1/2}} = \frac{0.664}{\sqrt{3.992E5}} = 0.00105$$

$$c_f = \frac{2\tau_w}{\rho U^2} \rightarrow \tau_w = \frac{c_f \rho U^2}{2} = \frac{(0.00105)(998)(1)^2}{2} = 0.524\ Pa \qquad \textbf{(1)}$$

$$C_D = \frac{1.328}{Re_L^{1/2}} = \frac{1.328}{\sqrt{3.992E5}} = 0.0021 \qquad \textbf{(1)}$$

$$C_D = \frac{D}{0.5\rho A U^2}$$

Find drag force considering A=bL and multiplying by 2 for both sides:

$$D = 2C_D\frac{1}{2}\rho \mathrm{bL}U^2 = (0.0021)(998)(0.2)(0.4)(1)^2 = 0.168\ N \qquad \textbf{(1)}$$