Name: ---------- **Quiz 6** **Time: 20 minutes**

ME:5160 **Fall 2022**

The exam is closed book and closed notes.

The thrust F of a propeller is generally thought to be a function of its diameter D and angular velocity Ω, the forward speed V, and the density ρ and viscosity μ of the fluid. Find suitable dimensionless groups for this problem and write the functional relationship between them.

Hint: Use ρ, V, D as repeated variables.

		Dimensions	
Quantity	**Symbol**	***MLTΘ***	***FLTΘ***
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω, Ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Thermal expansion coefficient	β	Θ^{-1}	Θ^{-1}

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Solution:

$$F = f(D, \Omega, V, \rho, \mu)$$

$$n = 6 \quad \textbf{(+1)}$$

$$F = \{MLT^{-2}\}; \quad D = \{L\}; \quad \Omega = \{T^{-1}\}$$

$$V = \{LT^{-1}\}; \quad \rho = \{ML^{-3}\}; \quad \mu = \{ML^{-1}T^{-1}\} \quad \textbf{(+1)}$$

$$j = 3 \rightarrow k = n - j = 6 - 3 = 3 \quad \textbf{(+1)}$$

The repeating variables are ρ, *V*, *D*. Using the Pi theorem, we find the three Pi groups:

$$\Pi_1 = \rho^a V^b D^c F = \{(ML^{-3})^a (LT^{-1})^b (L)^c (MLT^{-2})\} = \{M^0 L^0 T^0\} \quad \textbf{(+1)}$$

a= -1, b= -2, c= -2

$$\Pi_1 = \frac{F}{\rho V^2 D^2} \quad \textbf{(+1)}$$

$$\Pi_2 = \rho^a V^b D^c \Omega = \{(ML^{-3})^a (LT^{-1})^b (L)^c (T^{-1})\} = \{M^0 L^0 T^0\} \quad \textbf{(+1)}$$

a= -0, b= -1, c= 1

$$\Pi_2 = \frac{\Omega D}{V} \quad \textbf{(+1)}$$

$$\Pi_3 = \rho^a V^b D^c \mu = \{(ML^{-3})^a (LT^{-1})^b (L)^c (ML^{-1}T^{-1})\} = \{M^0 L^0 T^0\} \quad \textbf{(+1)}$$

a= -1, b= -1, c= 1

$$\Pi_3 = \frac{\mu}{\rho V D} \rightarrow \frac{\rho V D}{\mu} = Re \quad \textbf{(+1)}$$

Thus, the arrangement of the dimensionless variables is:

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

$$\frac{F}{\rho V^2 D^2} = f\left(\frac{\Omega D}{V}, \frac{\rho V D}{\mu}\right) \quad \textbf{(+1)}$$