

The exam is closed book and closed notes.

Water flows through a circular nozzle, exits into the air as a jet, and strikes a plate, as shown in below figure. The force required to hold the plate steady is 70 N. Assuming steady, one-dimensional flow and Turbulent pipe flow kinetic correction factor $\alpha = 1$. Consider the head loss of nozzle. (Use $K_c = 0.7$) estimate (a) the velocities at sections (1) and (2) and (b) the mercury manometer reading h .

$$[\rho_{water} = 998 \text{ kg/m}^3, \quad \rho_{Hg} = 13567.78 \text{ kg/m}^3]$$

$$\text{Continuity equation: } -\frac{d}{dt} \int_{CV} \rho dV = \int_{CS} \rho \underline{V}_R \cdot \underline{n} dA$$

$$\text{Momentum equation: } \sum F = \frac{d}{dt} \int_{CV} \rho \underline{V} dV + \int_{CS} \rho \underline{V} \underline{V}_R \cdot \underline{n} dA$$

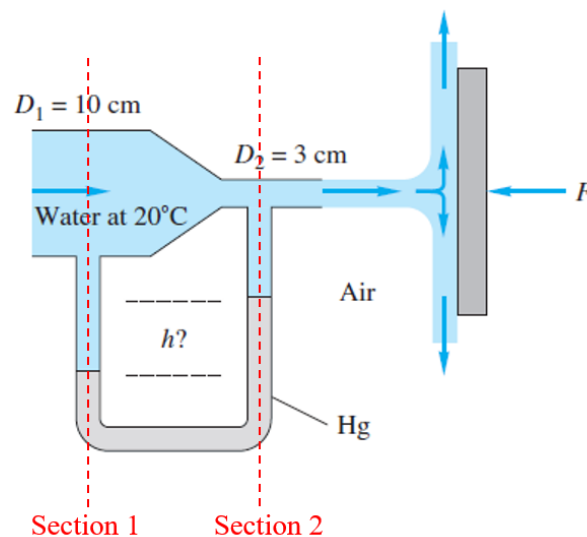
$$\left(\frac{p}{\rho g} + \frac{\alpha V^2}{2g} + z \right)_1 = \left(\frac{p}{\rho g} + \frac{\alpha V^2}{2g} + z \right)_2 + h_{loss} \quad \text{Turbulent pipe flow kinetic correction factor } \alpha = 1$$

$$\left[h_{loss} = K_c \frac{V_2^2}{2g} \right]$$

Hint: Calculate V_2 using linear momentum equation.

Calculate V_1 using continuity with V_2 .

Calculate pressure difference between section 1 and 2 considering head loss, then calculate h using both water and mercury density.



Solution:(a) Momentum equation in x direction:

$$\sum \underline{F} = \frac{d}{dt} \int_{CV} \rho \underline{V}_R dV + \int_{CS} \rho \underline{V}_R (\underline{V}_R \cdot \underline{n}) dA$$

$$\sum F_x - \dot{m}_{in} u_{in} = -\rho A_2 V_2^2 \quad (+3.5)$$

$$70N = -(998) \frac{\pi}{4} (0.03^2) (V_2^2) \quad \therefore V_2 = 9.96 \text{ m/s} \quad (+1)$$

Continuity:

$$V_1 A_1 = V_2 A_2, \quad V_1 = \frac{V_2 A_2}{A_1} = \frac{(9.96) \frac{\pi}{4} 0.03^2}{\frac{\pi}{4} 0.1^2} \quad \therefore V_1 = 0.9 \text{ m/s} \quad (+1)$$

(b) Calculate pressure difference using Bernoulli (Energy) equation

$$p_1 - p_2 = \frac{1}{2} \rho (V_2^2 - V_1^2) + \rho g h_{loss} \quad (+2.5)$$

$$p_1 - p_2 = \frac{1}{2} \rho (V_2^2 - V_1^2) + \rho g K_c \frac{V_2^2}{2g}$$

$$= \frac{1}{2} (998) (9.96^2 - 0.9^2) + (998) (9.81) \left(0.7 \frac{9.96^2}{2 \times 9.81} \right) = 83748.53 \text{ Pa} \quad (+1)$$

$$\therefore \Delta P = 83748.53 \text{ Pa}$$

$$\Delta P = \Delta(\rho) g h$$

$$h = \frac{\Delta P}{\Delta(\rho) g} = \frac{83748.53}{(13567.78 - 998) 9.81} \approx 0.679 \text{ m} \quad (+1)$$