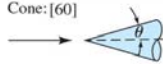
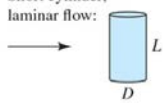
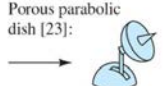
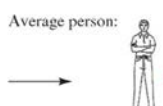


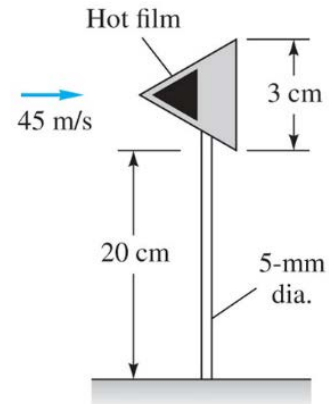
The exam is closed book and closed notes.

A hot-film probe is mounted on a cone-and-rod system in an airstream ( $\rho = 1.347 \text{ kg/m}^3$ ,  $\mu = 1.78\text{E-}5 \text{ kg/m.s.}$ ) of 45 m/s, as in the Figure below. Estimate the maximum cone vertex angle allowable if the flow-induced bending moment at the root of the rod is not to exceed 30 N.cm.

**Equations:**  $C_D = \frac{F_D}{\frac{1}{2}\rho AV^2}$

**Table:** Drag of three-dimensional bodies

Body	$C_D$ based on frontal area									
Cone: [60]		$\theta$ :	10°	20°	30°	40°	60°	75°	90°	
		$C_D$ :	0.30	0.40	0.55	0.65	0.80	1.05	1.15	
Short cylinder, laminar flow:		$L/D$ :	1	2	3	5	10	20	40	$\infty$
		$C_D$ :	0.64	0.68	0.72	0.74	0.82	0.91	0.98	1.20
Porous parabolic dish [23]:		Porosity:	0	0.1	0.2	0.3	0.4	0.5		
		$C_D$ (←):	1.42	1.33	1.20	1.05	0.95	0.82		
		$C_D$ (→):	0.95	0.92	0.90	0.86	0.83	0.80		
Average person:			$C_D A = 9 \text{ ft}^2$		$C_D A = 1.2 \text{ ft}^2$					



**Solution:**

KNOWN: wind velocity, body shapes and sizes, maximum bending moment

(1) FIND:  $\theta$

ASSUMPTIONS: smooth surfaces such that the values in the Table are valid

ANALYSIS:

For the rod:

$$Re_{D,rod} = \frac{\rho V D_{rod}}{\mu} = \frac{(1.347)(45)(0.005)}{(1.78E-5)} = 1.7E4 < 5E5 \rightarrow \text{Laminar} \quad (2.5)$$

From the Table:

$$\frac{L_{rod}}{D_{rod}} = \frac{(200)}{(5)} = 40 \rightarrow C_{D,rod} = 0.98 \quad (0.5)$$

$$F_{rod} = C_{D,rod} \left( \frac{1}{2} \rho A_{rod} V^2 \right)$$

$$A_{rod} = L_{rod} D_{rod} \quad (0.5)$$

$$F_{rod} = (0.98)(0.5)(1.347)(0.2)(0.005)(45)^2 = 1.34 \text{ N} \quad (0.5)$$

$$M_{rod} = F_{rod} \times r_{rod} \quad (0.5)$$

$$r_{rod} = 10 \text{ cm} \quad (0.5)$$

$$M_{rod} = (1.34)(10) = 13.4 \text{ N.cm}$$

For the cone:

$$F_{cone} = C_{D,cone} \left( \frac{1}{2} \rho A_{cone} V^2 \right)$$

$$A_{cone} = \frac{\pi}{4} D_{cone}^2 \quad (0.5)$$

$$M_{cone} = F_{cone} \times r_{cone}$$

$$r_{cone} = 20 + \frac{3}{2} = 21.5 \text{ cm} \quad (0.5)$$

$$M_{cone} = C_{D,cone} (0.5) (1.347) \left( \frac{\pi}{4} \right) (0.03)^2 (45)^2 (21.5) = (20.73) C_{D,cone} \quad (0.5)$$

$$M_{total} = M_{rod} + M_{cone} = 13.4 + 20.73 C_{D,cone} = 30 \text{ N.cm} \quad (0.5)$$

$$C_{D,cone} = \frac{(30) - (13.4)}{(20.73)} = 0.8 \quad (0.5)$$

(0.5)

From the Table:

$$\theta = 60^\circ \quad (0.5)$$