
The exam is closed book and closed notes.

The centerboard on a sailboat is 3 ft long parallel to the flow and protrudes 7 ft down below the hull into seawater at 20°C. (a) Using flat-plate theory for a smooth surface, estimate its drag (considering both faces of the centerboard) if the boat moves at 10 knots (16.9 ft/s). (b) Compute the boundary layer thickness at the end of the centerboard and (c) the water velocity at a point 0.01 ft normal to the end of the centerboard. Assume $Re_{x,tr}=5E5$.

Water density at 20°C: $\rho = 1.99 \text{ slug/ft}^3$

Water viscosity at 20°C: $\mu = 2.34E-5 \text{ slug/(ft s)}$

Equations:

- Turbulent Boundary Layer: $c_f = \frac{2\tau_w}{\rho U^2} \approx \frac{0.027}{Re_x^{1/7}}$; $C_D = \frac{D}{\frac{1}{2}\rho AU^2} = \frac{0.031}{Re_L^{1/7}} - \frac{1440}{Re_L}$;
velocity profile: $\frac{u}{U} \approx \left(\frac{y}{\delta}\right)^{1/7}$ where $\frac{\delta}{x} \approx \frac{0.16}{Re_x^{1/7}}$

Solution:

(a) Evaluate Re_L and the drag. Convert 10 knots to 16.9 ft/s.

$$Re_L = \frac{\rho UL}{\mu} = \frac{(1.99)(16.9)(3)}{(2.34E-5)} = 4.31E6 \quad +3$$

$$C_D = \frac{0.031}{Re_L^{1/7}} - \frac{1440}{Re_L} = \frac{0.031}{(4.31E6)^{1/7}} - \frac{1440}{4.31E6} = 0.003162 \quad +2$$

$$F_{drag} = \frac{C_D \rho}{2} V^2 bL(2) = 0.003162 \left(\frac{1.99}{2} \right) (16.9)^2 (3)(7)(2) = 37.74 lbf \quad +2$$

(b)

$$\delta_x = \frac{0.16(3ft)}{Re_x^{1/7}} = \frac{(0.16)(3)}{(4.31E6)^{1/7}} = 0.054132ft \quad +2$$

(c)

$$\frac{u}{U} \approx \left(\frac{y}{\delta} \right)^{1/7} = \left(\frac{0.01}{0.054132} \right)^{1/7} = 0.7856 \rightarrow u = 0.7856 \left(16.9 \frac{ft}{s} \right) = 13.28 \frac{ft}{s} \quad +1$$