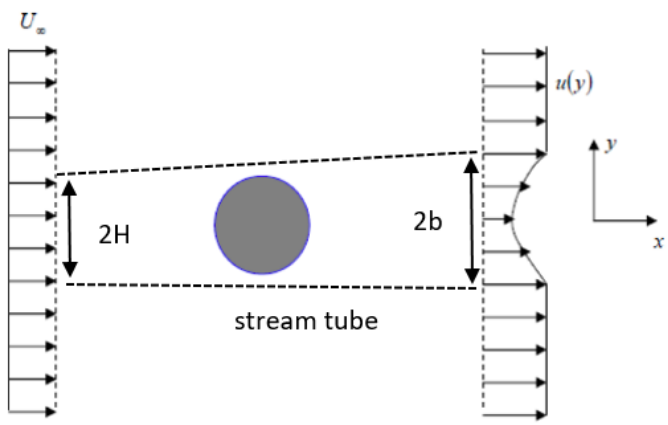
Name: ------------------ **Final Exam** **Time: 120min**

ME:5160 **Fall 2022**

The exam is closed book and closed notes.

1. Consider an experiment in which the drag on a two-dimensional body immersed in a steady incompressible flow can be determined from measurement of velocity distribution far upstream and downstream of the body as shown in Figure below. Velocity far upstream is the uniform flow U_∞, and that in the wake of the body is measured to be $u(y) = \frac{U_\infty}{2}\left(\frac{y^2}{b^2} + 1\right)$, which is less than U_∞ due to the drag of the body. Assume that there is a stream tube with inlet height of 2H and outlet height of 2b as shown in Figure below. (a) Determine the relationship between H and b using the continuity equation. (b) Find the drag per unit length of the body as a function of U_∞, b and ρ.

(Hint: Momentum Equation $\sum F_x = \int u\rho(\underline{V} \cdot \underline{n})dA$)



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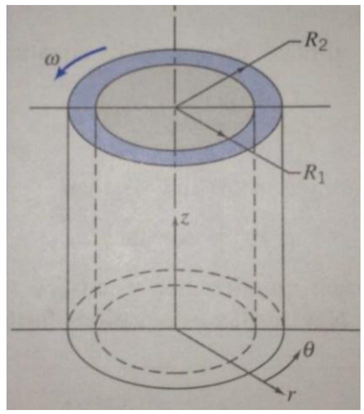
2. A Newtonian fluid of density ρ and viscosity μ fills the annular gap between concentric cylinders as shown blow. The inner cylinder of radius R_1 is stationary, and the outer cylinder of radius R_2 rotates steadily at a constant angular speed ω. The flow between the cylinders is laminar and incompressible. Assume steady, purely circulating motion, 2D flow, circumferentially symmetric pressure, and neglect gravity. Simplify the continuity and momentum equations and use appropriate boundary conditions to determine the velocity distribution in the gap.



The equations of motion of an incompressible Newtonian fluid with constant density and viscosity in cylindrical coordinates (r, θ, z) with velocity components (v_r, v_θ, v_z):

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

r-momentum:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = \rho g_r - \frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

θ-momentum:

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = \rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]$$

z-momentum:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

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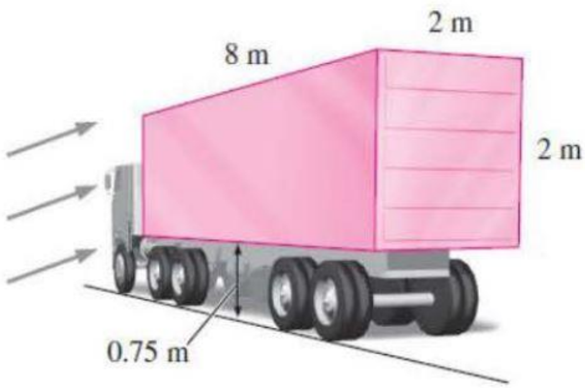
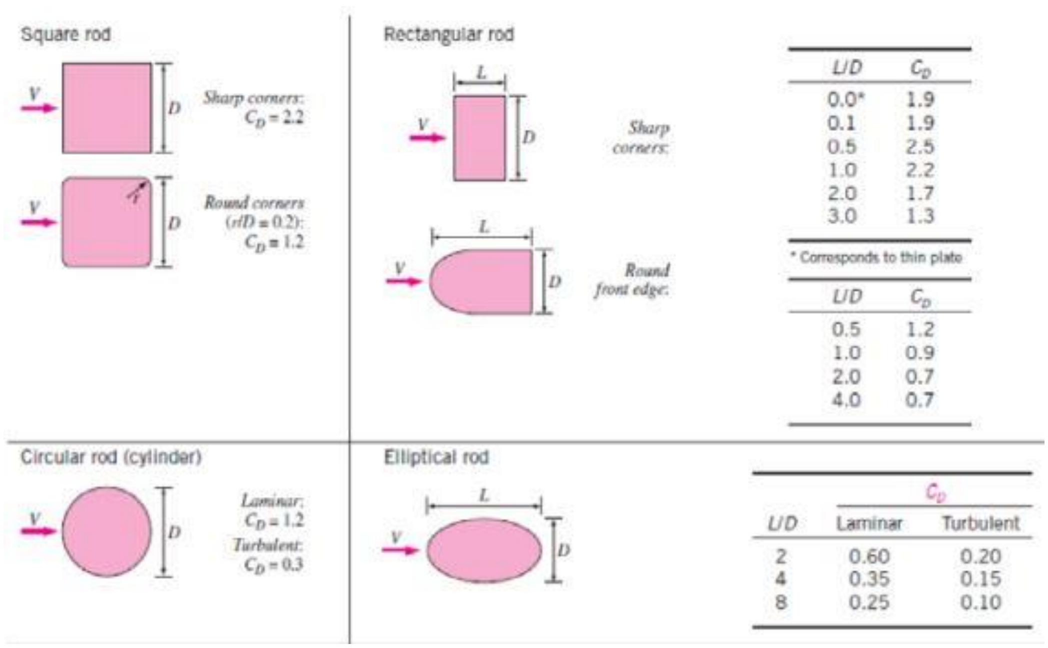
3. During major windstorms, high vehicles such as RVs and semis may be thrown off the road, especially when they are empty and in open areas. Consider a 5000-kg semi that is 8m long, 2m high, and 2m wide. The distance between the bottom of the truck and the road is 0.75m. The truck is exposed to winds from its side surface. (a) Determine the wind velocity that will tip the truck over to its side. Take the air density to be 1.1 kg/m^3 and assume the weight to be uniformly distributed. (b) If the wind arrives with a 45° angle on the side of the truck, does the tip velocity increase or decrease (suppose same C_D)? Why? (c) If you are to repeat the experiment for a 2 times smaller model using Reynolds similarity and the new semi weights 2000-kg, determine the new tipping velocity (consider that the distance between the bottom of the truck and the road is 0.5m).

Hint: $C_D = \frac{D}{0.5\rho A V^2}$ (use square rod) $\quad Re_{full\ scale} = \frac{\rho U(2m)}{\mu}$





L/D	C_D
0.0*	1.9
0.1	1.9
0.5	2.5
1.0	2.2
2.0	1.7
3.0	1.3

* Corresponds to thin plate

L/D	C_D
0.5	1.2
1.0	0.9
2.0	0.7
4.0	0.7

	C_D	
L/D	Laminar	Turbulent
2	0.60	0.20
4	0.35	0.15
8	0.25	0.10

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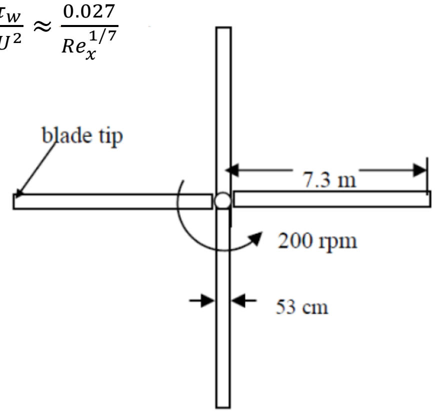
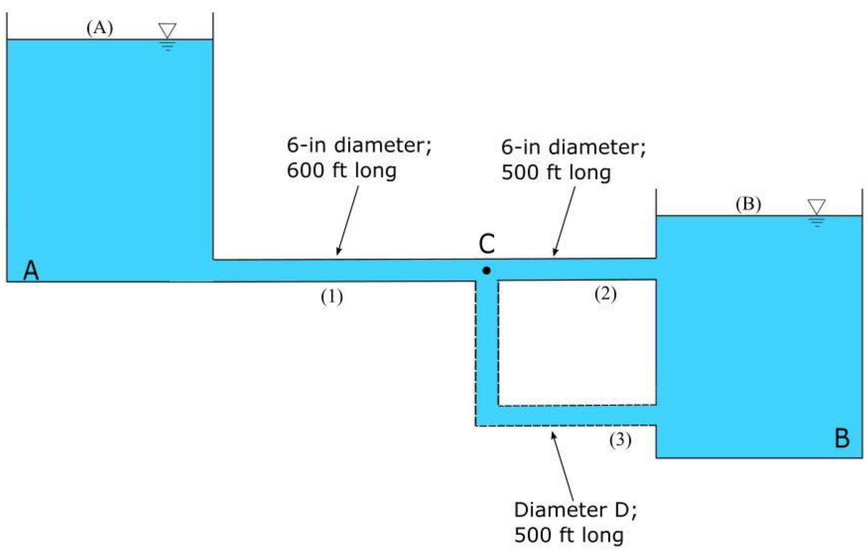
4. A helicopter rotor rotates at ω =20.94 rad/s in air (ρ=1.2 kg/m^3 and μ=1.8E-5 kg/m-s). Each blade has a chord length of 53 cm and extends a distance of 7.3 m from the center of the rotor hub. Assume that the blades can be modeled as very thin flat plates at a zero angle of attack. (a) At what radial distance from the hub center is the flow turbulent (Re_{crit} = 5E5). (b) Find the boundary layer thickness at the blade tip trailing edge (c) At what rotor angular velocity does the wall shear stress at the blade tip trailing edge become 80 N/m^2?

Hint: $Re = \frac{\rho U C}{\mu}$ where $U = r\omega$, C: *Chord Length*

Turbulent BL: $\frac{\delta}{x} \approx \frac{0.16}{Re^{1/7}}$, $c_f = \frac{2\tau_w}{\rho U^2} \approx \frac{0.027}{Re_x^{1/7}}$



5. The flowrate between tank A and tank B shown in the figure is to be increased by 30% (i.e. from Q to 1.30Q) by the addition of a second pipe (indicated by the dotted lines) running from node C to tank B. If the elevation of the free surface in tank A is 25ft above that in tank B, determine the diameter, D, of this new pipe. Neglect minor losses and assume that the friction factor for each pipe is 0.02.



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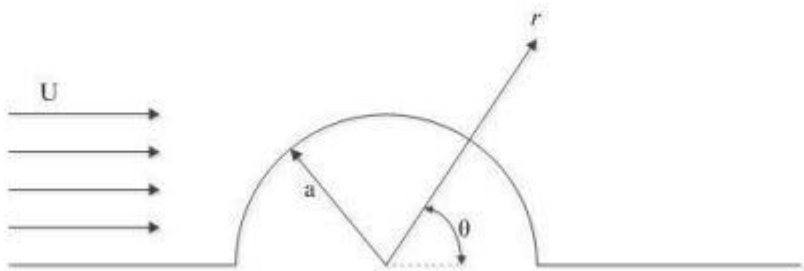
6. A practice facility for the Cal football team is to be covered by a semi-circular cylindrical dome, as shown in the sketch. The dome with radius a is placed normal to the cross flow of velocity U. Assume that the dome is very long (into the page) so you can ignore end effects.



a. The stream function and velocity potential for this flow $0 \le \theta \le \pi$ are given by

$$\Psi = Ur\left(1 - \frac{a^2}{r^2}\right)\sin\theta$$

$$\Phi = Ur\left(1 + \frac{a^2}{r^2}\right)\cos\theta$$

Use potential flow theory to find the tangential velocity along the surface of the dome $(r = a)$ as a function of the angle θ.

Hint: $u_r = \frac{\partial \Phi}{\partial r}, \quad u_\theta = \frac{1}{r}\frac{\partial \Phi}{\partial \theta}$

b. Choose a reference condition to be $p = p_0$ at z=0 (on the ground) far from the cylindrical dome. Use the Bernoulli equation to find the pressure field on the surface of the dome $(r = a)$ as a function of θ. Do NOT neglect hydrostatic variations.

c. If the vertical force on the dome per unit length (upwards) is given by $F_{lift} = \frac{5}{3}\rho U^2 a$, find the mass per unit length (m_c) required to keep the cylindrical dome on the ground when the wind blows at 30m/s, with air density ρ=1.2kg/m^3, and a=50m.

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1. Solution

a) Continuity:

$$2\rho H U_\infty = \rho \int_{-b}^{b} u(y)dy = \rho \int_{-b}^{b} \frac{U_\infty}{2}\left(\frac{y^2}{b^2}+1\right)dy \qquad \textbf{(2)}$$

$$2\rho H U_\infty = \rho \frac{U_\infty}{2}\int_{-b}^{b}\left(\frac{y^2}{b^2}+1\right)dy = \rho\frac{U_\infty}{2}\left(\frac{y^3}{3b^2}+y\right)\Big]_{-b}^{b} \qquad \textbf{(1)}$$

$$2H = \frac{1}{2}\left(\frac{b^3}{3b^2}+b+\frac{b^3}{3b^2}+b\right) = \frac{1}{2}\left(\frac{8}{3}b\right) = \frac{4}{3}b \qquad \textbf{(1)}$$

$$H = \frac{2b}{3} \qquad \textbf{(1)}$$

b) x-momentum:

$$\sum F_x = \int u\rho(\underline{V}\cdot\underline{n})dA \qquad \textbf{(2)}$$

Drag per unit length:

$$-F_D = -\rho U_\infty^2(2H) + \rho\int_{-b}^{b} u^2(y)dy \qquad \textbf{(1)}$$

$$F_D = \rho U_\infty^2(2H) - \rho\int_{-b}^{b}\left[\frac{U_\infty}{2}\left(\frac{y^2}{b^2}+1\right)\right]^2 dy = \rho U_\infty^2(2H) - \rho\frac{U_\infty^2}{4}\int_{-b}^{b}\left(\frac{y^2}{b^2}+1\right)^2 dy$$

Calculating integral:

$$\int_{-b}^{b}\left(\frac{y^2}{b^2}+1\right)^2 dy = \int_{-b}^{b}\left(\frac{y^4}{b^4}+\frac{2y^2}{b^2}+1\right)dy = \frac{y^5}{5b^4}+\frac{2y^3}{3b^2}+y\Big]_{-b}^{b} = 2\left(\frac{b}{5}+\frac{2b}{3}+b\right) = \frac{56}{15}b \qquad \textbf{(1)}$$

Entering into the momentum equation:

$$F_D = 2\rho H U_\infty^2 - \frac{1}{4}\rho U_\infty^2\left(\frac{56}{15}b\right) = \rho U_\infty^2\left(\frac{4}{3}b - \frac{14}{15}b\right) = \rho U_\infty^2\frac{2b}{5} \qquad \textbf{(1)}$$

2. Solution

ASSUMPTIONS: **(1)**

1. Steady flow ($\frac{\partial}{\partial t}=0$)
2. Purely circumferential flow (v_r=v_z=0)
3. 2D flow ($\frac{\partial}{\partial z}=0$)
4. circumferentially symmetric pressure ($\frac{\partial p}{\partial \theta}=0$)
5. Negligible gravity

ANALYSIS:

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

$$0(2) + \frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta) + 0(2) = 0$$

$$\frac{\partial v_\theta}{\partial \theta} = 0 \quad (6)$$ **(2)**

θ-momentum:

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = \rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]$$

$$\rho(0(1) + 0(2) + 0(6) + 0(2{,}3) + 0(2)) = 0(5) - 0(4) + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + 0(6) + 0(3) + 0(2)\right]$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) = 0$$ **(2)**

Integrate:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta) = C_1$$

$$\frac{\partial}{\partial r}(r v_\theta) = C_1 r$$ **(1)**

Integrate again:

$$r v_\theta = C_1\frac{r^2}{2} + C_2$$

$$v_\theta = C_1\frac{r}{2} + \frac{C_2}{r}$$ **(1)**

B.C.:

1) At $r = R_1$, $v_\theta = 0$ **(0.75)**
2) At $r = R_2$, $v_\theta = R_2\omega$ **(0.75)**

Replace:

$$0 = C_1\frac{R_1}{2} + \frac{C_2}{R_1}$$

$$R_2\omega = C_1\frac{R_2}{2} + \frac{C_2}{R_2}$$

And find:

$$C_1 = \frac{2\omega}{1 - \left(\frac{R_1}{R_2}\right)^2}$$ **(0.25)**

$$C_2 = \frac{-\omega {R_1}^2}{1 - \left(\frac{R_1}{R_2}\right)^2}$$ **(0.25)**

Substitute into the expression for v_θ:

$$v_\theta = \frac{\omega R_1}{1-\left(\frac{R_1}{R_2}\right)^2}\left(\frac{r}{R_1}-\frac{R_1}{r}\right) \quad \textbf{(1)}$$

3. Solution

(a)
From table: L/D=2/2 → $C_D = 2.2$ **(2)**
When the truck is first tipped, the wheels on the wind-loaded side of the truck will be off the ground, and thus all reaction forces from the ground will act on wheels on the other side. Taking the moment about an axis passing through these wheels and setting it equal to zero gives the required frag force and consequently the wind speed:

$$\sum M = 0 \rightarrow M_{drag} - M_{weight} = 0 \quad \textbf{(1)}$$

$$M_{drag} = F_D \times arm_{drag} \quad \textbf{(1)}$$

$$F_D = 0.5\rho A V^2 C_D = 0.5(1.1)(8)(2)V^2(2.2) = 19.36V^2 \quad \textbf{(1)}$$

$$arm_{drag} = \frac{2}{2} + 0.75 = 1.75m$$

$$M_{weight} = mg \times arm_{weight} \quad \textbf{(1)}$$

$$arm_{weight} = \frac{2}{2} = 1$$

$$\rightarrow 19.36V^2(1.75) - 49050(1) = 0$$

$$\rightarrow V = 38.05m/s \quad \textbf{(1)}$$

(b)

It the wind arrives with a 45° angle, the normal area decreases, reducing F_D and increasing the tipping velocity. **(1)**

(c)

Reynolds similarity:

$$\frac{\rho V L}{\mu} = \frac{\rho V_m L_m}{\mu} \quad \textbf{(1)}$$
$$V_m = \frac{VL}{L_m} = \frac{2V}{1} = 2V$$
$$M_{drag} = F_D \times arm_{drag}$$

$$F_D = 0.5\rho A V^2 C_D = 0.5(1.1)(4)(1)(2V)^2(2.2) = 19.36V^2$$

$$arm_{drag} = \frac{1}{2} + 0.5 = 1m$$

$$M_{weight} = mg \times arm_{weight}$$

$$arm_{weight} = \frac{1}{2} = 0.5$$

$$\rightarrow 19.36V^2(1) - 19620(0.5) = 0$$

$$\rightarrow V = 22.51m/s$$ **(1)**

4. Solution

a)

$$U = r\omega$$

$$Re = \frac{\rho U c}{\mu} = \frac{\rho r \omega c}{\mu}$$ **(1)**

$$Re_{crit} = \frac{\rho r_{crit} \omega c}{\mu}$$

$$r_{crit} = \frac{Re_{crit}\mu}{\rho\omega c} = \frac{(5E5)(1.8E-5)}{(1.2)(20.94)(0.53)} = 0.68\ m$$ **(1)**

b) At the tip trailing edge:

$$Re = \frac{\rho r_{tip} \omega c}{\mu} = \frac{(1.2)(7.3)(20.94)(0.53)}{(1.8E-5)} = 5{,}401{,}124$$ **(1)**

$$\frac{\delta}{x} \approx \frac{0.16}{Re^{1/7}}$$ **(1)**

$$\delta \approx \frac{0.16(0.53)}{(5401124)^{\frac{1}{7}}} = 0.00926\ m = 9.26\ mm$$ **(2)**

c)

$$\tau_w = 0.0135 \frac{\rho U^2}{Re_x^{1/7}} = \frac{0.0135\mu^{1/7}\rho^{6/7}U^{13/7}}{x^{1/7}}$$ **(2)**

Re-arrange to find *U*:

$$U = r_{tip}\omega = \left(\frac{\tau_w\, x^{1/7}}{0.0135\mu^{1/7}\rho^{6/7}}\right)^{7/13} \quad \textbf{(1)}$$

$$\omega = \frac{1}{(7.3)}\left(\frac{(80.0)(0.53)^{1/7}}{0.0135\ (1.8E-5)^{1/7}(1.2)^{6/7}}\right)^{7/13} = 29.88\ rad/s \quad \textbf{(1)}$$

5. Solution

With the single pipe:

$$\left(\frac{\mathrm{p}}{\rho \mathrm{g}} + \frac{\alpha \mathrm{V}^2}{2\mathrm{g}} + \mathrm{z}\right)_{\mathrm{A}} = \left(\frac{\mathrm{p}}{\rho \mathrm{g}} + \frac{\alpha \mathrm{V}^2}{2\mathrm{g}} + \mathrm{z}\right)_{\mathrm{B}} + h_f \quad \textbf{(2)}$$

Where $p_a = p_b = 0$, $V_a = V_b = 0$, $z_a = 25ft$, $z_b = 0$.

Thus,

$$z_a = f\frac{L_1 + L_2}{D_1}\frac{\mathrm{V_1}^2}{2\mathrm{g}} \quad \textbf{(1)}$$

$$25ft = 0.02\frac{(600+500)}{6/12}\frac{\mathrm{V_1}^2}{2(32.2)}$$

$$\mathrm{V_1} = 5.96\ ft/s$$

$$\mathrm{Q_1} = \mathrm{A_1 V_1} = \frac{\pi}{4}(0.5^2)5.96 = 1.188 ft^3/s \quad \textbf{(1)}$$

With the second pipe Q=1.3Q=1.54ft^3/s.

Therefore, $\mathrm{Q_1} = \mathrm{Q_2} + \mathrm{Q_3}$ and

$$\mathrm{V_1} = \frac{\mathrm{Q_1}}{\mathrm{A_1}} = \frac{1.52}{\frac{\pi}{4}(0.5^2)} = 7.84 ft/s \quad \textbf{(2)}$$

For fluid flowing from A to B through pipes 1 and 2:

$$z_a = h_{L1} + h_{L2} = f\frac{L_1}{D_1}\frac{\mathrm{V_1}^2}{2\mathrm{g}} + f\frac{L_2}{D_2}\frac{\mathrm{V_2}^2}{2\mathrm{g}} \quad \textbf{(1)}$$

$$z_a = 0.02\frac{600}{0.5}\frac{7.84^2}{2(32.2)} + 0.02\frac{500}{0.5}\frac{\mathrm{V_2}^2}{2(32.2)}$$

Hence, $\mathrm{V_2} = 2.6 ft/s$

And

$$Q_2 = A_2V_2 = \frac{\pi}{4}(0.5^2)2.60 = 0.511ft^3/s$$

Thus,

$$Q_3 = Q_1 - Q_2 = 1.03ft^3/s \quad \textbf{(1)}$$

For fluid flowing from A to B through pipes 1 and 3,

$$z_a = h_{L1} + h_{L3} = f\frac{L_1}{D_1}\frac{V_1{}^2}{2g} + f\frac{L_3}{D_3}\frac{V_3{}^2}{2g}$$

Where

$$V_3 = \frac{Q_3}{A_3} = \frac{1.03}{\frac{\pi}{4}(D_3{}^2)} = \frac{1.31}{D_3{}^2}$$

$$25ft = 0.02\frac{(600)}{0.5}\frac{7.84^2}{2(32.2)} + 0.02\frac{(500)}{D_3}\frac{\left(\frac{1.31}{D_3{}^2}\right)^2}{2(32.2)}$$

$$D_3 = 0.662ft \quad \textbf{(1)}$$

6. Solution

a. Find the tangential velocity along the surface of the dome

$$u_\theta = \frac{1}{r}\frac{\partial \Phi}{\partial \theta} = \frac{1}{r}\frac{\partial}{\partial \theta}\left(Ur\left(1+\frac{a^2}{r^2}\right)\cos\theta\right) \quad \textbf{(2)}$$

$$u_\theta = \frac{1}{r}\left(-Ur\left(1+\frac{a^2}{r^2}\right)\sin\theta\right)$$

At $r=a$, we have

$$u_\theta = -2U\sin\theta \quad \textbf{(1)}$$

b. Apply Bernoulli between any 2 points

$$p_0 + \frac{1}{2}\rho V_0{}^2 + \gamma z_0 = p_s + \frac{1}{2}\rho V_s{}^2 + \gamma z_s \quad \textbf{(3)}$$

s in on the surface.

$$p_0 + \frac{1}{2}\rho U^2 + 0 = p_s + \frac{1}{2}\rho(-2U\sin\theta)^2 + \gamma a\sin\theta$$

$$p_s = p_0 + \frac{1}{2}\rho U^2 - 2\rho(U\sin\theta)^2 - \gamma a\sin\theta \quad \textbf{(2)}$$

c.

$$\sum F_z = 0$$ **(1)**

$$F_{lift} - F_w = 0$$
$$\frac{5}{3}\rho U^2 a = m_c g$$
$$m_c = \frac{5}{3}(1.2)(30)^2(50)\left(\frac{1}{9.8}\right)$$
$$m_c = 9170 kg/m$$ **(1)**