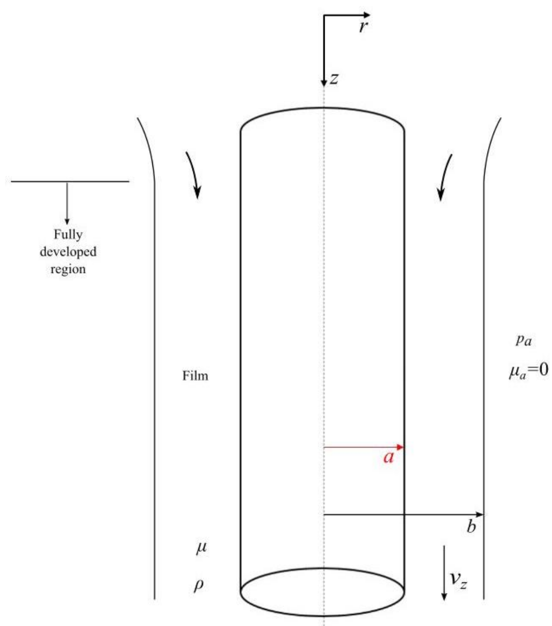
Name: ----------------- **Exam 2** **Time: 50 minutes**

ME:5160 **Fall 2022**

The exam is closed book and closed notes.

1. Consider a viscous film of liquid draining uniformly down the side of a vertical rod of radius a, as shown in the Figure. At some distance down the rod the film will approach a terminal or *fully developed* draining flow of constant outer radius b, with $v_z = v_z(r), v_\theta = v_r = 0$. Assume that the atmosphere offers no shear resistance to the film motion and that $\frac{\partial p}{\partial z} = 0$. (a) Using continuity, how that v_z is independent from z. (b) Derive a differential equation for v_z, stating and applying the proper boundary conditions and solve for the film velocity distribution. (c) Determine the value of the shear stress at r=a.



Boundary condition Hint

- At the film/air interface, the shear stress is zero ($\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$)

The equations of motion of an incompressible Newtonian fluid with constant density and viscosity in cylindrical coordinates (r, θ, z) with velocity components (v_r, v_θ, v_z):

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

r-momentum:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = \rho g_r - \frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]$$

θ-momentum:

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = \rho g_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]$$

z-momentum:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

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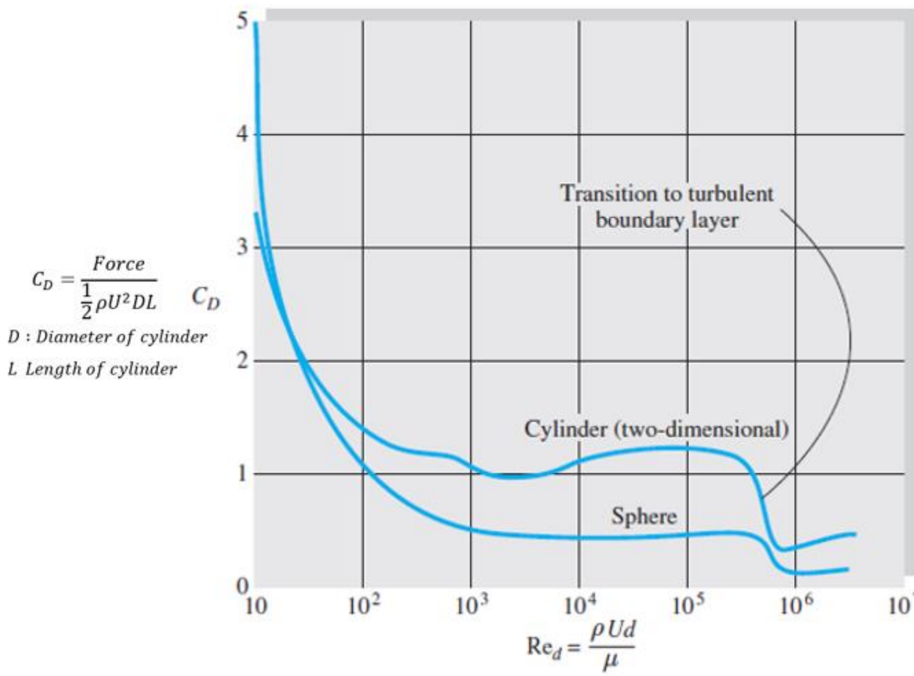
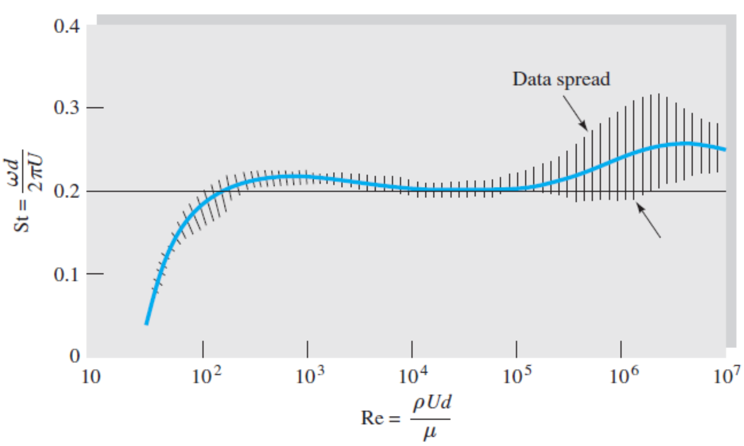
2. A ship is towing a sonar array which approximates a submerged cylinder 1 ft in diameter and 30 ft long with its axis normal to the direction of tow. If the tow speed is 12 kn (1 kn = 1.69 ft/s), (a) estimate the horsepower required to tow this cylinder. (b) What will be the frequency (Hz) of vortices shed from the cylinder? (Use below figures)

Hint: Free surface effect is negligible, so ***Fr*** *does not need to be considered*

For seawater at 20°C
ρ= *1.99 slug/ft*^{3} *and* $\mu = 2.23\times 10^{-5}$ *slug/ft·s*

550 lbf·ft/s = 1 Hp





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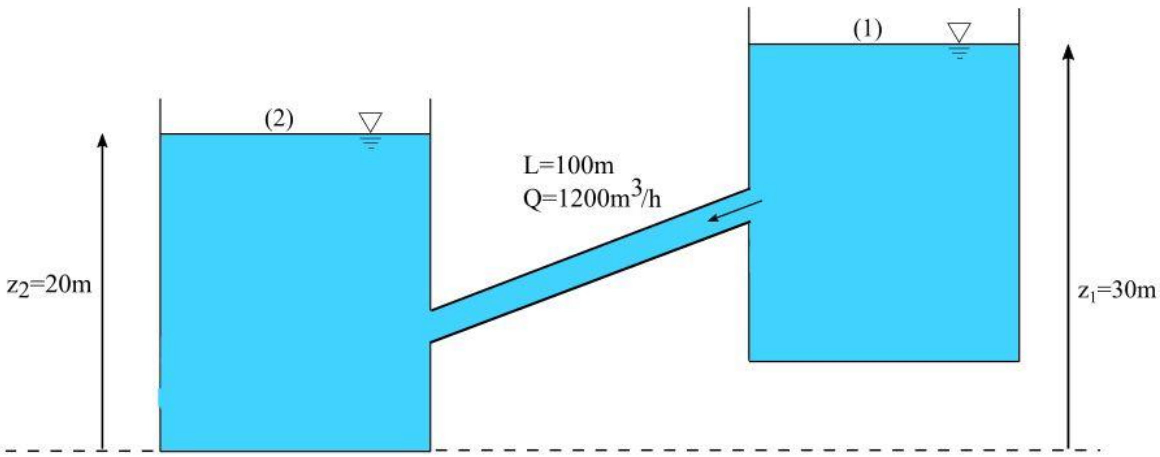
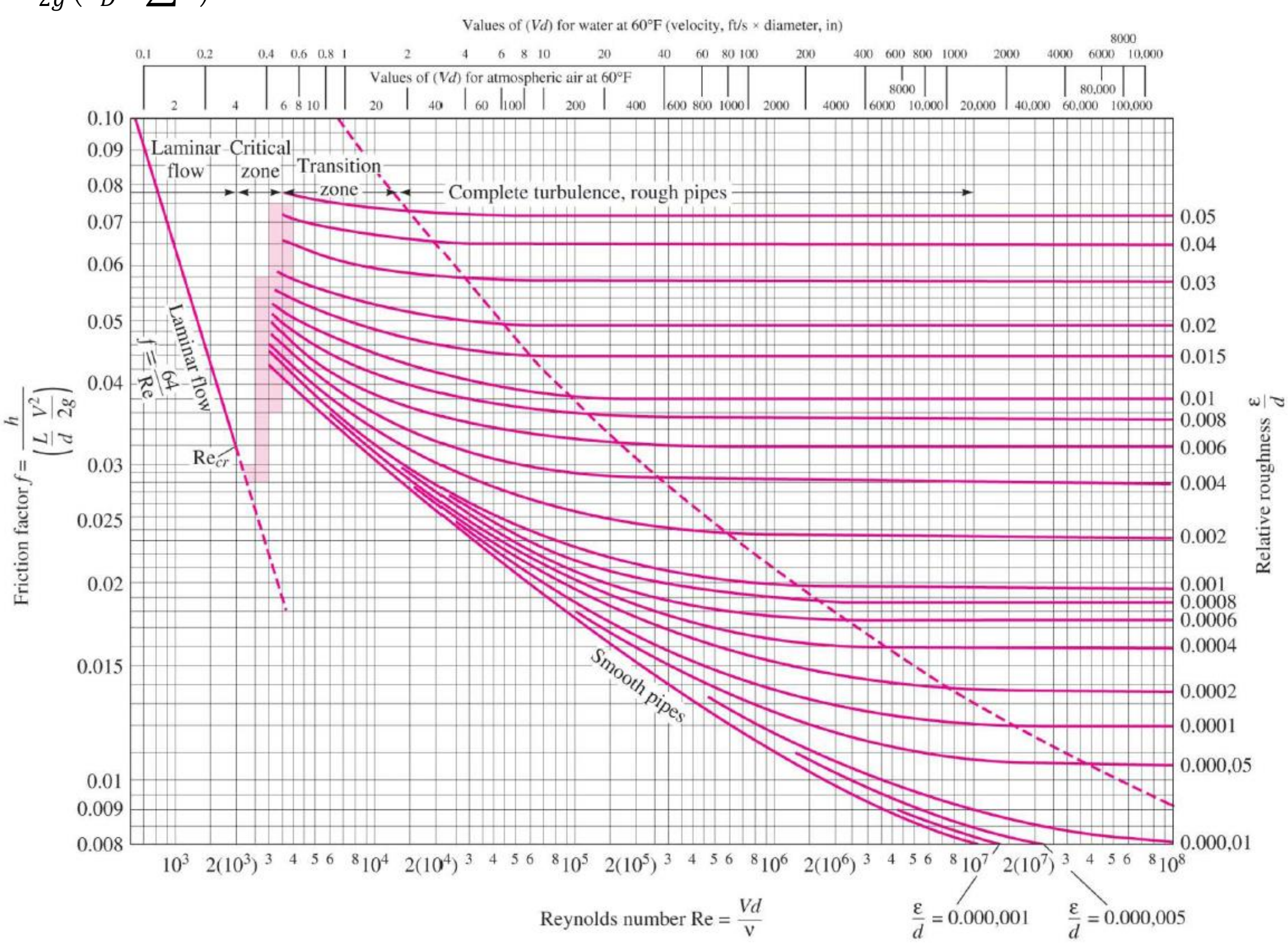
3. A petrochemical plant transfers oil at T=30 °C with a flowrate Q=1200 m^3/h through a steel pipe section with a length L=100 m. The oil density and viscosity are ρ=950kg/m^3 and μ=0.019kg/ms, respectively. Assume absolute roughness ε for steel as 0.06 mm. (a) Write the energy equation between the two tanks, and (b) determine the pipe diameter using *f*=0.03 as a first guess for the friction factor.



Energy equation $\left(\frac{P}{\rho g}+\frac{V^2}{2g}+z\right)_1=\left(\frac{P}{\rho g}+\frac{V^2}{2g}+z\right)_2+h_f+h_t$

$h_f=\frac{V^2}{2g}\left(f\frac{L}{D}+\sum K\right)$



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ME:5160 **Fall 2022**

Solution 1:

ASSUMPTIONS:
1. Steady flow ($\frac{\partial}{\partial t}$=0)
2. Incompressible flow (ρ=constant)
3. Purely axial flow (vr=vθ=0)
4. Circumferentially symmetric flow, so properties do not vary with θ ($\frac{\partial}{\partial \theta}$=0)
5. Zero pressure gradient ($\partial p/\ \partial$z =0)
6. Vertical motion (gz=g)

(a)

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

$$0(3) + 0(3) + \frac{\partial v_z}{\partial z} = 0 \qquad \textbf{(+1.5)}$$

z-momentum:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] \quad \textbf{(+0.5)}$$

$$\rho(0(1) + 0(3) + 0(3,4) + 0(continuity)) = \rho g - 0(5) + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + 0(4) + 0(continuity)\right] \textbf{(+1.5)}$$

$$\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = -\rho g$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = -\frac{\rho g}{\mu}r$$

$$r\frac{\partial v_z}{\partial r} = -\frac{\rho g}{2\mu}r^2 + c_1 \quad \textbf{(+1)}$$

$$\frac{\partial v_z}{\partial r} = -\frac{\rho g}{2\mu}r + \frac{c_1}{r}$$

$$v_z(r) = -\frac{\rho g}{4\mu}r^2 + c_1 \ln r + c_2 \quad \textbf{(+2)}$$

Boundary conditions:

$$v_z(a) = 0, \qquad \mu\frac{\partial v_z}{\partial r}(b) = 0 \quad \textbf{(+1.5)}$$

Apply BCs:

$$v_z(a) = 0 = -\frac{\rho g}{4\mu}a^2 + c_1 \ln a + c_2$$

$$\frac{\partial v_z}{\partial r} = -\frac{\rho g}{2\mu}r + \frac{c_1}{r}$$

$$\frac{\partial v_z}{\partial r}(b) = 0 = -\frac{\rho g}{2\mu}b + \frac{c_1}{b}$$

$$c_1 = \frac{\rho g}{2\mu}b^2 \quad \text{(+0.5)}$$

$$c_2 = \frac{\rho g}{4\mu}a^2 - \frac{\rho g}{2\mu}b^2 \ln a \quad \text{(+0.5)}$$

Film velocity distribution:

$$v_z(r) = -\frac{\rho g}{4\mu}r^2 + \frac{\rho g}{2\mu}b^2 \ln r + \frac{\rho g}{4\mu}a^2 - \frac{\rho g}{2\mu}b^2 \ln a$$

$$v_z(r) = \frac{\rho g}{2\mu}\left(b^2 \ln\left(\frac{r}{a}\right) - \frac{r^2}{2} + \frac{a^2}{2}\right) \quad \text{(+0.5)}$$

Shear stress:

$$\tau_w(r) = \mu\frac{\partial v_z}{\partial r} = -\frac{\rho g r}{2\mu} + \frac{\rho g b^2}{2\mu r}$$

$$\tau_w(r)|_{r=a} = \mu\frac{\partial v_z}{\partial r}\bigg|_{r=a} = -\frac{\rho g a}{2} + \frac{\rho g b^2}{2a} \quad \text{(+0.5)}$$

Solution 2:

(a)

12 knots = 20.28 ft/s

$$Re_D = \frac{\rho U D}{\mu} = \frac{1.99 \times 20.28 \times 1.0}{2.23 \times 10^{-5}}$$

$$= 1.8 \times 10^6 \quad \textbf{(+2)}$$

From the Figure(a), $C_D \approx 0.3$ **(+1)**

$$C_D = \frac{Force}{\frac{1}{2}\rho U^2 DL}$$

$$\therefore F = C_D \times \frac{1}{2}\rho U^2 DL = 0.3 \times \frac{1}{2} \times 1.99\,(20.28)^2 \times 1 \times 30$$

$$= 3683\ lbf \quad \textbf{(+2)}$$

Reguired horse power

$$\therefore P = F \times U = 3690 \times 20.28 = 74691\ \ lbf \cdot ft/s$$

$\rightarrow$ 74833.2/550 = 135.8 Hp **(+1)**

(b) From the Figure(b), $St \approx 0.24$

$$St = \frac{\omega D}{2\pi U} = 0.24$$

$$\omega = 0.24 \times \frac{2\pi U}{D} = 30.58 \quad \textbf{(+3)}$$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = 4.87 Hz \quad \textbf{(+1)}$$

Solution 3:

1. Apply energy equation between (1) and (2):

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_1=\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right)_2+h_f \qquad \textbf{(+1)}$$

$$(0+0+z)_1=(0+0+z)_2+h_f$$

$$\Delta h=f\frac{LV^2}{2gD} \qquad \textbf{(+3)}$$

2. Express the velocity as function of the flow rate:

$$V=f\frac{4Q}{\pi D^2} \qquad \textbf{(+1)}$$

Substitute in the expression for Δh:

$$\Delta h=f\frac{8LQ^2}{g\pi^2D^5}$$

Find the relation between f and D:

$$10m=f\frac{8(100m)(1200/3600)^2}{9.81(3.14)^2D^5}$$

$$f=10.88D^5$$

$$D=0.62\sqrt[5]{f} \qquad \textbf{(+1)}$$

Also, the relative roughness depends on D (unknown):

$$\frac{\varepsilon}{D}=\frac{0.06mm}{D} \qquad \textbf{(+1)}$$

Make a first guess for $f=0.03$:

$$D=0.307m$$

$$\frac{\varepsilon}{D}=\frac{0.06\cdot 10^{-3}m}{0.307m}=1.95\cdot 10^{-4}$$

Express Reynolds number using the flowrate:

$$Re_D=\frac{\rho UD}{\mu}=\frac{4\rho QD}{\mu\pi D^2}=\frac{4(950)(\frac{1200}{3600})}{0.019(3.14)D}=\frac{21231}{D}$$

Plug in the value of D obtained from the first guess:

$Re_D=\frac{21231}{0.307m}=69156$ **(+1)**

Using $\frac{\varepsilon}{D}$ and the new Re_D, enter the Moody's diagram to get a new guess for f.

$$f \sim 0.02$$

$$D = 0.284m$$

$$\frac{\varepsilon}{D} = \frac{0.06 \cdot 10^{-3} m}{0.284m} = 2.11 \cdot 10^{-4}$$

$$Re_D = \frac{21231}{0.284m} = 74757$$ **(+1)**

Using $\frac{\varepsilon}{D}$ and the new Re_D, enter the Moody's diagram to get a new guess for f.

$$f \sim 0.02$$ **(+1)**

The difference between Re_D after two successive iterations is sufficiently small, and the process is considered converged.