Name: ------------------ **Exam 1** **Time: 50 minutes**

ME:5160 **Fall 2022**

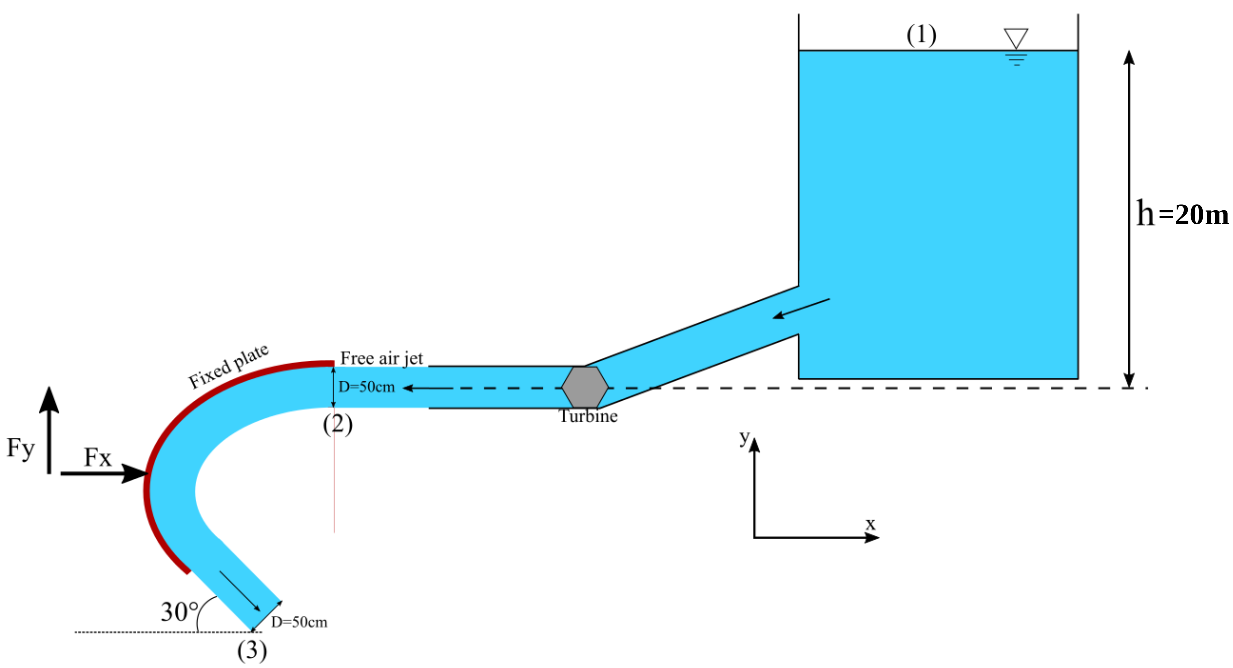
The exam is closed book and closed notes.

1. A reservoir of water (ρ = 998 kg/m^3) discharges through a conduct (D=50cm), where a turbine is placed to generate hydroelectric energy. System friction losses between section (1) and (2) are $h_f = K{V_2}^2/(2g)$. (a) Find an expression for h_t as a function of V_2. (b) Suppose that $h_t = 17.9m$ and $K = 3.5$, what is the correspondent value of V_2?

The exhaust water from the turbine is guided using a fixed plate, as shown in the Figure. (c) Determine the horizontal and vertical component of the force that the water jet exerts on the plate.

Hint:

Energy equation: $\left(\frac{p}{\rho g} + \frac{\alpha V^2}{2g} + z\right)_1 = \left(\frac{p}{\rho g} + \frac{\alpha V^2}{2g} + z\right)_2 + h_f + h_t$



---

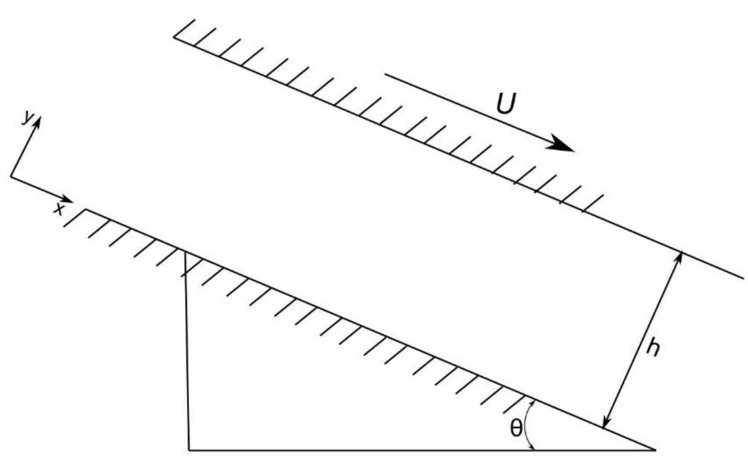
2. A viscous, incompressible fluid flows between two infinite, parallel plates distance h apart, as shown in the Figure. The upper plate moves at constant velocity U while the lower plate is at rest. Use the given coordinate system and assume that the flow is laminar, steady, fully developed ($\frac{\partial u}{\partial x} = 0$), and 2D. Assume pressure gradient is constant and gravity is not negligible. (a) Simplify the continuity equation and show that $v = 0$. Find (b) velocity distribution $u(y)$ and (c) wall shear stress in terms of $\mu, \rho, g, \frac{dp}{dx},\ h$ and θ.

Incompressible continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

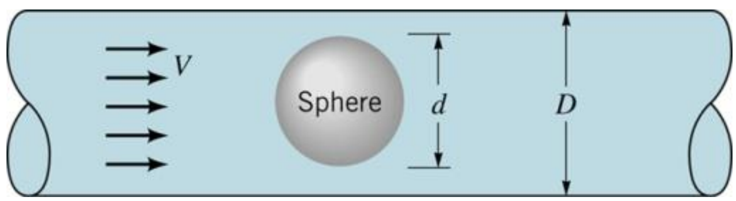
Incompressible Navier-Stokes Equations in Cartesian Coordinates:

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(u\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$
$$\rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\left(v\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$
$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\left(w\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$



---

3. The drag F, on a sphere located in a pipe through which a fluid is flowing is to be determined experimental. Assume that the drag is a function of sphere diameter, d, the pipe diameter, D, the fluid velocity, V, and the fluid density, ρ. (a) What dimensionless parameters would you use for this problem (don't use D as a repeating variable) ? (b) Some experiments using water indicated for d_m=0.2 in, D_m=0.5 in, and V_m=2 ft/s, the drag is $1.5\text{x}10^{-3}$ lb. Estimate the drag on a sphere located in a 2 ft dimeter pipe (D) trough which water is flowing with a velocity of 6 ft/s. The sphere diameter is such that geometric similarity is maintained.



Quantity	Symbol	Dimensions $MLT\Theta$	Dimensions $FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω, Ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Thermal expansion coefficient	β	Θ^{-1}	Θ^{-1}

Solution:

1. (a) Apply energy equation between (1) and (2):

$$\left(\frac{p}{\rho g}+\frac{\alpha V^2}{2g}+z\right)_1=\left(\frac{p}{\rho g}+\frac{\alpha V^2}{2g}+z\right)_2+h_f+h_t \quad \textbf{(+2)}$$

$$(0+0+z)_1=\left(0+\frac{V^2}{2g}+z\right)_2+h_f+h_t$$

$$h_t=h-\frac{{V_2}^2}{2g}-K\frac{{V_2}^2}{2g}=h-\frac{{V_2}^2}{2g}(1+K) \quad \textbf{(+2)}$$

(b) Substitute K=3.5, $h_t=17.9m\rightarrow V_2=3m/s.$ **(+1)**

(c) Using continuity: $A_2V_2=A_3V_3$, but $D_2=D_2\rightarrow V_2=V_3$ **(+1)**

$$\sum \boldsymbol{F}=\frac{d}{dt}(\int_{CV}\boldsymbol{V}\rho dV)+\int_{CS}\boldsymbol{V}\rho(\boldsymbol{V_r}\cdot\boldsymbol{n})dA$$

Assuming fixed control volume, $\boldsymbol{V_r}=\boldsymbol{V}$**,** constant density and zero acceleration

$$\sum \boldsymbol{F}=\int_{CS}\boldsymbol{V}\rho(\boldsymbol{V}\cdot\boldsymbol{n})dA$$

Assuming V and ρ uniform over $A_2$and A_3 and projecting in the x-direction:

$$\sum F_x=-F_x=\int_{CS}u\rho(\boldsymbol{V}\cdot\boldsymbol{n})dA=\ -V_2\rho(\boldsymbol{V_2}\cdot\boldsymbol{n_2})A_2+V_3\cos(30°)\,\rho(\boldsymbol{V_3}\cdot\boldsymbol{n_3})A_3$$

$$-F_x=\rho{V_2}^2A_2+\rho{V_3}^2\cos(30°)\,A_3=\rho{V_2}^2A_2(1+\cos(30°)) \quad \textbf{(+1)}$$

$$F_x=-\frac{998\cdot 3^2\cdot\pi\cdot(0.5m)^2}{4}(1.866)=-3289N \quad \textbf{(+1)}$$

Projecting in the y-direction:

$$\sum F_y=-F_y=\int_{CS}u\rho(\boldsymbol{V}\cdot\boldsymbol{n})dA=\ -V_3\sin(30°)\,\rho(\boldsymbol{V_3}\cdot\boldsymbol{n_3})A_3$$

$$-F_y=-\rho{V_3}^2\sin(30°)\,A_3 \quad \textbf{(+1)}$$

$$F_y=\frac{998\cdot 3^2\cdot\pi\cdot(0.5m)^2}{4}(0.5)=881N \quad \textbf{(+1)}$$

---

2. Assumptions:

1. Steady flow ($\frac{\partial}{\partial t} = 0$)
2. 2D flow (w=0)
3. Incompressible flow (ρ =constant)
4. Fully developed flow ($\frac{\partial u}{\partial x} = 0$)
5. Pressure gradient is constant (∇P=constant)

(a) Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \textbf{(+1)}$$

$$\underset{(4)}{0} + \frac{\partial v}{\partial y} + \underset{(2)}{0} = 0 \quad \textbf{(+1)}$$

$$v(y) = constant$$

$v(0)$ =0 from BCs **(+1)**

Therefore, v=0 everywhere. **(+1)**

(b) x-momentum

$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(u\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) \quad \textbf{(+1)}$$

$$\rho g \sin\theta - \frac{\partial p}{\partial x} + \mu\left(\underset{(4)}{0} + \frac{\partial^2 u}{\partial y^2} + \underset{(2)}{0}\right) = \rho(\underset{(1)}{0} + \underset{(4)}{0} + \underset{(a)}{0} + \underset{(2)}{0}) \quad \textbf{(+1)}$$

Integrate twice:

$$u(y) = \left(\frac{\partial p}{\partial x} - \rho g \sin\theta\right)\frac{y^2}{2\mu} + c_1 y + c_2 \quad \textbf{(+1)}$$

Boundary conditions:

$$\text{at } y = 0: u(y) = 0 \rightarrow c_2 = 0 \quad \textbf{(+0.5)}$$

---

at y=h: u(y)=U $\rightarrow c_1 = \frac{U}{h} - \frac{h}{2\mu}\left(\frac{\partial p}{\partial x} - \rho g \sin\theta\right)$ **(+0.5)**

Replace and find:

$$u(y) = \left(\frac{\partial p}{\partial x} - \rho g \sin\theta\right)\frac{y^2}{2\mu} + \left(\frac{U}{h} - \frac{h}{2\mu}\left(\frac{\partial p}{\partial x} - \rho g \sin\theta\right)\right)y \quad \textbf{(+1)}$$

(c) Wall shear stress:

$$\tau_w = \mu\frac{\partial u}{\partial y}\bigg|_{y=0} = \left(\frac{\partial p}{\partial x} - \rho g \sin\theta\right)y + \left(\frac{U}{h} - \frac{h}{2\mu}\left(\frac{\partial p}{\partial x} - \rho g \sin\theta\right)\right) = \frac{U}{h} - \frac{h}{2\mu}\left(\frac{\partial p}{\partial x} - \rho g \sin\theta\right) \quad \textbf{(+1)}$$

Solution 3

a) Pi theorem

$$F = f(d, D, V, \rho)$$

$$F \doteq MLT^{-2}, d \doteq L, D \doteq L, V \doteq LT^{-1}, \rho \doteq ML^{-1}T^{-3}$$

$$k - r = 5 - 3 = 2 \quad \boxed{+1}$$

Repeating variables d, V, ρ

$$\Pi_1 = d^a V^b \rho^c F = (L)^a (LT^{-1})^b (ML^{-3})^c (MLT^{-2}) = M^0 L^0 T^0$$

$$M: c + 1 = 0$$

$$L: a + b - 3c + 1 = 0$$

$$T: -b - 2 = 0$$

$$\therefore a = -2, b = -2, c = -1$$

$$\Pi_1 = \frac{F}{\rho V^2 d^2} \quad \boxed{+3}$$

$$\Pi_2 = d^a V^b \rho^c D = (L)^a (LT^{-1})^b (ML^{-3})^c (L) = M^0 L^0 T^0$$

$$M: c = 0$$

$$L: a + b - c + 1 = 0$$

$$T: -b = 0$$

$$\therefore a = -1, b = 0, c = 0$$

$$\Pi_2 = \frac{D}{d}$$ +3

$$\therefore \frac{F}{\rho V^2 d^2} = \phi\left(\frac{D}{d}\right)$$

b) Similarity

$$\frac{D_m}{d_m} = \frac{D}{d}$$

$$d = D\frac{d_m}{D_m} = 2\,ft\frac{0.2\,in}{0.5\,in} = 0.8\,ft$$ +1

$$\frac{F}{\rho V^2 d^2} = \frac{F_m}{\rho_m V_m^2 d_m^2}$$

$$F = \frac{\rho}{\rho_m}\left(\frac{V}{V_m}\right)^2\left(\frac{d}{d_m}\right)^2 F_m = (1)\left(\frac{6\text{ ft/s}}{2\text{ ft/s}}\right)^2\left(\frac{0.8\,ft}{0.2/12\,ft}\right)^2(1.5\times10^{-3}\text{ lb}) = \mathbf{31.1\ lb}$$ +2