The exam is closed book and closed notes.

The horizontal lawn sprinkler in the Figure below has a total inlet water flow rate of 4.0 gal/min (0.008912 ft³/s) introduced vertically (in z direction) through the center. Water density is \( \rho = 1.94 \) slug/ft³ and friction is negligible. Estimate (a) the retarding torque required to keep the arms from rotating and (b) the rotation rate (rev/min) if there is no retarding torque.

Angular Momentum Equation:

\[
\sum M_p = \frac{d}{dt} \int_C (r \times \mathbf{v}) \rho d\mathbf{v} + \int_S (r \times \mathbf{v}) \rho \mathbf{v} \cdot \mathbf{n} \ dA
\]
Solution:

KNOWN: $\rho$, $Q$, $D$, $R$

FIND: $T$, $\omega$

ASSUMPTIONS: non-deforming CV, steady incompressible flow, uniform inlet/outlet

ANALYSIS:

Continuity:

$$\dot{m}_{in} = \rho Q = (1.94)(0.008912) = 0.017289 \frac{lb}{s}$$

For each arm is:

$$\dot{m}_{out} = \frac{\dot{m}_{in}}{2} = 0.008645 \frac{lb}{s}$$

$$V_{out} = \frac{Q}{\pi \frac{D^2}{4}} = \frac{(0.008912 \frac{ft^3}{s})/2}{(3.14)(0.25 \frac{ft}{12})^2} = 13.1 \frac{ft}{s}$$

Angular momentum:

$$\sum M_o = \frac{d}{dt} \int_{CV} (r \times V) \rho dV + \int_{CS} (r \times V) \rho \nu \cdot \vec{n} dA$$

Reduce for steady flow and non-deforming CV with 1D (uniform) inlets/outlets:

$$\sum M_o = \sum (r \times V)_{out} \dot{m}_{out} - (r \times V)_{in} \dot{m}_{in}$$

(a) Arms not rotating:

$$T_0 = \left[ R_i \times V_{out} j \right] \dot{m}_{out} + \left[ (\mathbf{R})_i \times (-V_{out}) j \right] \dot{m}_{out} - \left[ 0 \mathbf{i} \times V_{in} k \right] \dot{m}_{in}$$

$$T_0 = 2RV_{out} \dot{m}_{out} k = 2 \left( \frac{6}{12} \right) (13.1)(0.008645)K = 0.1132 \frac{ft.lbf}{s}$$
(b) No retarding torque:

\[ 0 = \left[ R \hat{i} \times (V_{out} - R\omega) \hat{j} \right] m_{out} + \left[ (-R) \hat{i} \times (-V_{out} + R\omega) \hat{j} \right] m_{out} - \left[ 0 \hat{i} \times V_{in} k \right] m_{out} \]  

\[ 0 = 2R(V_{out} - R\omega) m_{out} \]  

\[ (V_{out} - R\omega) = 0 \]  

\[ \omega = \frac{V_{out}}{R} = \frac{(13.1)}{(6/12)} = 26.2 \text{ rad/s} = \frac{(26.2)(60)}{(2\pi)} = 250.19 \text{ rev/min} \]