The exam is closed book and closed notes.

A necked-down section in a pipe flow, called a venturi, develops a low throat pressure which can aspirate fluid upward from a reservoir, as in the Figure below. Using Bernoulli’s equation with no losses, derive an expression for the velocity $V_1$ (in terms of $h$, $D_1$, $D_2$, and $g$) which is just sufficient to bring reservoir fluid into the throat.

Hint: the fluid in the tube is stationary, such that: $p_a - p_1 = \rho gh$

Bernoulli’s Equation:

$$\frac{P_1 + \frac{V_1^2}{2g}}{\rho g} + z_1 = \frac{P_2 + \frac{V_2^2}{2g}}{\rho g} + z_2$$
Solution:

KNOWN: \( h, D_1, D_2 \)

FIND: \( V_1 \)

ASSUMPTIONS: frictionless, steady, incompressible flow

ANALYSIS:

Continuity:

\[
V_1 A_1 = V_2 A_2 \\
V_1 \left( \frac{\pi}{4} D_1^2 \right) = V_2 \left( \frac{\pi}{4} D_2^2 \right) \\
V_2 = V_1 \left( \frac{D_1}{D_2} \right)^2
\]

Bernoulli:

\[
\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2
\]

Substitute:

\[
p_1 = p_a - \rho gh; \ z_1 = z_2; \ p_2 = p_a; \ V_2 = V_1 \left( \frac{D_1}{D_2} \right)^2
\]

Solve for \( V_1 \):

\[
V_1 = \sqrt{\frac{2gh}{1 - \left( \frac{D_1}{D_2} \right)^4}}
\]