In regions far from the entrance, fluid flow through a circular pipe is one-dimensional and the velocity profile for laminar flow is given by

\[ u(r) = u_{max} \left( 1 - \frac{r^2}{R^2} \right) \]

where \( R \) is the radius of the pipe, \( r \) is the radial distance from the center of the pipe, and \( u_{max} \) is the maximum flow velocity, which occurs at the center. Obtain (a) a relation for the drag force applied by the fluid on a section of the pipe of length \( L \) and (b) the value of the drag force for water flow at 20°C with \( R = 0.095 \) m, \( L = 17 \) m, \( u_{max} = 2.5 \) m/s, and \( \mu = 0.0010 \) kg/m \( \cdot \) s.

**Equation:** \( \tau = \mu \frac{du}{dy} \)
Solution

KNOWN: \( u(r) \)

FIND: \( F_D \)

ASSUMPTIONS: The flow is one-dimensional; the fluid is Newtonian

ANALYSIS:

(a)

The shear stress at the pipe surface can be expressed as:

\[
\tau_w = \mu \frac{du}{dy}_{\text{wall}} = -\mu \left. \frac{du}{dr} \right|_{r=R} \quad +1
\]

\[
\tau_w = -\mu u_{\text{max}} \frac{d}{dr} \left( 1 - \frac{r^2}{R^2} \right) \quad +2 = -\mu u_{\text{max}} \left. \frac{-2r}{R^2} \right|_{r=R} = \frac{2\mu u_{\text{max}}}{R} \quad +1
\]

The friction drag force exerted by the fluid on the inner surface of the pipe becomes:

\[
F_D = \tau_w A = \left( \frac{2\mu u_{\text{max}}}{R} \right) (2\pi RL) = 4\pi \mu L u_{\text{max}} \quad +1
\]

(b)

Substituting,

\[
F_D = 4\pi \mu L u_{\text{max}} = 4\pi (0.0010 \text{ kg/m} \cdot \text{s})(17 \text{ m})(2.5 \text{ m/s}) = 0.534 \text{ N} \quad +0.5
\]