The exam is closed book and closed notes.

The pressure rise, $\Delta p$, across a centrifugal pump in Figure can be expressed as $\Delta p = f(D, \omega, \rho, Q)$, where $D$ is the impeller diameter, $\omega$ the angular velocity of the impeller (unit for $\omega$ is $T^{-1}$), $\rho$ the fluid density, and $Q$ the volume rate of flow through the pump. (a) By using dimensional analysis find the pi terms. (b) A model pump having a diameter of 8 in. is tested in a laboratory using water ($\rho = 998 \text{ kg/m}^3$). When operated at an angular velocity of $40\pi \text{ rad/s}$ the model pressure rise as a function of $Q$ is shown in Figure. Use this curve to predict the pressure rise across a geometrically similar pump (prototype) for a prototype flowrate of 6 ft$^3$/s. The prototype has a diameter of 12 in. and operates at an angular velocity of $60\pi \text{ rad/s}$. The prototype fluid is also water.

![Centrifugal pump diagram](image1)

![Pressure rise vs. flowrate](image2)
Solution:

Format (+3)

(a)

<table>
<thead>
<tr>
<th>$\Delta p$</th>
<th>$D$</th>
<th>$\omega$</th>
<th>$\rho$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ML^{-1}T^{-2}$</td>
<td>$L$</td>
<td>$T^{-1}$</td>
<td>$ML^{-3}$</td>
<td>$L^3T^{-1}$</td>
</tr>
</tbody>
</table>

$k - r = 5 - 3 = 2 \quad (+1)$

Pi terms

$$\Pi_1 = D^a \omega^b \rho^c \Delta p \doteq (L)^a (T^{-1})^b (ML^{-3})^c (ML^{-1}T^{-2}) \doteq L^0 T^0 M^0$$

Thus, $a = -2, b = -2, c = -1.$

$$\therefore \Pi_1 = \frac{\Delta p}{\rho \omega^2 D^2} \quad (+1.5)$$

$$\Pi_2 = D^a \omega^b \rho^c Q \doteq (L)^a (T^{-1})^b (ML^{-3})^c (L^3T^{-1}) \doteq L^0 T^0 M^0$$

Thus, $a = -3, b = -1, c = 0.$

$$\therefore \Pi_2 = \frac{Q}{\omega D^3} \quad (+1.5)$$

(b) Similarity

$$Q \omega D^3 = Q_m \omega_m D_m^3$$

$$Q_m = \left(\frac{\omega_m}{\omega}\right) \left(\frac{D_m}{D}\right)^3 Q$$

$$Q_m = \left(\frac{40\pi}{60\pi}\right) \left(\frac{8}{12}\right)^3 (6) = 1.19 \text{ ft}^3/\text{s} \quad (+1.5)$$

Prediction equation

$$\frac{\Delta p}{\rho \omega^2 D^2} = \frac{\Delta p_m}{\rho_m \omega_m^2 D_m^2}$$

$$\Delta p = \left(\frac{\rho}{\rho_m}\right) \left(\frac{\omega}{\omega_m}\right)^2 \left(\frac{D}{D_m}\right)^2 \Delta p_m$$

From Fig., $\Delta p_m = 5.5 \text{ psi} \text{ at } Q_m = 1.19 \text{ ft}^3/\text{s}.$

$$\therefore \Delta p = (1) \left(\frac{60\pi}{40\pi}\right)^2 \left(\frac{12}{8}\right)^2 (5.5) = 27.8 \text{ psi} \quad (+1.5)$$