Problem 1:
A weightless, homogeneous, isotropic, infinite plate with a crack of length \(2a\) is subjected to a far-field applied stress \(\sigma^\infty\), as shown in the figure below. With the crack-tip A as the origin of the coordinate system, the linear-elastic crack-tip stress field (i.e., when \(r \to 0\)) can be obtained as

\[
\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \theta \left[ 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right], \quad 0 \leq |\theta| \leq \pi
\]

\[
\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \theta \left[ 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right], \quad 0 \leq |\theta| \leq \pi
\]

\[
\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{3\theta}{2}
\]

where \(K_I = \sigma^\infty \sqrt{\pi a}\) is the mode-I stress intensity factor. For the plate material with uniaxial yield strength \(S_y\), let \(r_y(\theta)\) denote the plastic zone size as a function of \(\theta\). Using the linear-elastic stress field above and assuming no redistribution of stresses due to crack-tip plasticity,

(a) Determine the plastic zone size \(r_y(\theta)\) in terms of \(K_I\) and \(S_y\) for (1) plane stress and (2) plane strain conditions.

(b) From the results of (a) and \(\nu = 1/3\), sketch the plastic zone boundary in terms of \(r_y(\theta) \cos \theta \left[ K_I^2 / (\pi S_y^2) \right]\) vs. \(r_y(\theta) \sin \theta \left[ K_I^2 / (\pi S_y^2) \right]\) plot at crack tip A for \(0 \leq |\theta| \leq \pi\). Which stress state gives conservative prediction of the plastic zone size? Comments.

Assume von Mises yield criterion for your analysis.

Problem 2:
Solve Problem 1 assuming Tresca yield criterion. Compare results of Problems 1 and 2. Comments.
**Problem 3:**
Consider an uncracked, linear-elastic \((E, \nu)\) specimen in plane stress, which is subject to uniaxial tensile stress of magnitude \(\sigma\), as shown in the figure below. Let \(\Gamma\) denote a closed circular contour of radius \(R\) indicated in the figure. Confirm that the \(J\)-integral for this uncracked body is

\[
J = \int_{\Gamma} \left[ Wdy - T \cdot \frac{\partial u}{\partial x} d\Gamma \right] = 0 .
\]

\[
T = \sigma n
\]

\[
\sigma = \begin{bmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_y & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_z
\end{bmatrix} ; \quad n = \begin{bmatrix}
n_x \\
n_y \\
n_z
\end{bmatrix}
\]

\[
u = \begin{bmatrix}
u_x \\
u_y \\
u_z
\end{bmatrix}
\]